A FULL -WAVE ANALYSIS OF 2-D MICROSTRIP GRATING ON A MULTILAYERED BIANISOTROPIC SUBSTRATE

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Abstract

An integral equation method with numerical solution is presented for the analysis of infinite 2D grating embedded into bianisotropic multilayered medium. Dyadic Green functions for shielded and open structure are derived using 4 \Theta 4 transfer matrix method. This method is verified by examining various results for microstrip resonator on isotropic and on ferrite substrates and for regular periodic strip line on the chiral substrate. The results of testing of the algorithm convergence are presented. Resonant frequencies of the 2-D grating on uniaxial crystal substrate are computed versus phase shifts per periods of grating. Influence of the chirality parameter on the resonant properties is studied.

1. INTRODUCTION

Development of the millimeter-wave integrated circuits (MIC's) and microwave monolitic integrated circuits (MMIC's) technology has resulted into growing interest of multilayered planar transmission lines, resonators and antennas, which consist of layers of anisotropic and gyrotropic materials [1]. Recently much attention has been focused on composite materials, the so-called complex media. The isotropic chiral media [2] - [5], anisotropic chiral media [6], chiroferrites [7, 8], omega-medium and more complex bianisotropic materials [9] have been studied.

Until now the number of publications dealing with planar (microstrip, slot and coplanar) regular lines using of arbitrary type anisotropic substrates is restricted. The finite-element [10], transmission lines [11] and integral equations (IE) [12]-[14] methods have been used to determine the propagation characteristics of such lines.

The analysis of planar discontinuities is a more complicated three-dimensional problem, and only particular cases of anisotropy and biisotropy have been considered for microstrip resonators [15] and gratings [16]. Thus, the microstrip and slot resonators on the uniaxial crystal and on the ferrite substrates have been studied in [15], where the anisotropy axis and magnetization axis have been taken to be directed only along one of the coordinate axes. Only the isotropic chiral medium has been considered in [16] as a substrate for 2-D microstrip grating .

In this paper an analysis is presented for the infinite 2-D microstrip grating embedded into multilayered bianisotropic medium (see Fig. 1).

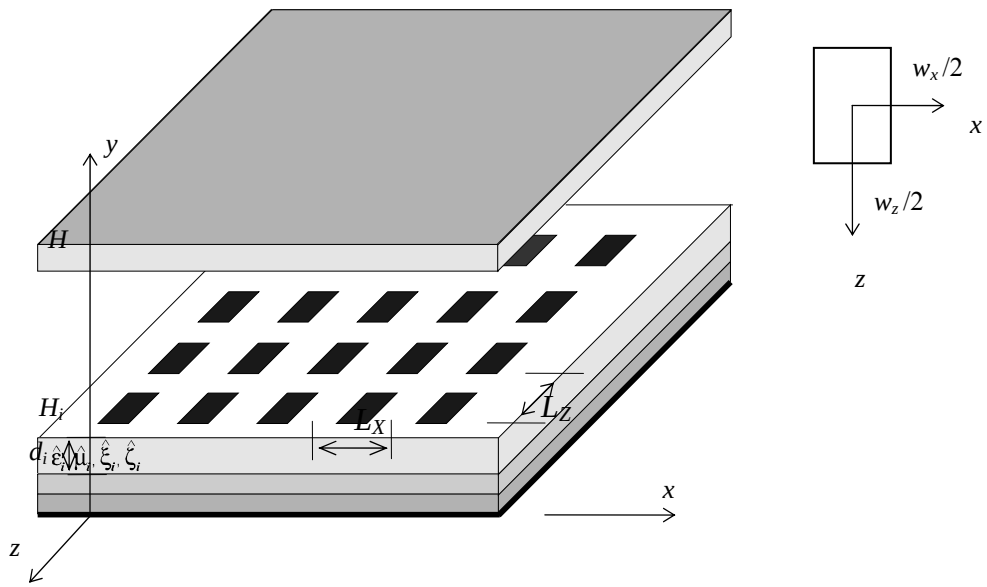

Figure 1. 2-D microstrip grating, embedded into multilayered bianisotropic medium.

This structure may contain an arbitrary number of layers both above the grating and below it. Integral equations method is used, which seems to be the most appropriate one concerning problems with planar geometry. A dyadic Green function for multilayered bianisotropic media is constructed using a 4 x 4 transfer matrix method [17]. This method is a great convenience for structures with planar geometries and it has been applied to microstrip lines on anisotropic substrates in [18]-[20]. The convergence properties of the proposed algorithm are investigated and its accuracy is verified by comparing the results with those of other authors. Then the resonant properties of the 2-D grating are studied.

2. THEORY

A two-dimensional (2-D) periodic grating is considered with a period of L_x along the x axis and with a period of L_z along z axis. The quantities w_x and w_z are the sizes of rectangular grating elements along x and z axes, respectively. As shown in Fig. 1, the substrate structure consists of N layers with thickness d_m (m = 1,...N) and is bounded by electric wall at y = 0 . The plane y = H may be an electric wall or an interface with an isotropic dielectric. The grating is placed on the interface between i -

th and i +1 - th layers. The thickness of conductors is assumed to be negligible and the dielectric loss is absent.

In this paper the algorithm developed in Ref. [21] for a regular periodic stripline on the multilayered anisotropic substrate is extended to the case of bianisotropic media and three-dimensional structure.

In order to represent the field components in the layered structure the transfer matrix method is used. Let us consider the m - th layer with thickness d_m. It is characterized by relative permittivity tensor ε_m, permeability tensor μ_m, and the two cross - coupling tensors ζ_m and ξ_m. All of these tensors have a general form, i.e. all their components may be nonzero.

The source-free Maxwell equations can be written as

$$\begin{aligned} \nabla \times \vec{E} &= -j(k_0 \rho \mu_m \vec{H} + \omega \zeta_m \vec{E}) \\ \nabla \times \vec{H} &= j(k_0 \varepsilon_m / \rho \vec{E} + \omega \xi_m \vec{H}) \end{aligned} \tag{1}$$

where $\rho = 120\pi$ Ohm is the wave impedance of vacuum, $k_0 = \omega\,(\mu_0\,\varepsilon_0)^{1/2}$.

Let us denote the column-vector of tangential components of electromagnetic field for the m - th layer as $X^{(m)} = [E_x, E_z, H_x, H_z]_{col} = [X_1, X_2, X_3, X_4]_{col}$. Using the Fourier expansion of $X^{(m)}$

$$X^{(m)}(x, y, z) = \sum_{n=-\infty}^{\infty} \sum_{p=-\infty}^{\infty} X_{np}^{(m)}(y) e^{-j(\beta_n x + \gamma_p z)}$$

where $X_{np} = [X_{1np}, X_{2np}, X_{3np}, X_{4np}]_{col}$, $\beta_n = (\psi + 2\pi n) / L_x$, $\gamma_p = (\varphi + 2\pi p) / L_z$ and ψ, φ are the field phase shifts per periods L_x and L_z , respectively, one can eliminate E_y, H_y components from the Maxwell equations. The expressions of E_y, H_y in terms of the tangential field components are given in Appendix by (A.1). Substituting E_y, H_y into Eq. (1), the Maxwell equations may be written as a set of four ordinary linear differential equations

$$\frac{d}{dy} X_{np}^{(m)} = j[A_{np}^{(m)}] X_{np}^{(m)}. \tag{2}$$

The expressions for the components of matrix [A] are given in Appendix by (A.2). The solution of the Cauchy problem for Eq. (2) has the form:

$$X_{np}^{(m)}(d_m) = \exp(j[A_{np}^{(m)}]d_m)X_{np}^{(m)}(0)$$

where $[T_{np}^{(m)}] = \exp(j [A_{np}^{(m)}] d_m)$ is the transfer matrix or the matrix propagator of the m - th layer in spectral domain. The transfer matrix of each layer may be computed using spectral representation of the matrix function. When the matrix [A] has a simple structure, the matrix exponent is expressed as [22]

$$\exp(j[A_{np}^{(m)}]d_m) = \sum_{l=1}^{s} \exp(j\lambda_l d_m) \prod_{k \neq l}^{s} \frac{([A_{np}^{(m)}] - \lambda_k[I])}{(\lambda_l - \lambda_k)}$$

where s is the number of different eigenvalues λ_κ of the matrix $A_{np}^{(m)}$ and [I] is the unit matrix. The transfer matrix of N - layered structure may be obtained as follows:

$[B] = [T_N] \times [T_{N-1}] \times \ldots [T_2] \times [T_1]$.

Denote by $[B^t]$ and $[B^l]$ the transfer matrices for the structures, which contain the layers with $H_i \leq y \leq H$ and $0 \leq y \leq H_i$, respectively. Then at the $y = H_i$ plane the following conditions for Fourier components of EM field should be satisfied:

for Fourier components of EM field should be satisfied:

$$X_{np}^{+}(H_i) - X_{np}^{-}(H_i) = V_{np}; \quad (k = 1, 2) \tag{3}$$

where the column vector $V_{np} = [0, 0, -I_{znp}, I_{xnp}]_{col}$ Her X_{np}^{+}, X_{np}^{-} are the vectors of EM field components which belong to the top and bottom layers with respect to the interface $y = H_i$ considered, I_{znp} , I_{xnp} are the Fourier coefficients of the current density components J_z (x, z)

$$\vec{I}_{np} = \frac{1}{L_x L_z} \int_{-L_x/2}^{L_x/2} \int_{-L_z/2}^{L_z/2} \vec{J}(x, z) e^{j(\beta_n x + \gamma_p z)} \, dx dz. \tag{4}$$

Now conider a shielded structure. Let us express the vectros X_{np}^{+-} on the $y = H_i$ via the x and z components of magnetic field on the top and bottom shields

$$X_{k\,np}^{+}(H_i) = B_{k3}^{t} X_{3\,np}(H) + B_{k4}^{t} X_{4\,np}(H) \quad (k = 1 \div 4) \tag{5}$$

$$X_{k\,np}^{-}(H_i) = B_{k3}^{l} X_{3\,np}(0) + B_{k4}^{l} X_{4\,np}(0) \quad (k = 1 \div 4). \tag{6}$$

Denoting by [D] the matrix composed of the column vectors (B^t_3, B^t_4, $-B^l_3$, $-B^l_4$), where B^t_k, B^l_k are the k - th column-vectors of matrices [B^t], [B^l] respectively, and substituting (5), (6) into (3) one can obtain for each Fourier-component the set of nonhomogeneous linear algebraic equations (SLAE)

$$[D]\mathrm{Y}_{np} = \mathrm{V}_{np} \quad n = -\infty, \ldots \infty, \; p = -\infty, \ldots \infty \tag{7}$$

where Y_{np} = [$X_{3np}(0)$, $X_{4np}(0)$, $X_{3np}(H)$, $X_{4np}(H)$]$_{col}$ is the column - vector of variables. The solutions $X_{knp}(0)$ and $X_{knp}(H)$ (k = 3, 4) of Eq. (7) are expressed via I_{xnp}, I_{znp} using Cramer's rule

$$X_{k\,np}(0) = \frac{\mathrm{Det}([D^0_k])}{\mathrm{Det}([D])}; \quad X_{k\,np}(H) = \frac{\mathrm{Det}([D^H_k])}{\mathrm{Det}([D])}; \quad (k = 3, 4). \tag{8}$$

The matrices [D^0_3,] and [D^0_4] are obtained by replacing the third and fourth columns in the matrix [D] with the column-vector V_{np} , respectively. The matrices [D^H_3,] and [D^H_4] obtained by replacing the first and the second columns in the matrix [D] with the same column-vector respectively. Substituting (8) into (7) we obtain

$$\begin{aligned} X_{1\,np}(H_i) &= I_{z\,np}\frac{C_{3\,np}B^l_{11} - C_{1\,np}B^l_{13}}{D_{np}} + I_{x\,np}\frac{C_{4\,np}B^l_{11} - C_{2\,np}B^l_{12}}{D_{np}} \\ X_{2\,np}(H_i) &= I_{z\,np}\frac{C_{3\,np}B^l_{21} - C_{1\,np}B^l_{23}}{D_{np}} + I_{x\,np}\frac{C_{4\,np}B^l_{21} - C_{2\,np}B^l_{23}}{D_{np}} \end{aligned} \tag{9}$$

where D_{np} = Det ([D]), C_{1np} = det([C1]), C_{2np} = Det([C2]), C_{3np} = Det([C3]), C_{4np} = Det(C4]). Expressions for matrices [C1], [C2], [C3], [C4] are given in Appendix, see Eq. (A.3).

As a next step, consider a case, when the upper shield is absent. Nonradiative modes may be calculated using the technique described above. For this purpose one should place the upper shield far from the strip grating. However, it is required to construct an algorithm valid for calculations of the radiative losses. In addition, the Green function constructed for open structures may be used for the problems of antennas with 2-D grating.

If the grating considered is bounded with an isotropic half-space y > H , Eq. (5) should be changed. Defining $k_x = \beta_n$; $k_z = \gamma_p$, we express H_x and H_z for isotropic medium via E_x E_z as follows:

$H_x = t_x E_x + t_z E_z$, $H_z = q_x E_x + q_z E_z$

where

$$t_x = \frac{k_x k_z}{k_y k_0 \rho \mu}; \quad t_z = \frac{1}{k_y \rho}\left(k_0 \varepsilon - \frac{k_x^2}{k_0 \mu}\right)$$

$$q_x = \frac{1}{k_y \rho}\left(-k_0 \varepsilon + \frac{k_z^2}{k_0 \mu}\right); \quad q_z = -\frac{k_x k_z}{k_y k_0 \rho \mu}$$

$$k_y = \sqrt{k_0^2 \varepsilon \mu - k_x^2 - k_z^2}$$

Here ε and μ are the permittivity and the permeability of the isotropic half - space y > H. The branches of the complex function k_y should be selected so that the condition $Re(k_y)$ is satisfied, which corresponds to the case when the radiative waves leave the grating surface. Thus for components of vector X^+_{np} one can obtain

$$X^+_{knp}(H_i) = X_{1np}(H)(B^t_{k1}+t_x B^t_{k3}+q_x B^t_{k4})+X_{2np}(H)(B^t_{k2}+t_z B^t_{k3}+q_z B^t_{k4}) \quad (k=3,4) \tag{10}$$

Substituting (10) and (6) into (3) one can obtain SLAE with the variables $X_{1np}(H)$, $X_{2np}(H)$, $X_{3np}(0)$ and $X_{4np}(0)$

$$\begin{aligned}
&X_{1np}(H)(B^t_{k1}+t_x B^t_{k3}+q_x B^t_{k4})+X_{2np}(H)(B^t_{k2}+t_z B^t_{k3}+q_z B^t_{k4})-\\
&-B^l_{k3}X_{3np}(0)-B^l_{k4}X_{4np}(0)=0\\
&(k=1,2)\\
&X_{1np}(H)(B^t_{31}+t_x B^t_{33}+q_x B^t_{34})+X_{2np}(H)(B^t_{32}+t_z B^t_{33}+q_z B^t_{34})-\\
&-B^l_{33}X_{3np}(0)-B^l_{34}X_{4np}(0)=-I_{znp}\\
&X_{1np}(H)(B^t_{41}+t_x B^t_{43}+q_x B^t_{44})+X_{2np}(H)(B^t_{42}+t_z B^t_{43}+q_z B^t_{44})-\\
&-B^l_{43}X_{3np}(0)-B^l_{44}X_{4np}(0)=I_{xnp}.
\end{aligned} \tag{11}$$

The solution $X_{knp}(0)$, (k = 3, 4) and $X_{knp}(H)$ (k = 1, 2) of Eq. (1) are expressed using Cramer's rule via the same Eq. (8) with k = 1, 2 for $X_{knp}(H)$. The matrix [D] for the case of open structure is composed of the column-vectors $(B^t_1 + t_x B^t_3 + q_x B^t_4)$,

$(B^t_2 + t_z B^t_3 + q_z B^t_4)$, $(-B^l_3)$, $(-B^l_4)$. The matrices $[D_1^H]$, $[D_2^H]$, $[D_3^H]$, $[D_4^0]$ are obtained by replacing the 1 ÷ 4 columns in the matrix [D] with the column-vector V_{np} , respectively. Substituting (8) into (11) we obtain the same expressions as (9), where matrices [C1], [C2], [C3], [C4] are given in Appendix by (A.4). In the following all considerations relate to both shielded and open structures.

Using the conditions $E_x(x, H_i) = 0$, $E_z(x, H_i) = 0$ with $|x| \leq w / 2$ and taking into account (9) we obtain the pair of infinite equations:

$$\begin{aligned} &\sum_{n=-\infty}^{\infty}\sum_{p=-\infty}^{\infty}(R_{zz}^{np} I_{z\,np} + R_{zx}^{np} I_{x\,np}) e^{-j(\beta_n x+\gamma_p z)} = 0 \\ &\sum_{n=-\infty}^{\infty}\sum_{p=-\infty}^{\infty}(R_{xz}^{np} I_{z\,np} + R_{xx}^{np} I_{x\,np}) e^{-j(\beta_n x+\gamma_p z)} = 0 \end{aligned} \qquad |x| \leq w_x/2; \quad |z| \leq w_z/2 \tag{12}$$

where

$$R_{zz}^{np} = (C_{3np}B_{21}^l - C_{1np}B_{23}^l)/D_{np}; \quad R_{zx}^{np} = (C_{4np}B_{21}^l - C_{2np}B_{23}^l)/D_{np}$$
$$R_{xz}^{np} = (C_{3np}B_{11}^l - C_{1np}B_{13}^l)/D_{np}; \quad R_{xx}^{np} = (C_{4np}B_{11}^l - C_{2np}B_{13}^l)/D_{np}.$$

Taking into account (4), the equation (12) may be written as the vector Fremdholm integral equation of impedance type:

$$\int_{-w_x/2}^{w_x/2}\int_{-w_z/2}^{w_z/2} \hat{G}(x, z|x', z')\vec{J}(x', z')\, dx' dz' = 0 \quad |x| \leq w_x/2; \; |z| \leq w_z/2 \tag{13}$$

where the tensor Green function G(x, z | z', z') has the form:

$$G(x, z|x', z') = \sum_{n=-\infty}^{\infty}\sum_{p=-\infty}^{\infty}\begin{pmatrix} R_{zz}^{np} & R_{zx}^{np} \\ R_{xz}^{np} & R_{xx}^{np} \end{pmatrix} e^{-j[\beta_n(x-x')+\gamma_p(z-z')]}$$

The integral equation is solved using the Galerkin method. The weighted Cehbyshev polinomials of the first kind $T_k(x)$ and the second kind $U_l(x)$ are used as basis functions

$$J_z(x,z) = \left(1-(2x/w_x)^2\right)^{-1/2}\left(1-(2z/w_z)^2\right)^{1/2}\sum_{k=0}^{\infty}\sum_{l=1}^{\infty} a_{kl}T_k(2x/w_x)U_l(2z/w_z)$$
$$J_x(x,z) = \left(1-(2x/w_x)^2\right)^{1/2}\left(1-(2z/w_z)^2\right)^{-1/2}\sum_{k=0}^{\infty}\sum_{l=1}^{\infty} b_{lk}U_l(2x/w_x)T_k(2z/w_z). \tag{14}$$

After applying the Galerkin procedure to Eq. (13) we obtain the homogeneous SLAE:

$$\sum_s a_s\left(\sum_{n=-\infty}^{\infty}\sum_{p=-\infty}^{\infty} R_{zz}^{np} Z_{xk}^n Z_{xk'}^n Y_{zl}^p Y_{zl'}^p\right) + \sum_r b_r\left(\sum_{n=-\infty}^{\infty}\sum_{p=-\infty}^{\infty} R_{zx}^{np} Z_{xk'}^n Y_{xl}^n Z_{zk}^p Y_{zl'}^p\right) = 0$$
$$\sum_s a_s\left(\sum_{n=-\infty}^{\infty}\sum_{p=-\infty}^{\infty} R_{xz}^{np} Z_{xk}^n Y_{xl'}^n Z_{zk'}^p Y_{zl}^p\right) + \sum_r b_r\left(\sum_{n=-\infty}^{\infty}\sum_{p=-\infty}^{\infty} R_{xx}^{np} Y_{xl}^n Y_{xl'}^n Z_{zk}^p Z_{zk'}^p\right) = 0$$
$$k,k' = 0,\ldots K; \quad l,l' = 1,\ldots K. \tag{15}$$

where s ≡ {kl} and r ≡ {lk} Here

$$Z_{xk}^n = J_k(\beta_n w_x/2); \qquad Z_{zk}^p = J_k(\gamma_p w_z/2)$$
$$Y_{xk}^n = J_k(\beta_n w_x/2)/\beta_n; \quad Y_{zk}^p = J_k(\gamma_p w_z/2)/\gamma_p$$

where $J_k(x)$ is the Bessel function of k - th order.

The resonant frequencies f r are found by an iterative root search from the determinant of SLAE (15).

3. NUMERICAL RESULTS

The algorithm and computer program developed were compared against the earlier published results. Numerical results for 2-D grating embedded into three-layered structure with $w_x << L_x$, $w_z << L_z$ were compared to data available in [23],[15], with selected data points listed in Table 1. For the computations, the permittivity of layers 1 and 3 was assumed to be that of air. The second layer is an isotropic dielectric with $\varepsilon = 2.2$. Other parameters of the structure are:

$d_1 = 0.125$, $d_2 = 0.127$, $d_3 = 1.298$, $w_x = 0.5$, $L_x = 10$, $L_z = 40$ mm, $\psi = \pi$, $\varphi = \pi$.

Next the proposed algorithm is verified by comparing of the results with [15] for resonator on ferrite substrate (see Fig. 2).

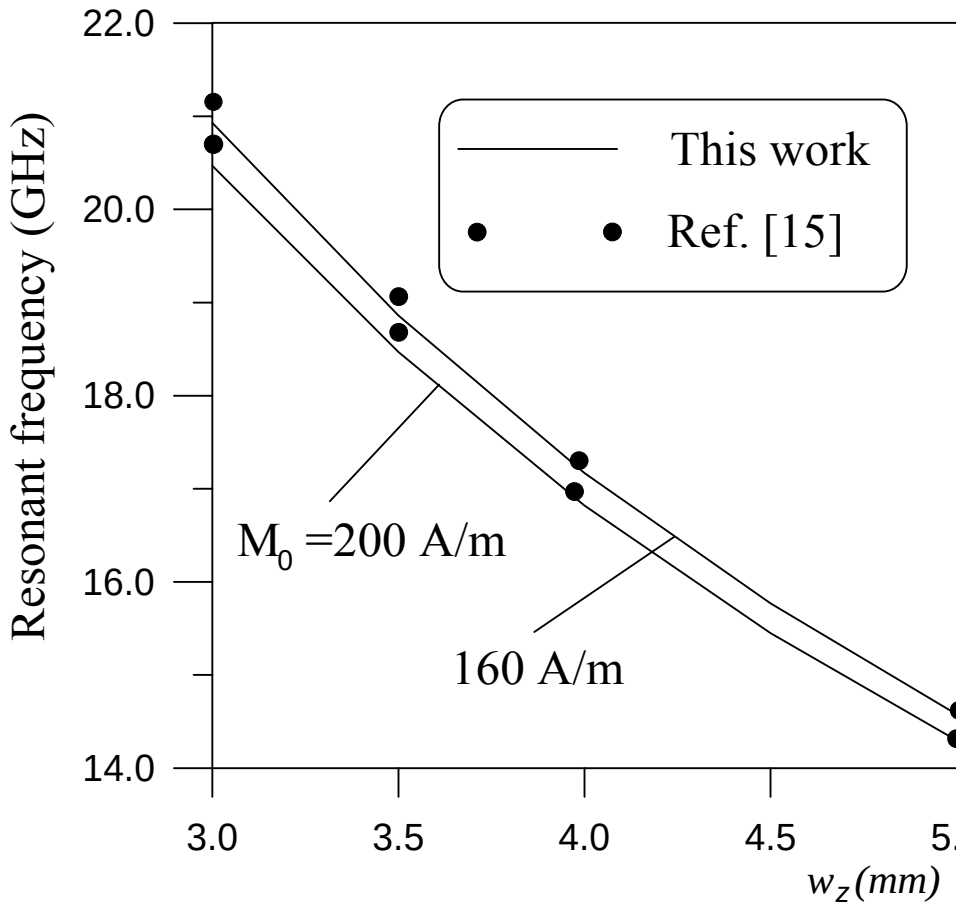

Figure 2. Comparison with the results of [15]. f_r for microstrip 2-D grating on ferrite substrate with L_x=3.556; L_z=14.2; w_x=0.3556; d_1=3; d_2=0.127; d_3=3.985mm; ε_2=11.5; $\psi=\pi$; φ=0.

The three-layered structure is considered, where the second layer is ferrite while the first and the third layers are air. According to [15], the permeability tensor of ferrite has the form:

$$\hat{\mu} = \begin{bmatrix} \mu & 0 & j\mu_a \\ 0 & 1 & 0 \\ -j\mu_a & 0 & \mu \end{bmatrix}$$

w_z (mm)	f_r(GHz)	f_r from [15]	f_r from [23]
8	16.11	16.27	15.8
10	13.28	13.30	12.7
12	11.36	11.25	10.9

Table I. Comparison with strip resonator.

M	20	30	40	50	60	70
f_r	54.1805	54.2975	54.3570	54.3962	54.4220	54.4418

Table II. Convergence investigation

where $\mu = (\omega^2_\perp - \omega^2) / (\omega^2_H - \omega^2)$; $\mu_\alpha = \omega\omega_M / (\omega^2_H - \omega^2)$; $\omega_H = 2\pi\gamma H_0$, $\omega_M = 2\pi\gamma M_0$; $\omega_\perp = (\omega_H(\omega_H + \omega_M))^{1/2}$; $H_0 = 357.94$ Oe (3000 A / m) is the inner field, $\gamma = 2.8$ MHz / Oe is the gyromagnetic ratio. Saturation magnetization M_0 is being 2005 Oe (160000 A / m) and 2506 Oe (200000 A/m).

As seen from Table I and Fig. 2, good agreement was obtained for both structures, validating the numerical solution to the three-dimensional boundary-value problems. The difference may be explained by the fact that infinite grating is not fully equivalent to a single resonator. To compare the results we need to take large L/ w , but the convergence of the series in Green function is decreased in this case.

Since there are no results for the resonant frequencies of the planar gratings or the resonators on bianisotropic substrate, the results [12], [24] are used, where the propagation constant of the dominant mode in a single strip line on chiral (biisotropic) substrate is computed. The characteristics of the dominant mode in the periodic strip line with $w_x / L_x << 1$ are assumed to be close to those for the single strip line (see, Fig. 3), where $\xi = - j (\varepsilon_0 \mu_0)^{1/2} \kappa$, $\zeta = -\xi$.

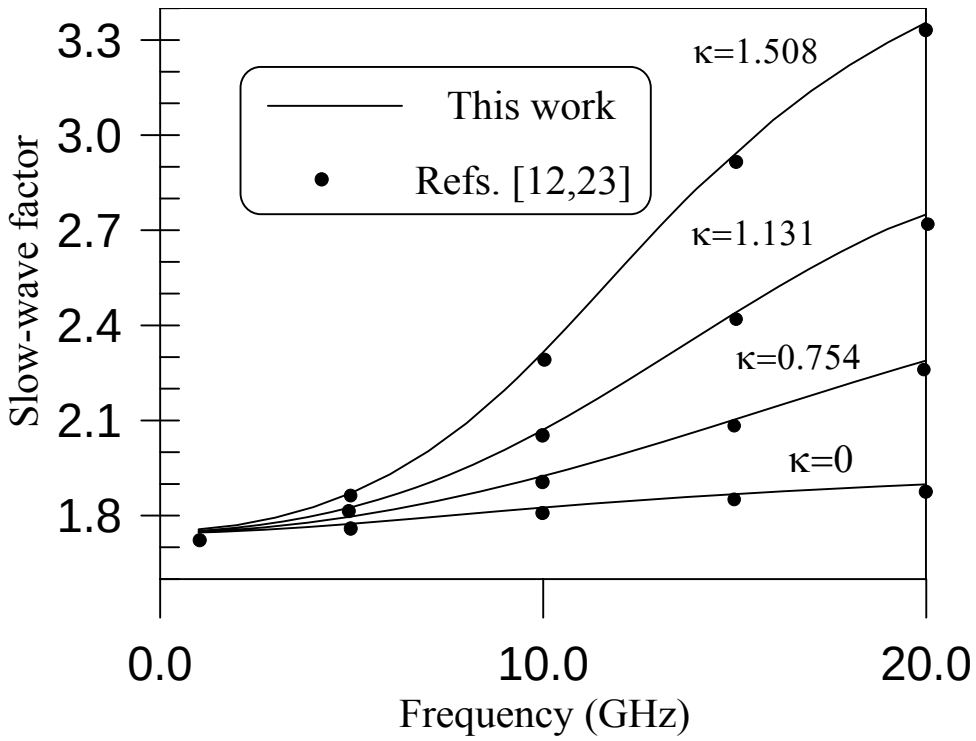

Figure 3. Comparison with the results of [12,23]. Dispersion curve for a periodic strip line with period L_x=23mm on a chiral substrate for various values of the chirality parameter κ. $N=2$, $\varepsilon_1=4+\kappa^2$, $\varepsilon_2=\mu_1=\mu_2=1$, $d_1=w_x=3$ mm, $d_2=30$ mm.

The results obtained by testing the dependence of the algorithm convergence upon the number M of the Fourier components taken to be into account are given in Table II. The numbers of Fourier components in x and in z directions are taken equal to M . The parameters of the structure are: N = 3, L_x = 1; L_z = 1; w_x = 0.5; w_z = 0.35; d_1 = 0.5; d_2 = 0.1; d_3 = 1 mm; ψ = 0; $\varphi = \pi$. The layers 1 and 3 were assumed to be air. The second layer is an uniaxial crystal which anisotropy axis lies in xOz plane and is directed along z -axis. $\varepsilon_\perp$ = 11.5; $\varepsilon_\parallel$ = 7, where $\varepsilon_\perp$ and $\varepsilon_\parallel$ are the components of the permittivity tensor of crystal across and along the axis of anisotropy.

The appropriate choice of basis which takes into account the current singularity at the edges of strips, results in a very good convergence upon the order of Galerkin's matrix Q . For lowest type of oscillation with M = 40 we have f_r = 54.357 with Q = 4 and f_r = 54.329 with Q = 12.

Finally, some new results for a shielded microstrip grating on an anisotropic chiral substrate are presented in Fig. 4. The parameters of the structure are the same as in Table II with the exception that the crystal has chiral property. Anisotropy axis lies in

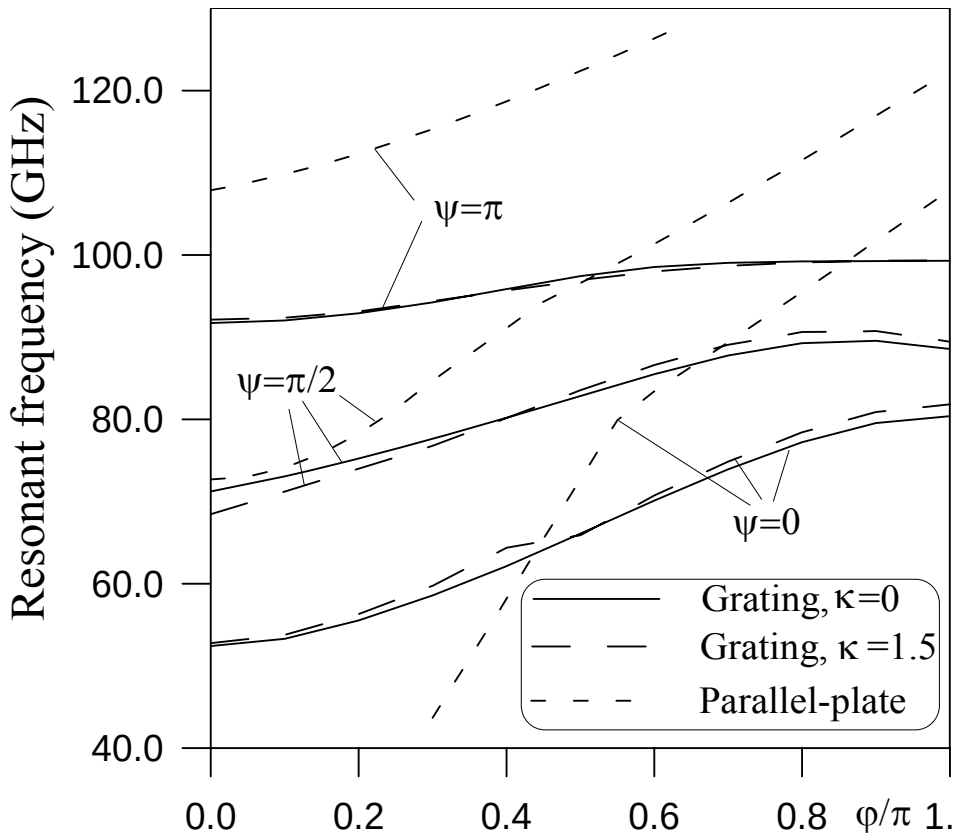

Figure 4. Resonant frequencies of microstrip grating and parallel-plate three-layered resonator versus phase shift φ per period L_z.

xOz plane and forms the angles $\theta = 45^o$ and $\theta = 0$ with z axis. We assume that $\xi = -j\,(\varepsilon_0\mu_0)^{1/2}$, $\zeta = -\xi$. The lowest resonant frequency of 2 - D grating versus phase φ for different phase shifts ψ and κ are given in Fig. 4. It can be seen that the presence of chirality decreases resonant frequency. The influence of the anisotropy axis direction on resonant properties is small.

In addition the lowest resonant frequency of 2-D parallel-plate with the same phase shifts is shown. It is equal to zero when $\psi = \varphi = 0$. The resonant frequencies of parallel-plate mode are poles of the equation for computing of the grating resonant frequencies.

4. CONCLUSION

An efficient spectral-domain full-wave algorithm for solving the problems with microstrip discontinuities, embedded into multilayered bianisotropic medium is described. The integral equation method leads to the matrix problem of small dimension Q . The order Q = 4 of matrix is enough for the calculation of the resonant frequency of the lowest type of oscillation. The value of Q needed is defined by the

type of oscillation calculated. Limitations of method, which appear for the high-type oscillations consideration, are connected with the necessity to take into account the large number of basis functions in the approximation of currents. The resonant properties of the 2-D microstrip grating on the chiral anisotropic substrate are studied. The algorithm presented may be used for calculations of both 1-D and 2-D infinite gratings and resonators, embedded into complex media, which are characterized by material parameter tensors ε, μ, ξ, ζ, having arbitrary form. The algorithm described may be extended over the problem of wave diffraction on 1-D and 2-D infinite gratings. This requires to specify the fields of incident and reflected waves on the interface with isotropic half-space in the model of open structure. After that a nonhomogeneous vector integral equation for deriving the reflected waves may be obtained. The algorithm proposed may be improved by the acceleration of series convergence in Green dyadic.

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APPENDIX A

The field components E_y and H_y may be expressed via E_x, E_z, H_x, H_z, as follows

$$E_y = u_x E_x + u_z E_z + v_x H_x + v_z H_z$$
$$H_y = s_x E_x + s_z E_z + t_x H_x + t_z H_z \qquad \text{(A.1)}$$

where

$$u_x = (-\xi_{22} k_z / k_0 + \xi_{22}\zeta_{21} - \mu_{22}\varepsilon_{21})/q$$
$$u_z = (\xi_{22} k_x / k_0 + \xi_{22}\zeta_{23} - \mu_{22}\varepsilon_{23})/q$$
$$v_x = \rho(-\mu_{22} k_z / k_0 + \xi_{22}\mu_{21} - \mu_{22}\xi_{21})/q$$
$$v_z = \rho(\mu_{22} k_x / k_0 + \xi_{22}\mu_{23} - \mu_{22}\xi_{23})/q$$
$$s_x = \frac{1}{\rho}(\varepsilon_{22} k_z / k_0 + \varepsilon_{21}\zeta_{22} - \varepsilon_{22}\zeta_{21})/q$$
$$s_z = \frac{1}{\rho}(-\varepsilon_{22} k_x / k_0 + \varepsilon_{23}\zeta_{22} - \varepsilon_{22}\zeta_{23})/q$$
$$t_x = (\zeta_{22} k_z / k_0 + \xi_{21}\zeta_{22} - \varepsilon_{22}\mu_{21})/q$$
$$t_z = (-\zeta_{22} k_x / k_0 + \xi_{23}\zeta_{22} - \varepsilon_{22}\mu_{23})/q$$
$$q = \mu_{22}\varepsilon_{22} - \zeta_{22}\xi_{22}.$$

The component of matrix [A] which are obtained from the Maxwell equations (1) are expressed as follows:

$$
\begin{aligned}
A_{11} &= \omega\zeta_{31} - k_x u_x + \omega\zeta_{32} u_x + k_0 \rho \mu_{32} s_x \\
A_{12} &= \omega\zeta_{33} - k_x u_z + \omega\zeta_{32} u_z + k_0 \rho \mu_{32} s_z \\
A_{13} &= k_0 \rho \mu_{31} - k_x v_x + \omega\zeta_{32} v_x + k_0 \rho \mu_{32} t_x \\
A_{14} &= k_0 \rho \mu_{33} - k_x v_z + \omega\zeta_{32} v_z + k_0 \rho \mu_{32} t_z \\
A_{21} &= -(\omega\zeta_{11} + k_z u_x + \omega\zeta_{12} u_x + k_0 \rho \mu_{12} s_x) \\
A_{22} &= -(\omega\zeta_{13} + k_z u_z + \omega\zeta_{12} u_z + k_0 \rho \mu_{12} s_z) \\
A_{23} &= -(k_0 \rho \mu_{11} + k_z v_x + \omega\zeta_{12} v_x + k_0 \rho \mu_{12} t_x) \\
A_{24} &= -(k_0 \rho \mu_{13} + k_z v_z + \omega\zeta_{12} v_z + k_0 \rho \mu_{12} t_z) \\
A_{31} &= -(k_0 \varepsilon_{31}/\rho + k_x s_x + k_0 \varepsilon_{32} u_x/\rho + \omega\xi_{32} s_x) \\
A_{32} &= -(k_0 \varepsilon_{33}/\rho + k_x s_z + k_0 \varepsilon_{32} u_z/\rho + \omega\xi_{32} s_z) \\
A_{33} &= -(\omega\xi_{31} + k_x t_x + k_0 \varepsilon_{32} v_x/\rho + \omega\xi_{32} t_x) \\
A_{34} &= -(\omega\xi_{33} + k_x t_z + k_0 \varepsilon_{32} v_z/\rho + \omega\xi_{32} t_z) \\
A_{11} &= k_0 \varepsilon_{11}/\rho - k_z s_x + \varepsilon_{12} u_x/\rho + \omega\xi_{12} s_x \\
A_{12} &= k_0 \varepsilon_{13}/\rho - k_z s_z + \varepsilon_{12} u_z/\rho + \omega\xi_{12} s_z \\
A_{13} &= \omega\xi_{11} - k_z t_x + k_0 \varepsilon_{12} v_x/\rho + \omega\xi_{12} t_x \\
A_{14} &= \omega\xi_{13} - k_z t_z + k_0 \varepsilon_{12} v_z/\rho + \omega\xi_{12} t_x.
\end{aligned}
$$

The matrices [C1], [C2], [C3], [C4] for the shielded 2 - D grating are given by the expressions:

$$
[C1] = \begin{bmatrix} B^t_{13} & B^t_{14} & -B^l_{14} \\ B^t_{23} & B^t_{24} & -B^l_{24} \\ B^t_{43} & B^t_{44} & -B^l_{44} \end{bmatrix}
$$

$$
[C2] = \begin{bmatrix} B^t_{13} & B^t_{14} & -B^l_{14} \\ B^t_{23} & B^t_{24} & -B^l_{24} \\ B^t_{33} & B^t_{34} & -B^l_{34} \end{bmatrix}
$$

$$
[C3] = \begin{bmatrix} B^t_{13} & B^t_{14} & -B^l_{13} \\ B^t_{23} & B^t_{24} & -B^l_{23} \\ B^t_{43} & B^t_{44} & -B^l_{43} \end{bmatrix}
$$

$$
[C4] = \begin{bmatrix} B^t_{13} & B^t_{14} & -B^l_{13} \\ B^t_{23} & B^t_{24} & -B^l_{23} \\ B^t_{33} & B^t_{34} & -B^l_{33} \end{bmatrix}
$$

The matrices [C1], [C2], [C3], [C4] for the open 2 - D grating are given by the expressions:

$$[C1] = \begin{bmatrix} B^t_{11} + t_x B^t_{13} + q_x B^t_{14} & B^t_{12} + t_z B^t_{13} + q_z B^t_{14} & -B^l_{14} \\ B^t_{21} + t_x B^t_{23} + q_x B^t_{24} & B^t_{22} + t_z B^t_{23} + q_z B^t_{24} & -B^l_{24} \\ B^t_{41} + t_x B^t_{43} + q_x B^t_{44} & B^t_{42} + t_z B^t_{43} + q_z B^t_{44} & -B^l_{44} \end{bmatrix}$$

$$[C2] = \begin{bmatrix} B^t_{11} + t_x B^t_{13} + q_x B^t_{14} & B^t_{12} + t_z B^t_{13} + q_z B^t_{14} & -B^l_{14} \\ B^t_{21} + t_x B^t_{23} + q_x B^t_{24} & B^t_{22} + t_z B^t_{23} + q_z B^t_{24} & -B^l_{24} \\ B^t_{31} + t_x B^t_{33} + q_x B^t_{34} & B^t_{32} + t_z B^t_{33} + q_z B^t_{34} & -B^l_{34} \end{bmatrix}$$

$$[C3] = \begin{bmatrix} B^t_{11} + t_x B^t_{13} + q_x B^t_{14} & B^t_{12} + t_z B^t_{13} + q_z B^t_{14} & -B^l_{13} \\ B^t_{21} + t_x B^t_{23} + q_x B^t_{24} & B^t_{22} + t_z B^t_{23} + q_z B^t_{24} & -B^l_{23} \\ B^t_{41} + t_x B^t_{43} + q_x B^t_{44} & B^t_{42} + t_z B^t_{43} + q_z B^t_{44} & -B^l_{43} \end{bmatrix}$$

$$[C4] = \begin{bmatrix} B^t_{11} + t_x B^t_{13} + q_x B^t_{14} & B^t_{12} + t_z B^t_{13} + q_z B^t_{14} & -B^l_{13} \\ B^t_{21} + t_x B^t_{23} + q_x B^t_{24} & B^t_{22} + t_z B^t_{23} + q_z B^t_{24} & -B^l_{23} \\ B^t_{31} + t_x B^t_{33} + q_x B^t_{34} & B^t_{32} + t_z B^t_{33} + q_z B^t_{34} & -B^l_{33} \end{bmatrix}$$