A MULTILAYER MODEL FOR THE RECONSTRUCTION OF SEA BOTTOM ACCOUSTIC PARAMETERS†

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Abstract

A new algorithm for the reconstruction of the two basic acoustic parameters of the sea floor (density and sound speed) is presented in this communication. A multilayer model for the ocean bottom is assumed and the data consist of reflection coefficient measurements (magnitude and phase) in the sea water for obliquely incident acoustic waves. Since the reconstruction of the two unknown parameters mentioned above is saught, an empirical formula between them must be used in the reconstruction algorithm. The algorithm is based on the use of a recursive formula for the local reflection coefficient above each interface of the sea bottom model. Finally, accurate reconstruction results, based on synthetically obtained reflection data, are obtained.

1. INTRODUCTION

During the last decades a continuous interest for the development of acoustic reconstruction algorithms in one dimension (inverse scattering for stratified media) exists [1-7]. In these publications, either both the density profile *and* sound speed profile of acoustic inhomogeneous (stratified) media are sought from reflection coefficient data [1,3] , or the sound speed profile alone is sought [2,4,6,7] , in cases that the density is assumed known or constant (it is obvious that the former problem is more complicated than the latter one, so that in general more data are needed for its implementation). In those references, the interest is either for the reconstruction of general acoustic media [3,5-7], or for the oceanic environment, in particular [1,2,4]. For the latter case, extensive discussion on acoustic propagation and other related topics can also be found in the literature [8,9]. Furthermore, for the same environment of interest, reconstruction algorithms have been published for the inversion of the

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sound speed profile of sea water [4], the acoustic parameters of the ocean bottom [1], or for the sound speed of both a stratified ocean *and* bottom [2]. Here we concentrate in the inversion of the sea bottom acoustic parameters (density and sound speed), and a multilayer model is used for the approximation of the ocean floor structure.

Appropriate multilayer models for a suitable representation of the ocean floor can be found in the literature [10]. In this communication, a three - layer model will be used, which is most appropriate for rather deep water applications [10]. The layers are assumed to be liquid [10] (it is sometimes customary [10] to assume that the third (bottom) layer is solid, so that it exhibits shear properties. This case will be examined by the authors in the near future). More layers for the representation of the sea bottom [10] can also be accomodated by our proposed method of inversion, as it will be realized in Section II (not shown in this Communication). Baseband acoustic data will be used in this manuscript with appropriate bandwidth (up to about 1 kHz or 10 kHz, depending on the required reconstruction accuracy, as it will be explained in Section III). For the case of bandpass data the reader can consult Ref. [13], where the corresponding electromagnetic problem is considered. Assuming a time - harmonic acoustic source, both the magnitude *and* the phase of the reflection coefficient must be measured in the above bands, with appropriate sampling. Then, a standard Fast Fourier Transform (FFT) algorithm [11,12] must be used, according to the proposed inversion technique, in order to calculate the corresponding time - domain reflection.

The proposed reconstruction algorithm is based on a recursive use of the local reflection coefficient above each interface of the multilayer model. A variant of it has recently been introduced in the literature [13] for the inversion of the permittivity (å) of multilayer dielectric models from *electromagnetic* (EM) reflection coefficient data [note that in the formulation of that EM inverse scattering problem the permeability (ì) of the entire multilayer structure was assumed constant and equal to the permeability in vacuum]. Since in realistic problems of ocean bottom acoustic parameter estimation *both* the density *and* the sound speed profile are in general unknown, our model uses an empirical formula [10] relating these two unknown parameters (see Section II). Then, according to the proposed method, the unknown sea bottom parameters of each layer (density, sound speed), as well as the

corresponding layer height, are sequentially calculated, beginning with the first (upper) layer and moving downward (see Section II).

The formulation of the proposed inverse scattering algorithm is now in order.

2. FORMULATION OF THE ACOUSTIC SEA BOTTOM INVERSE SCATTERING PROBLEM - DESCRIPTION OF THE RECONSTRUCTION ALGORITHM

2.1. Method mathematics

The multilayer model of the sea floor is shown in Fig. 1, where $\grave{e}_{\ddot{\imath}}$ is the angle of incidence and $\tilde{n}_n$, c_n , h_n (n=1 to N) are the unknown parameters to be determined. Following Hashish [13] and assuming time - harmonic acoustic source, the total reflection coefficient due to the obliquely incident acoustic wave on the interface between layers of order (n-1) and n, is defined by

$$R_{n-1,n} = \frac{A^r}{A^i} \qquad (n = 1 \text{ to } N) \qquad (1)$$

where A^r is the (complex) amplitude of the reflected acoustic wave and A^i the corresponding amplitude of the incident wave. It can be shown that $R_{n-1,n}$ can be expressed as [9,10,13,14] :

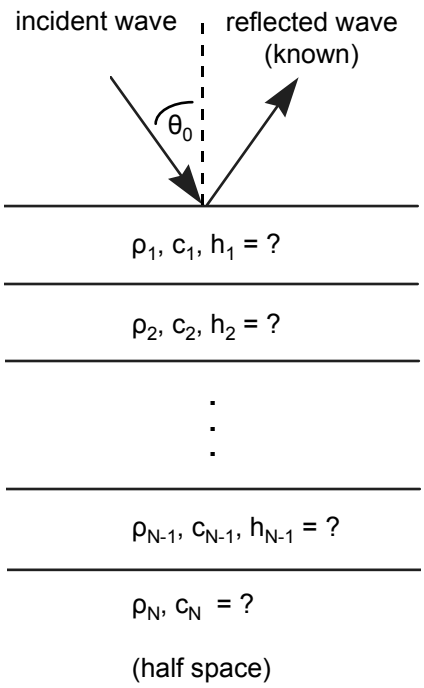


Figure 1. Multilayer acoustic model for the sea bottom representation considered in this work.

$$R_{n-1,n} = \frac{r_{n-1,n} + R_{n,n+1}\exp(-2j\hat{a}_n \cos\grave{e}_n h_n)}{1 + r_{n-1,n} R_{n,n+1}\exp(-2j\hat{a}_n \cos\grave{e}_n h_n)} \qquad (n = 1 \text{ to } N) \quad (2)$$

(recursive formula) where

$$r_{n-1,n} = \frac{Z_n \cos\grave{e}_{n-1} - Z_{n-1}\cos\grave{e}_n}{Z_n \cos\grave{e}_{n-1} + Z_{n-1}\cos\grave{e}_n} \qquad (n = 1 \text{ to } N) \quad (3)$$

is the reflection coefficient corresponding to the (n-1,n) interface [usually called "Fresnel reflection coefficient" in Electromagnetics or Optics] and $Z_n = \tilde{n}_n c_n$ is the acoustic impedance of the n-th layer. Furthermore, $\grave{e}_{n-1}$ is the angle of incidence at the interface (n-1,n), $\grave{e}_n$ is the angle of transmission at the same interface and $\hat{a}_n = \grave{u}/c_n$ ($f = \grave{u}/2\eth$ is the operating frequency). Finally, note that for the last interface it is $R_{N-1,N} = r_{N-1,N}$.

Concentrating now to the reflection coefficient in the sea water (n=1) we may obtain (see [13] for the details of the mathematical derivation) :

$$R_{0,1}(\grave{u}) = r_{0,1} + (1 - r_{0,1}^2)\, r_{1,2}\exp(-j\grave{u}T_1) + \ldots = r_{0,1} + t_{0,1}\, r_{1,2}\, t_{1,0}\exp(-j\grave{u}T_1) + \ldots \quad (4)$$

where $t_{i,j}$ represents the transmission coefficient from layer i to layer j [9, ch. 12], $T_1 = 2h_1\cos\grave{e}_1/c_1$ and missing terms correspond to reflections occuring later in time. In other words, the first two terms at the right - hand - side of Eqn. (4) correspond to the two primary reflections, i.e. direct reflections from the interfaces (0,1) and (1,2) respectively, as also seen by inspection of the corresponding time - domain equation, Eqn. (6), below. Indeed, the corresponding to $R_{0,1}(\grave{u})$ time - domain reflection is defined by

$$\bar{R}_{0,1}(t) = \frac{1}{2\eth}\int_{-\infty}^{\infty} R_{0,1}(\grave{u})\exp(j\grave{u}t)\, d\grave{u} \quad (5)$$

Following [13] we can calculate the Fourier transform of Eqn. (4) to obtain :

$$\bar{R}_{0,1}(t) = r_{0,1}\,\ddot{a}(t) + (1 - r_{0,1}^2)\, r_{1,2}\,\ddot{a}(t - T_1) + \ldots = r_{0,1}\,\ddot{a}(t) + t_{0,1}\, r_{1,2}\, t_{1,0}\,\ddot{a}(t - T_1) + \ldots \quad (6)$$

For the sea bottom acoustic reconstruction problem under consideration, Eqn. (3) takes the form, for the case of a liquid - liquid interface [9, ch. 4], [10] :

$$r_{n-1,n} = \frac{\tilde{\rho}_n c_n \cos\theta_{n-1} - \tilde{\rho}_{n-1} c_{n-1} \cos\theta_n}{\tilde{\rho}_n c_n \cos\theta_{n-1} + \tilde{\rho}_{n-1} c_{n-1} \cos\theta_n} \qquad (n = 1 \text{ to } N) \qquad (7)$$

while Snell's law, for the same kind of interface, takes the form [9, ch. 4], [10] :

$$\frac{\sin\theta_{n-1}}{c_{n-1}} = \frac{\sin\theta_n}{c_n} \qquad (n = 1 \text{ to } N) \qquad (8)$$

Finally, the empirical formula needed for the reconstruction algorithm, relating the sound speed c_n at the n-th layer with the density $\tilde{\rho}_n$ at the same layer, is given by [10] :

$$c_n = f(\tilde{\rho}_n) = 2{,}390 - 1.358\,\tilde{\rho}_n + 5.246 \cdot 10^{-4}\,\tilde{\rho}_n^2 \qquad (9)$$

2.2. Reconstruction algorithm

(a) Determination of the parameters of the first layer.

In the reconstruction problem under consideration, the quantities $r_{0,1}$ and T_1 can be directly calculated from the measured data $R_{0,1}(\omega)$ by taking its Fourier transform, Eqn. (4). In particular, regarding the reconstruction of the parameters of the first layer, Eqn. (7) yields :

$$r_{0,1} = \frac{\tilde{\rho}_1 c_1 \cos\theta_0 - \tilde{\rho}_0 c_0 \cos\theta_1}{\tilde{\rho}_1 c_1 \cos\theta_0 + \tilde{\rho}_0 c_0 \cos\theta_1} \qquad (10)$$

Furthermore, Snell's law, Eqn. (8), for n=1 yields :

$$\frac{\sin\theta_0}{c_0} = \frac{\sin\theta_1}{c_1} \qquad (11)$$

while empirical relation (9) yields, for n=1 :

$$c_1 = f(\tilde{\rho}_1) = 2{,}390 - 1.358\,\tilde{\rho}_1 + 5.256 \cdot 10^{-4}\,\tilde{\rho}_1^2 \qquad (12)$$

For the inverse scattering problem under consideration, Eqns. (10) - (12) contain three (3) unknowns, the quantities θ_1, $\tilde{\rho}_1$, c_1 (quantities $\tilde{\rho}_0$, c_0 , θ_0 and $r_{0,1}$ are known). Since the number of available equations equals the number of unknowns, we expect that a solution can be found. Indeed, combining Eqns. (10) and (12) we obtain :

$$r_{0,1} = \frac{\tilde{n}_1 f(\tilde{n}_1)\cos\theta_0 - \tilde{n}_0 c_0 \cos\theta_1}{\tilde{n}_1 f(\tilde{n}_1)\cos\theta_0 + \tilde{n}_0 c_0 \cos\theta_1} \tag{13}$$

Furthermore, eliminating the unknown parameter θ_1 between Eqns. (11) and (13), we easily obtain :

$$f(\tilde{n}_1) = \frac{(1+r_{0,1})\tilde{n}_0 c_0 / (1-r_{0,1})\cos\theta_0}{[\tilde{n}_1^2 + \tilde{n}_0^2(1+r_{0,1})^2 \tan^2\theta_0 / (1-r_{0,1})^2]^{1/2}} = \frac{\alpha}{[\tilde{n}_1^2 + \hat{\alpha}^2]^{1/2}} \tag{14}$$

where α, $\hat{\alpha}$ are known constants. Eqn. (14) can be numerically solved for $\tilde{n}_1$, using standard root - finding numerical techniques [12]. Since $f(\tilde{n}_1)$, Eqn. (12), is a polynomial of second order, Eqn. (14) is expected to yield, in general, two solutions. However, it can be easily realized that *only one root* exists in the range of interest (i.e. for possible values of sediment density). Subsequently, c_1 is determined from Eqn. (12), and θ_1 from Eqn. (11). Finally, the height h_1 of the first layer is determined from the (known) delay T_1 of the second reflection in the time - domain data, Eqn. (6) [see relation below equation (4)] :

$$h_1 = \frac{c_1 T_1}{2\cos\theta_1} \tag{15}$$

(b) Proceeding to subsequent layers.

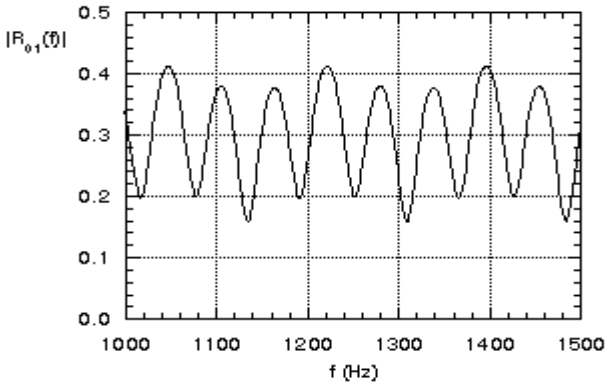
By solving Eqn. (2) for $R_{n,n+1}(\omega)$, we obtain :

$$R_{n,n+1}(\omega) = \exp(2j\hat{\alpha}_n \cos\theta_n h_n) \cdot \frac{R_{n-1,n}(\omega) - r_{n-1,n}}{1 - r_{n-1,n} \cdot R_{n-1,n}(\omega)} \qquad (n = 1 \text{ to } N-1) \tag{16}$$

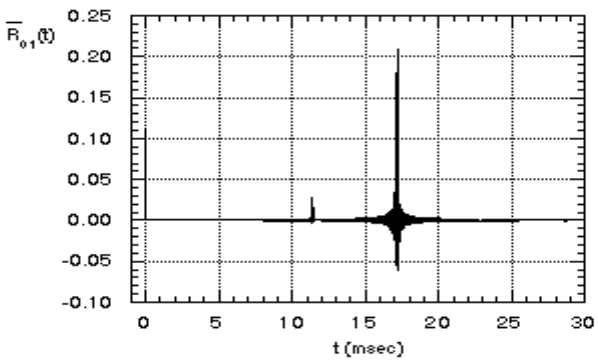
At first stage, $R_{1,2}(\omega)$ is calculated from the knowledge of $R_{0,1}(\omega)$, $r_{0,1}$, θ_1, h_1, $\hat{\alpha}_1 = \omega/c_1$. Then, the same procedure described above for the determination of the parameters of the first layer is repeated here, except that $R_{1,2}(\omega)$, calculated from Eqn. (16), is used instead of $R_{0,1}(\omega)$. In a similar fashion we proceed to the calculation of the acoustic parameters and the height of subsequent layers (n = 2,3,...,N).

3. NUMERICAL RECONSTRUCTION RESULTS FROM SIMULATED SYNTHETIC REFLECTION DATA

Here we present numerical reconstruction results based on the algorithm presented in the previous Section. The sea bottom model that we use here consists of three sediment layers [10], as shown in Table 1. In this Table, the values of the sound speed, as calculated from empirical formula (9), are also shown, as well as the corresponding to this calculation relative error. By solving the direct scattering problem related to these models, i.e. by using the recursive formula (2) and moving from the substrate to the upper layers, the total reflection coefficient in the water $R_{0,1}$(ù) can be calculated [Fig. 2(a)]. Subsequently, by applying a standard Fast Fourier Transform (FFT) routine [12] the corresponding time - domain reflection (impulse response) $R_{0,1}(t)$ is obtained [Fig. 2(b)]. Here totally 1024 samples of $R_{0,1}$(ù) were used in the range 0 Hz to 10 kHz (including the corresponding negative frequencies). In Fig. 2(b) we can observe the primary reflection from the water - layer 1 interface at $t = 0$, as well as the primary reflection from the layer 1 - layer 2 interface at $t \approx 12$ msec (this reflection is also observed and measured in the water). Note also that, as described in Section II, *only these two reflections* are needed in the inversion algorithm described above, namely the strength $r_{0,1}$ of the reflection at $t = 0$, as well as the time delay T_1, $T_1 \approx 12$ msec, at which the next (impulsive - type) reflection arrives [reflections arriving later in time may some times be stronger in magnitude, as is the case, for example, in fig. 2(b), depending on the contrast of acoustic parameters of subsequent layers. These reflections do not directly play a role in the inversion process, at least for the reconstruction of the acoustic parameters of the layer under consideration].



(a)



(b)

Figure 2

Figure2. (a) Total reflection coefficient in the water (magnitude) for the case of the first sea bottom model considered in this work (angle of incidence θ_0 equal to 30°). Here only a part of the available reflection coefficient data is shown in the range from 1 to 1.5 KHz. (b) Corresponding time - domain reflection obtained by the FFT algorithm.

	ρ(kg/m^3)	c(m/sec)	f(ρ), Eqn. (9)	relative error, use of Eqn. (9)	h(m)
water	1025	1510			infinite
layer1	1280	1490	1511	-1.427E-02	10
layer2	1350	1515	1513	1.463E-03	5
substrate	1980	1750	1758	-4.458E-03	infinite

Table 1. Sea bottom model considered in this work (see text for details).

Table 2 shows the results of reconstruction for the sea bottom model considered in this work. In particular, Table 2 shows the relative errors of reconstruction due to the proposed algorithm itself, assuming that the values ρ_n and c_n are related exactly by empirical formula (9). We can see from Table 2 that the relative error of reconstruction under the above assumption is less than 0.1% (similar results are found for the reconstruction of the layer heights h_1 and h_2 , not shown here. This remark holds through the entire paper presented here). Of course, the values c_n and ρ_n are never exactly connected through any empirical formula, and this introduces an additional reconstruction error. This is shown in Table 3, where the values ρ_n and c_n of the *initial* model are used for the generation of the synthetic reflection data, without the use of empirical formula (9). Then the relative errors in Table 3 are of order 2% or less. It should be noted that these errors are of the same order with the initial relative errors introduced by the use of the empirical formula, Table 1. From these results, it appears that the use of formula (9) is the most important source of reconstruction error. This means that if a more accurate relation connecting the values c_n with $\tilde{n}_n$ (n=1 to N) were a priori available, at least for the particular sea floor model under consideration, then the reconstruction error would be significantly smaller. Finally, note that the choice of the angle of incidence $\grave{e}_0$ in Tables 2- 4 should be such that no total reflection from any interface occurs [i.e. $\sin\grave{e}_n < 1$ for all $\grave{e}_n$, as calculated by Snell's law, Eqn. (8)]. Finally, Table 4 shows the *effect of the reduction of the frequency range* in which reflection coefficient data are available on the quality of the reconstructed sea bottom parameters. Note that empirical formula (9) *is* used here during the generation of the synthetic reflection data, which consist

of 1024 samples in the range 0 - 1 KHz (including the corresponding negative frequencies). The reconstruction error was found to be 2% or less [Table 4]. Then, eventhough the lack of acoustic data in medium and high frequencies (e.g. in the range 1 KHz to 10KHz) in general deteriorates the quality of reconstructions, the relative error still remains at acceptable levels, as described in the example of Table 4

angle θ_0(deg)	ρ_1	c_1	ρ_2	c_2	ρ_3	c_3
0	-9.864E-05	1.223E-06	-7.302E-04	-3.841E-05	-4.214E-04	-3.417E-04
5	2.085E-04	-2.704E-06	1.125E-03	5.785E-05	6.537E-04	5.292E-04
10	-1.310E-04	1.631E-06	-7.952E-04	-4.185E-05	-4.517E-04	-3.663E-04
15	2.464E-04	-3.196E-06	1.213E-03	6.230E-05	6.788E-04	5.495E-04
20	-1.176E-04	1.461E-06	-8.051E-04	-4.238E-05	-4.361E-04	-3.537E-04
25	2.990E-04	-3.882E-06	1.125E-03	5.783E-05	5.799E-04	4.695E-04
30	3.300E-04	-4.288E-06	-1.098E-03	-5.798E-05	-5.258E-04	-4.264E-04
35	-1.398E-04	1.741E-06	-1.104E-03	-5.833E-05	-4.800E-04	-3.893E-04

Table 2. Relative errors for reconstructed sea bottom parameters of the model considered in Table 1. Here reflection coefficient data are available in the range 0 to 10 KHz. Note that in this Table the generation of the reflection coefficient data is based on the assumption that the values of sea bottom layer density ρ_n and corresponding speed c_n are related through the empirical formula (9), i.e. the values c_n corresponding to the 3rd column of Table 1 are used.

angle θ_0(deg)	ρ_1	c_1	ρ_2	c_2	ρ_3	c_3
0	1.412E-02	-1.457E-02	-1.634E-03	1.376E-03	2.315E-03	-2.580E-03
5	1.453E-02	-1.458E-02	-2.342E-04	1.451E-03	3.150E-03	-1.906E-03
10	1.456E-02	-1.458E-02	-1.680E-03	1.374E-03	2.377E-03	-2.530E-03
15	1.550E-02	-1.461E-02	-1.843E-04	1.453E-03	3.316E-03	-1.772E-03
20	1.606E-02	-1.463E-02	-1.789E-03	1.368E-03	2.574E-03	-2.371E-03
25	1.780E-02	-1.468E-02	-8.361E-05	1.459E-03	3.657E-03	-1.497E-03
30	1.959E-02	-1.475E-02	-2.216E-03	1.345E-03	2.827E-03	-2.166E-03
35	2.192E-02	-1.483E-02	-1.486E-03	1.384E-03	3.502E-03	-1.622E-03

Table 3. Same as in Table 2, except that in this Table the values ρ_n and c_n of the original sea bottom models are used, i.e. use of the values c_n corresponding to the 2nd column of Table 1.

angle	ρ_1	c_1	ρ_2	c_2	ρ_3	c_3

θ_0(deg)						
0	2.792E-03	-3.999E-05	6.642E-03	3.184E-04	4.230E-03	3.407E-03
5	3.599E-03	-5.319E-05	5.464E-03	2.660E-04	3.542E-03	2.855E-03
10	1.025E-03	-1.367E-05	-2.339E-03	-1.254E-04	-9.403E-04	-7.630E-04
15	-3.146E-03	3.438E-05	-1.305E-02	-7.877E-04	-7.034E-03	-5.758E-03
20	3.465E-03	-5.094E-05	-9.564E-03	-5.564E-04	-5.008E-03	-4.088E-03
25	6.232E-04	-8.178E-06	-3.158E-03	-1.709E-04	-1.302E-03	-1.057E-03
30	-5.059E-03	4.979E-05	1.114E-02	5.022E-04	5.513E-03	4.432E-03
35	-5.350E-03	5.177E-05	-1.731E-02	-1.092E-03	-7.251E-03	-5.937E-03

Table 4. Same as in Table 2, except that in this Table the reflection coefficient data are available in the range from 0 to 1 KHz.

4. DISCUSSION

Here we developed and presented a novel method for the inversion of the basic acoustic parameters (density, sound speed) of the sea bottom from reflection coefficient data in the sea water. Since only bandlimited reflection coefficient data (magnitude and phase) are required in a *single* direction of incidence, and both the density *and* sound speed values are to be reconstructed from these data (along with the corresponding layer height), some a priori relation connecting these two parameters at each layer should be known. In this work a usually used empirical formula between these two acoustic parameters in sea bottom sediments [10] is taken into account.

It was found that the proposed method yields accurate reconstructions of the sea floor parameters, the accuracy of inversion being of order 0.1% or better, with an additional error coming from the specific empirical formula used (see Sections II and III for details). Obvious virtues of the proposed method are that it is conceptually simple and straightforward, and that it requires reflection coefficient data at a single angle of incidence. Furthermore, the proposed algorithm is very efficient in its computer implementation. This last effect is due to the fact that only very simple and efficient numerical routines, such as the FFT and root finding routines, are used , as well as simple algebraic expressions (see Section II).

Future directions of this research may include : inclusion of shear properties in the substrate (assumed solid), and examination of the effect of the presence of noise in the performance of the proposed method (note that this effect has already been examined in the corresponding electromagnetic inverse problem [13] with very promising

results). Furthermore, the effects of using realistic (not necessarily time - harmonic) acoustic sources on the quality of the method presented here, can also be examined. In these cases it is apparent that more advanced signal processing techniques will be necessary, before the data are used as the input to the proposed inversion algorithm.

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REFERENCES

[1] D. J. Thomson, "An inverse method for reconstructing the density and sound speed profiles of a layered ocean bottom", IEEE Journal of Oceanic Engineering, Vol. OE - 9, No. 1, pp. 18 - 25, 1984.

[2] D. C. Stickler and P. A. Deift, "Inverse problem for a stratified ocean and bottom", J. Acoust. Soc. Am., 70 (6), pp. 1723 - 1727, Dec. 1981.

[3] S. Coen, "Velocity and density profiles of a layered acoustic medium from common source - point data", Geophysics, Vol. 47, No. 6, pp. 898 - 905, 1982.

[4] J. A. DeSanto, "Oceanic sound speed profile inversion", IEEE Journal of Oceanic Engineering, Vol. OE - 9, No. 1, pp. 12 - 17, 1984.

[5] D. Lesselier, "Optimization techniques and inverse problems : probing of acoustic impedance profiles in time domain", J. Acoust. Soc. Am., 72 (4), pp. 1276 - 1284, Oct. 1982.

[6] G. N. Balanis, "Inverse scattering : determination of inhomogeneities in sound speed", J. Math. Phys., 23 (12), pp. 2562 - 2568, Dec. 1982.

[7] P. B. Abraham and H. E. Moses, "Exact solutions of the one - dimensional acoustic wave equations for several new velocity profiles : transmission and reflection coefficients", J. Acoust. Soc. Am., 71 (6), pp. 1391 - 1399, 1982.

[8] C. A. Boyles, Acoustic Waveguides, Applications to Oceanic Science, John Wiley and Sons Inc., New York, 1984.

[9] O. F. Hastrup, "Acoustic bottom reflectivity", Report no. SR - 115, Saclant Undersea Research Centre, June 1989.

[10] I. Ioannides and M. Taroudakis, "Acoustic propagation models for application in Greek Sea Environment", Research Report, Dept. Of Naval Engineering, National Technical University of Athens, Greece, December 1986.

[11] E.O. Brigham, The Fast Fourier Transform and its applications, Prentice Hall, New Jersey, U.S.A, 1988.

[12] W. H. Press et. al., Numerical Recipes, Cambridge University Press, Cambridge, United Kingdom, 1986.

[13] E. A. Hashish and H. M. Raafat, "Reconstruction of multilayered dielectric models using oblique transverse electric or transverse magnetic incidence", Optical Engineering, Vol. 34, No. 4, pp. 1172 - 1177, 1995.

[14] P. Lorrain and D. R. Corson, Electromagnetic Fields and Waves, 2nd edition, W. H. Freeman and Co., San Francisco, 1970, Ch. 12.