Statistical Moments of EM Waves in Random Media with Anisotropic Absorption

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Abstract

The evolution of statistical moments of the angular power spectrum for a semi-infinite absorbing medium, with large-scale random inhomogeneities is nvestigated. Nonsymmetrical structures are considered. Firstly the case of a normal illumination of the medium with anisotropic absorption is considered, while later an oblique illumination of the medium with regular absorption is taken. This approach is employed to derive systems of differential equations for the statistical moments. Then the systems were solved numerically. The most interesting result is the conclusion that under the asymmetrical dissipation of scattered waves the width and asymmetry of the spectrum can grow with the growing of absorbing properties of the medium.

1. INTRODUCTION

It is well known that the absorption does not only attenuate EM waves in random media, but changes their statistical characteristics essentially [1,2]. Especially grating effects can be observed in case of anisotropic absorption of scattered waves.

In this work evolution of statistical moments of the angular power spectrum of waves in the semi-infinite absorbing medium with large-scale random inhomogeneities is investigated. Two cases of incident wave directions were considered corresponding to normal and oblique directions. The last problem corresponds to the cases of light propagation in ocean, infrared light in clouds and near infrared light in biological tissues. Within the scope of the first problem the following cases were considered: normal wave propagation in magnetised plasma when electron gyrofrequency is considerably higher than the other characteristic frequencies; quasilongitudinal propagation of waves and in general case based on expression for refraction index of high-frequency waves in magnetised plasma [3].

2. EM WAVES IN MAGNETISED PLASMA

Problem and Method

Let's consider plane monochromatic wave which falls along the normal direction on a boundary of semi-infinite layer of collisional magnetised turbulent plasma (Fig. 1). Polarisation of the wave coincides with polarisation of one of the normal waves of the plasma. External magnetic field B^0 forms an angle θ with the initial direction of k^0 wave vector of the EM wave.

In the following shift of centre of gravity, variances and third central moment of angular spectrum of the wave power in the plasma is computed.

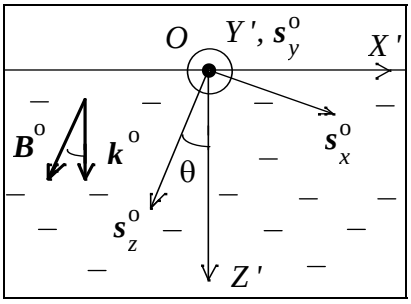

Figure 1.

Let's introduce Cartesian system of spatial coordinates $X'Y'Z'$ as shown in Fig. 1. Z' axis coincides with the boundary interior normal. The direction cosines of k are conveniently measured in the basis s_x^0, s_y^0, s_z^0 with $s_z^0 \parallel B^0$ and $s_y^0 \parallel OY$.

It is assumed that random inhomogeneities of refraction index are smooth in the scale of the wavelength. Variations of the normal wave intensity can be described by radiation transfer equation [4,5]. Under conditions of strong absorption and anisotropic scattering the angular spectrum will be narrow at arbitrary distance from the boundary. Therefore the transfer equation can be written in small-angle approximation [2,5]

$$[(s_z \cos\theta + s_x \sin\theta)\frac{\partial}{\partial \sigma z'} + 1 + \kappa(s)/\sigma\,]I(z',s) =$$

$$= \int\int_{-\infty}^{\infty} I(z',s')\,\chi(|s_x - s_x'|/\sqrt{\eta}\,,\,|s_y - s_y'|)\,ds'/\,4\pi\sqrt{\eta}\ , \qquad (1)$$

where $I(z',s)$ - angular power spectrum of the normal wave or intensity of the wave on depth z' in direction of unit vector (s_x, s_y, s_z); σ - scattering coefficient [4,5]; $\kappa(s)$ - absorption coefficient [3-5]; $s = (s_x, s_y)$; $\chi(s,s')$ - indicatrix of scattering which satisfies to the norming condition

$$\int\int_{-\infty}^{\infty} \chi(s,s')\,ds = 4\pi\ ; \qquad \eta = 1 - \nu^2\,,\ \nu = m_{10}/m_{00}\,, \qquad (2)$$

$$m_{nk} = \int\int_{-\infty}^{\infty} s_x^{\,n}\, s_y^{\,k}\, I(z',s)\,ds \qquad (3)$$

- angular moment of the spectrum. The assumption of smooth inhomogeneities permits negligence of the normal waves interaction [4-6].

Let's introduce the following boundary condition:

$$I(0,s) = I_0\,\delta(\,s_x - \sin\theta)\,\delta(\,s_y\,)\ . \qquad (4)$$

The assumption that all the radiation propagates in narrow solid angle near direction of the spectrum centre of gravity ($|s_y|$, $|s_x - \nu| << 1$) permits use the following approximation:

$$s_z = \sqrt{1 - s_x^2 - s_y^2} = N_0 + N_1 s_x - N_2 s_x^2 - N_3 s_y^2 , \tag{5}$$

where

$$N_0 = (1 - \nu^2 / 2\eta) / \sqrt{\eta} \quad , \quad N_1 = \nu^3 / \eta \sqrt{\eta} \quad , \quad N_3 = 1 / 2\sqrt{\eta} \quad , \quad N_2 = N_3 / \eta \quad .$$

and

$$\kappa(s) \approx \kappa + \kappa_x' (s_x - \nu) + [\kappa_x'' (s_x - \nu)^2 + \kappa_y'' s_y^2] / 2 \quad , \tag{6}$$

where

$$\kappa = \kappa(\nu,0) \quad , \quad \kappa_x' = \left.\frac{\partial \kappa(\boldsymbol{s})}{\partial s_x}\right|_{\nu,0} , \quad \kappa_x'' = \left.\frac{\partial^2 \kappa(\boldsymbol{s})}{\partial s_x^2}\right|_{\nu,0} , \quad \kappa_y'' = \left.\frac{\partial^2 \kappa(\boldsymbol{s})}{\partial s_y^2}\right|_{\nu,0} . \tag{7}$$

From (1) with (5) and (6) it is followed that angular moments (3) satisfy the infinite system of equations

$$\begin{aligned}
&n_0 m_{00}' + n_1 m_{10}' - n_2 m_{20}' - n_3 m_{02}' + A m_{00} + B m_{10} + \kappa_x'' m_{20} / 2\sigma + \kappa_y'' m_{02} / 2\sigma = 0 , \\
&n_0 m_{10}' + n_1 m_{20}' - n_2 m_{30}' - n_3 m_{12}' + A m_{10} + B m_{20} + \kappa_x'' m_{30} / 2\sigma + \kappa_y'' m_{12} / 2\sigma = 0 , \\
&n_0 m_{20}' + n_1 m_{30}' - n_2 m_{40}' - n_3 m_{22}' + A m_{20} + B m_{30} + \kappa_x'' m_{40} / 2\sigma + \kappa_y'' m_{22} / 2\sigma = \\
&\qquad = \eta \langle s^2 \rangle m_{00}/2 , \\
&n_0 m_{02}' + n_1 m_{12}' - n_2 m_{22}' - n_3 m_{04}' + A m_{02} + B m_{12} + \kappa_x'' m_{22} / 2\sigma + \kappa_y'' m_{04} / 2\sigma = \\
&\qquad = \langle s^2 \rangle m_{00} / 2 , \\
&n_0 m_{30}' + n_1 m_{40}' - n_2 m_{50}' - n_3 m_{32}' + A m_{30} + B m_{40} + \kappa_x'' m_{50} / 2\sigma + \kappa_y'' m_{32} / 2\sigma = \\
&\qquad = 3\eta \langle s^2 \rangle m_{10} /2 \quad ,...,
\end{aligned} \tag{8}$$

where

$$n_i = N_i \cos\theta , \quad i = 0, 2, 3 \quad ,$$

$$n_1 = (N_1 + \mathrm{tg}\,\theta) \cos\theta \quad ,$$

$$A = [\kappa - \nu (\kappa_x' - \nu \kappa_x'' / 2)] / \sigma \quad , \quad B = (\kappa_x' - \nu \kappa_x'') / \sigma \quad ,$$

$$\langle s^2 \rangle = \int\int_{-\infty}^{\infty} s_x^2 \, \chi(s,s') \, ds / 2\pi \quad . \tag{9}$$

Single primes in (8) denote derivatives with respect to $\sigma z'$.

To this end five equations of the system in (8) is kept. This means that the influence on the spectrum of indicatrix moments higher than the second one can be neglected (9). This is possible when the scattering in the plasma can be described by a function rapidly decreasing towards large angles.

Simple algebraic transformations allow to derive closed-loop system of differential equations for the centre of gravity shift ν (2) and the following integral characteristics of the spectrum:

$$v_x = m_{20} / m_{00} - \nu^2 \ , \qquad v_y = m_{02} / m_{00} \ , \qquad \mu = m_{30} / m_{00} - 3\nu\, v_x - \nu^3 \tag{10}$$

(variances and third central moment). The number of unknowns can be reduced due to the assumption about statistical independence of variables s_x and s_y:

$$m_{nk} = m_{n0}\, m_{0k} / m_{00}$$

and besides that it was supposed that the spectrum at large angles of scattering is approximated by a Gaussian function. As the result firth central moment can be neglected and firth noncentral moment can be expressed through the previous those in the equations

$$m_{40} / m_{00} = 3v_x^2 + 6\nu^2 v_x + \nu^4 \ , \qquad m_{04} / m_{00} = 3v_y^2 \ ,$$

$$m_{50} / m_{00} = 5\mu\,(v_x + \nu^2) + 15\nu\, v_x^2 + 10\nu^3 v_x + \nu^5 \ .$$

The system was solved numerically with the use of Runge-Kutta and Gauss methods [7,8].

The derived solution is valid under the following conditions:

$$\langle s^2 \rangle \ll 1 \ , \qquad \nu_\infty \ll 1 \ , \tag{11}$$

$$\nu_\infty^2 = \sigma \langle s^2 \rangle / 2\ \kappa'' \ , \tag{12}$$

$$\kappa'' = \left.\frac{\partial^2 \kappa(\boldsymbol{s})}{\partial s_x^{\,2}}\right|_{0,0} = \left.\frac{\partial^2 \kappa(\boldsymbol{s})}{\partial s_y^{\,2}}\right|_{0,0} \ . \tag{13}$$

The second of (11) inequalities means that the width of the spectrum should be small in deep (steady-state) regime. More so it is required that the width of the spectrum at arbitrary distance should not exceed considerably the deep one.

Results and Discussions

a) Let's consider first of all the simplest case when electron gyrofrequency ω_{Be} is considerably higher than the other characteristic frequencies ω , ν_{eff} , $\omega_p \ll \omega_{Be}$, where ω - the wave frequency, ν_{eff} - collisions effective frequency, ω_p - plasma frequency.

If the incident wave E vector is in the B^0, k^0 plane, then under the conditions

$$\nu_{eff} \ll \omega \ , \qquad \nu_{eff}\, v\, s^2 \ll \omega\,(1 - v)\,(1 - v + v\, s^2) \ ,$$

where $v = (\omega_p / \omega)^2$, the absorption coefficient in the sense of geometrical optics approximation [3,6] is

$$\kappa(s) = \nu_{eff}\, v\, s^2 / c\sqrt{1 - v}\,(1 - v + v\, s^2)^{3/2} \ , \tag{14}$$

with c being light velocity in vacuum. It should be mentioned that in this case normal waves do not interact, because of the second normal wave (E vector of which is perpendicular to B^0, k^0 plane) is not scattered on the plasma inhomogeneities (the wave propagates like in vacuum).

During the propagation centre of gravity of the spectrum turns toward B^0 the external magnetic field direction (Fig. 2). The reason is that the absorption coefficient (14) diminishes to zero in that direction. Propagation results in a growth of variances due to the scattering. It is very interesting that the variance v_x grows nonmonotone (for v_y this effect is less pronounced). Moreover the value of v_x variance in intermediate distances can exceed the asymptotic (large-depth) one.

The effect is the result of a competitive behaviour of two factors. On the one hand if the angle between wave vector k and the external magnetic field B^0 is decreased the absorption coefficient (14) diminishes. This causes increase of the scattered wave intensity if geometrical paths of all the waves could be equal to each other. On the other hand the nonscattered wave come to z=const plane by a shorter way that the scattered one. If the absorption coefficients of the waves could be equal then nonscattered wave will have grater amplitude than the scattered one. It is obvious the equilibrium of the above factors takes place in transition to large depths. Numerical calculations showed that the effect of exceeding of deep value by the variance appears when $\theta \neq 0$, and this effect grows with increasing of θ and anisotropy of absorption.

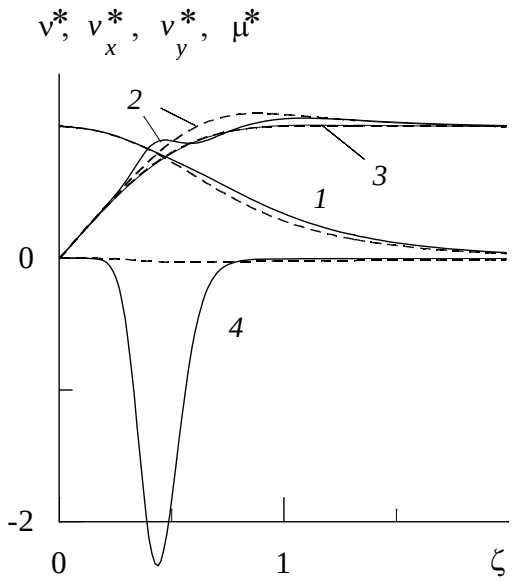

Figure 2. Evolution of wave parameters when electron gyrofrequency is considerably grater than the other characteristic frequencies. $\theta = 15^{O}$, $v = 0.7$, $\langle s^2 \rangle = 0.003$; solid lines - $\nu_{eff}/\sigma c = 5$, dashed-0.5; numbers denote: 1- $\nu^* = \nu/\sin\theta$, 2 - $v_x^* = v_x/\eta v_\infty$, 3 - $v_y^* = v_y/\eta v_\infty$, 4 - $\mu^* = \mu/(\eta v_\infty)^{3/2}$. $z = v_\infty \kappa'' z/2$.

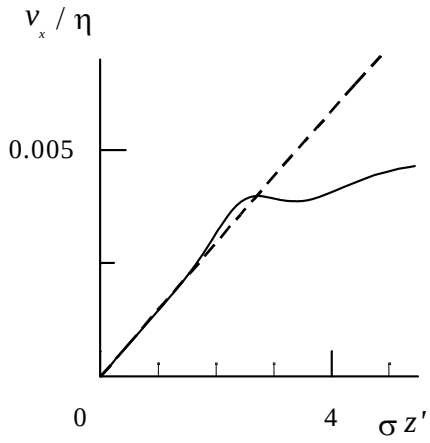

Figure 3. Parameters and notation are the same as in Fig. 2.

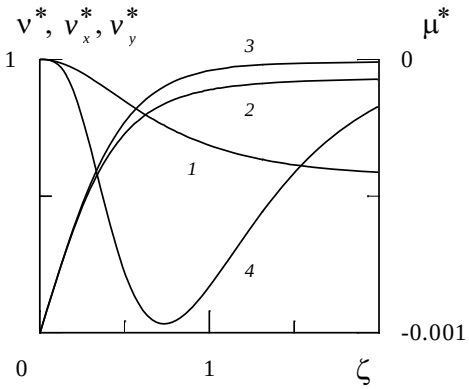

Figure 4. Qasilongitudional propagation. $\theta = 15^O$, $v = 0.7$, $<s^2> = 0.003$, $\nu_{eff} / \sigma c = 5$, $u = 0.3$ with the preceding notation.

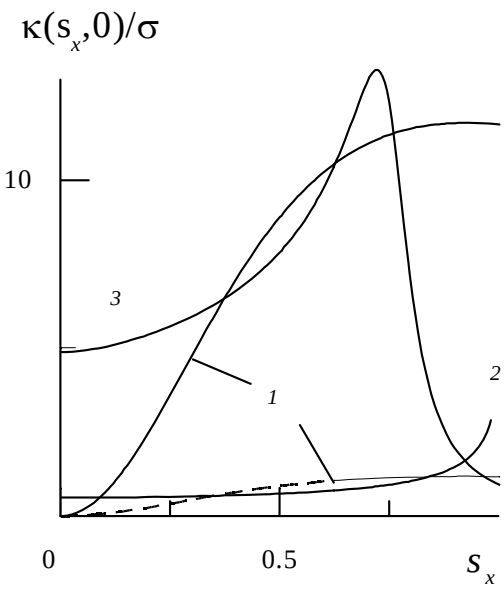

Figure 5. 1 - parameters correspond to those in Fig. 2, 2 - Fig. 4, 3 - $B^0 \parallel s_x^0$.

In transitional to deep regime region an increasing of the spectrum asymmetry takes place (the coefficient μ^* in Fig. 2). In deep regime the asymmetry diminishes to zero ($\mu^* \to 0$). The curves in

Fig. 3 exhibit the randomising effect of absorption [1,2]. The spectrum width increasing with the plasma absorption is observed in the range $1.5 < \sigma z' < 2.5$.

b) A similar analysis can be made for the case of EM wave propagation in plasma with weaker than in above described case external magnetic field. Let's consider the case of quasilongitudional propagation of unusual [3] wave with absorption coefficient

$$\kappa(s) = \nu_{eff}\, v / c \sqrt{1 - v + \sqrt{u(1-s^2)}}\ \left(1 + \sqrt{u(1-s^2)}\ \right)^{3/2}, \tag{15}$$

where $u = (\omega_{Be}/\omega)^2$. It should be mentioned that vistlers have the same coefficient of absorption, so described below results could be apply for them.

For illustration the case of $v < 1$ is considered (Fig. 4). The absorption anisotropy causes a similar as in above case shift of the centre of gravity towards B^0 the external magnetic field direction. But ν does not diminish to zero in deep regime. It is the result of slow relative variation of the absorption coefficient (15) over the range of angles $[0, \theta]$ (Fig. 5). In this case the waves scattered both along magnetic field B^0 and at small angles attenuate rapidly.

The nonmonotone variation of v_x variance is absent in this case because of too smooth dependence of the absorption coefficient on angle (Fig. 5). The difference between deep values of v_x and v_y is the result of the difference between derivatives $\left.\frac{\partial^2 \kappa(s)}{\partial s_x^{\,2}}\right|_{\nu_\infty,0}$ and $\left.\frac{\partial^2 \kappa(s)}{\partial s_y^{\,2}}\right|_{\nu_\infty,0}$, where ν_∞ - value of ν in deep regime.

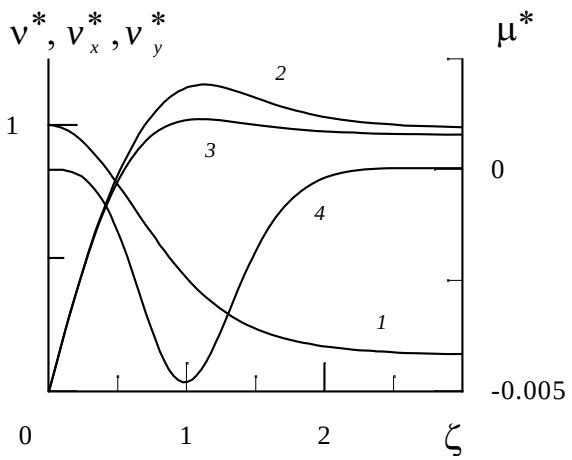

Figure 6. Numbers notation corresponds to figure 2.

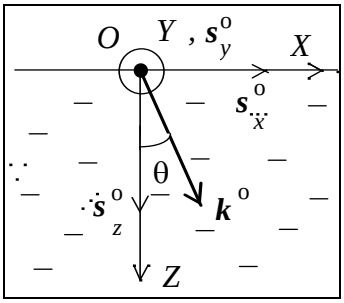

Figure 7.

c) The described above cases do not cover all the situations of anisotropic absorption of EM waves in magnetised plasma no doubt. But there are common peculiarities. It was confirmed by additional analysis on the basis of described above method but with numerical calculation of parameters (7). For the latter the expression for refraction index of high-frequency waves [3] was used.

The results for usual [3] wave parameters $\theta = 15^{O}$, $<s^2> = 0.003$, $\omega/\sigma\, c = 1$, $\nu_{eff} / \omega = 0.1$, $v = 7$, $u = 2$ are shown in Fig. 6. Geometry of the problem is shown in Fig. 1, but B^0 field is directed along s_x^0 vector and θ angle is counted from s_z^0 direction. This geometry permits to choose the most anisotropic part of absorption curve (curve 3 in Fig. 5).

The results presented in Fig. 6 confirm the discovered above peculiarities of EM waves propagation. The presence absorption in $s_x^0 = 0$ direction (Fig. 5) is the reason of the fact that $\nu_\infty \neq 0$. The large relatively variation of absorption coefficient in the range of angles [0,θ] explains nonmonotone dependence of the variances on distance. It should be underlined that in this case the last effect is observed for v_y variance too. As it is expected μ^* (the asymmetry) has maximum in the transitional to deep regime distances.

3. SIMILAR EFFECTS IN MEDIA WITH REGULAR ABSORPTION

Problem and Method

Similar effects take place in random media with regular absorption.

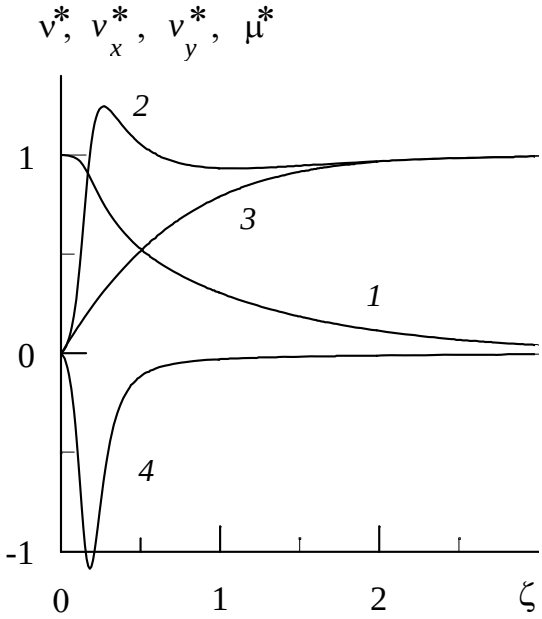

Figure 8. Evolution of wave parameters under oblique illumination of random medium with regular absorption. θ = 37°, Λ = 0.2 , $<s^2>$ = 0.0012; z = $v_\infty \kappa z$,; numbers denote: 1 - $\nu^* = \nu / \sin\theta$, 2 - $v_x^* = v_x / v_\infty$, 3 - $v_y^* = v_y / v_\infty$, 4 - $\mu^* = \mu/(\eta v_\infty)^{3/2}$.

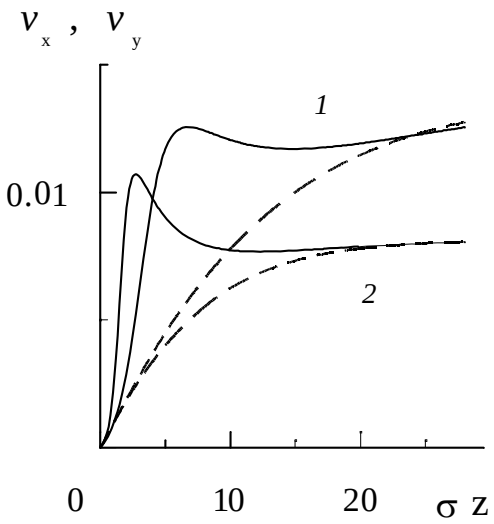

Figure 9. The same case as in Fig. 9 but $<s^2>$ = 0.003 . Dashed curve - v_y , solid - v_x ,. numbers denote: 1 - Λ = 0.2 , 2 - 0.08.

Let's consider plane monochromatic wave which is refracted at θ angle on a boundary of semi-infinite layer of random medium with the regular absorption (Fig. 7). k^0 denotes initial direction of the wave vector of EM wave. *XYZ* - Cartesian system of spatial coordinates. *Z* axis coincides with the boundary interior normal. The direction cosines of the vector *k* are measured in the basis s_x^0, s_y^0, s_z^0 (Fig. 7).

Let's suppose that the medium inhomogeneities are large in the scale of wavelength. Under condition of a narrow angular power spectrum the transfer equation can be written in the small-angle approximation:

$$[(N_0 + N_1 s_x - N_2 s_x^2 - N_3 s_y^2)\frac{\partial}{\partial \tau} + 1]I(\tau,s) =$$

$$= \Lambda \int\int_{-\infty}^{\infty} I(\tau,s')\,\chi(|s_x - s_x'|/\sqrt{\eta}\,,\,|s_y - s_y'|)\,ds'/4\pi\sqrt{\eta}\quad , \qquad (16)$$

where $I(\tau,s)$ - angular power spectrum of the normal wave or the wave intensity on optical depth $\tau=(\sigma+\kappa)z$ in direction of unit vector (s_x, s_y, s_z); σ - scattering coefficient; κ - absorption coefficient; $\Lambda=\sigma/(\sigma+\kappa)$ - albedo of single scattering, $s = (s_x, s_y)$; $\chi(s,s')$ - scattering indicatrix, definitions of η and ν coincides with the introduced above but angular moment of the light intensity is the following:

$$m_{nk} = \int\int_{-\infty}^{\infty} s_x^n s_y^k I(\tau,s)\,ds \qquad (17)$$

From the (16) infinite system of equations for the angular moments is derived

$$N_0 m_{00}' + N_1 m_{10}' - N_2 m_{20}' - N_3 m_{02}' + a\, m_{00} = 0\,,$$

$$N_0 m_{10}' + N_1 m_{20}' - N_2 m_{30}' - N_3 m_{12}' + a\, m_{10} = 0\,,$$

$$N_0 m_{20}' + N_1 m_{30}' - N_2 m_{40}' - N_3 m_{22}' + a\, m_{20} = \eta\, b\, m_{00}\,, \qquad (18)$$

$$N_0 m_{02}' + N_1 m_{12}' - N_2 m_{22}' - N_3 m_{04}' + a\, m_{02} = b\, m_{00}\,,$$

$$N_0 m_{30}' + N_1 m_{40}' - N_2 m_{50}' - N_3 m_{32}' + a\, m_{30} = 3\eta b\, m_{10}\,, \quad ,...,$$

where $a = 1-\Lambda$, $b = \Lambda\langle s^2\rangle/2$, primes in (18) denote derivatives with respect to τ.

The system truncated up to fife equations was solved numerically. The mentioned above assumptions are retained.

The derived solution is valid under the conditions (11) but

$$\nu_\infty^2 = \sigma \langle s^2\rangle / \kappa \qquad (19)$$

and it is demanded that width of the angular spectrum at arbitrary distance should not exceed considerably the deep one (given by (19)).

Results

The most interesting effects arise in the case of strong dissipation (Fig. 8). In this case asymmetry of the scattered waves attenuation causes increasing of the angular distribution width in some interval of distances. The effect accompanies with growing of asymmetry of the distribution and its shift toward the boundary normal direction.

Effect of increasing of the wavefield fluctuations caused by absorption is demonstrated in Fig. 9. There is shown that the angular spectrum width grows with increasing of the medium absorbing properties.

4. CONCLUSIONS

Using the method, there were derived qualitatively the same evolutions of the statistical moments with distances for both the problems.

The most interesting result is establishment that in some interval of distances the angular power spectrum variances can grow with the growing of dissipative properties of the media. At the same time there were registered nonmonotone changes of the variances with distance. Maximum values of the variances in intermediate distances in the case of strong dissipation can exceed the asymptotic values. Growing of the spectrum asymmetry in the intermediate distances accompanies the mentioned above effects. All the effects are absent both in the case of wave propagation in plasma along the magnetic field and under normal illumination of the medium with regular absorption.

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