APPLICATION OF "WARM RAYS" METHOD IN DESCRIPTION OF WAVE DIFFRACTION IN RANDOMLY INHOMOGENEOUS ATMOSPHERE

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Abstract

The process of optical wave propagation through a randomly inhomogeneous atmosphere is attended by gigantic random fluctuations developing in the wave intensity. It is known wave diffraction is the reason of limitation of intensity, but it is also known traditional methods for calculation of statistical moments of intensity are difficult enough.

To take proper account of diffraction influence on optical turbulence effects and at the same time to simplify the problem the "warm rays" method is offered. The method is based on the analogy between the evolution of the density of a warm gas of noninteractive particles and monochromatic optical waves propagating behind a random phase screen, which in many cases is a good model of real atmosphere.

The method allows to obtain a simple in structure expression for the correlation function of intensity fluctuations. To confirm the adequacy of applied assumptions verification is performed by an example of the dynamic phase screen. The diffraction influence on the integral probability function of intensity is also considered. The influence of parameter characterizing the relation of focusing and diffraction of waves is also analyzed.

1. INTRODUCTION

It is well known when optical waves propagate through a chaotic media, the inhomogeneities of refractive index focuse the rays. The process is attended by gigantic random fluctuations of wave intensity. Knowledge of the distribution of intensity is of interest for a variety of reasons. Indeed, the practical use of waves propagation requires knowledge of the probability of a high-intensity spike or a low-intensity fade at some time and place. Ideally, one desires a theory determining the complete probability density function of intensity, but this goal has proven to be quite difficult. Many theoretical studies [1-5] have concentrated on the variance of intensity with an effort directed toward an understanding of the general pattern of higher moments.

Since various inhomogeneities act as many different type of lenses with random properties, they create a random distribution of caustics and foci, where the wave amplitude tends to become infinite [5-6] and all higher moments of intensity are

unlimited. However this occurs only in geometrical approximation. In real media the restriction of caustic peculiarities of intensity is due to diffraction. At the same time, the exact mathematical description of the diffraction effects is considerably difficult aim [6].

In this paper an approximate method is proposed to take into account the diffraction limitation of intensity.

2. MATHEMATICAL FORMULATION

For simplicity only two-dimensional problem is analysed here. However it is easy to extend this method to real three-dimensional case. Consider a scalar wave field of the form $E(x,z)exp(ikz)$, where k is wave number and x is a coordinate perpendicular to the propagation distance z. Then the complex amplitude of the wave can be described by the parabolic wave equation [6], namely,

$$2ik\frac{\partial E}{\partial z}+\frac{\partial^2 E}{\partial x^2}+2k^2\eta E=0, \qquad E(x,z=0)=E_0(x), \qquad (1)$$

where η is the fluctuating part of refractive index. It is conveniently to consider the method for the example of a random phase screen which in many cases is a good model of real atmosphere [7]. Let an optical wave propagating along the z axis with the intensity I_0 falls on a random phase screen located in a plane z=0. The geometry analyzed is shown on Fig.1. For the present instance we may suppose the fluctuation part of a medium's index of refraction is equal to zero everywhere, but, instead of it, the random boundary condition on the screen appears.

Then define the Wigner function [8]

$$f(x,v,z)=\frac{1}{2\pi}\int_{-\infty}^{\infty}E(x+q/2k,z)E^*(x-q/2k,z)\exp(-ivq)dq \qquad (2)$$

which is the ray intensity of a wave propagating with an angle v to the z axis. This allows to get from (1) and (2) the Liouvelle equation [9]

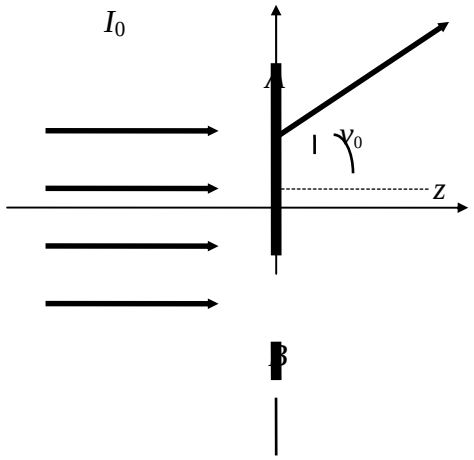


Figure 1. The geometry of the considered problem. *AB* is a phase screen.

$$\frac{\partial f}{\partial z} + v \frac{\partial f}{\partial x} = 0 \tag{3}$$

with the boundary condition

$$f(x,v,z=0) = I_0 f_0(x,v) \tag{4}$$

and full intensity in the fixed point (x,z) is

$$I(x,z) = \int_{-\infty}^{\infty} f(x,v,z)dv. \tag{5}$$

One can notice that just the same as (3) Liouvelle equation describes an evolution of density of noninteractive particles [9] in the phase space (x,v), where x and v are a coordinate and velocity of a particle, respectively. The limitation factor for the gas density is a thermal scattering of velocities. The absence of this scattering gives the unlimited peculiarities in the density field. In our case it is the diffraction that restricts the intensity fluctuations. So, we will use this analogy and call our method the ”warm rays” method.

Now we shall consider an initial angle v_0 under which a ray goes out the screen is a random function with known statistical properties.

Let plane wave with the intensity I_0 falls on the phase screen setted in the plane z=0 and contributed the distortion $k\Psi(x)$. Then the boundary condition (4) is

$$f(x,v,z=0)=\frac{I_0}{2\pi}\int_{-\infty}^{\infty}\exp\left\{ik\left[\Psi(x+q/2k)-\Psi(x-q/2k)\right]-ivq\right\}dq.$$

To find the wave intensity behind the screen the functions are used

$$v_0(x)=\Psi'(x), \qquad \Phi(x,q,k)=k\int_{x-q/2k}^{x+q/2k}\left[v_0(a)-v_0(x)\right]da. \qquad (6)$$

In terms of this functions one can write

$$f_0(x,v)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\exp\left[i\Phi(x,q,k)\right]\exp\left\{-i\left[v-v_0(x)\right]q\right\}dq,$$

or, in another form

$$f_0(x,v)=f_{\mathrm{T}}\left[x,v-v_0(x)\right],$$

where $f_{\mathrm{T}}(x,v)$ is given by

$$f_{\mathrm{T}}(x,v)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\exp\left[i\Phi(x,q,k)-ivq\right]dq. \qquad (7)$$

This function f_{T} takes the diffraction into account and gives the diffractional smoothing of peculiarities. Moreover, it helps to find the following exact expression for the wave intensity behind the screen

$$I(x,z)=\int_{-\infty}^{\infty}du\int_{-\infty}^{\infty}daf_{\mathrm{T}}(a,u)\delta\left[x-uz-X(z,a)\right]. \qquad (8)$$

Here

$$X(z,a) = a + v_0(a)z$$

is an equation of a geometrical ray coming out the point with coordinates (x=a, z=0) on the screen, and $\delta(q)$ is delta-function.

It should be emphasized here that (6), (7) and (8) give the closed form of exact solution of our problem. Unfortunately it isn't possible to simplify this solution if the phase on the screen changes on arbitrary law. So we shall consider one concrete example to show how the ”warm rays” method can be putted into practise.

3. RESULTS AND DISCUSSION

Let a plane wave falls on the screen with phase φ evenly distributed on the interval [-π,π]

$$k\Psi(x) = k\psi_0 \sin(\kappa x + \varphi). \tag{9}$$

Calculating in consecutive order $v_0(x)$, $\Phi(x,q,k)$ from (6) and $f_T(x,v)$ from (7), one can find the intensity $I(x,z)$. The additional assumption the wavelength λ is smaller than the smallest screen dimensions are allows to use the expansion of $\Phi(x,q,k)$ in a series with a parameter κ/k. The first-order approximation makes it possible to obtain the following form for the intensity field

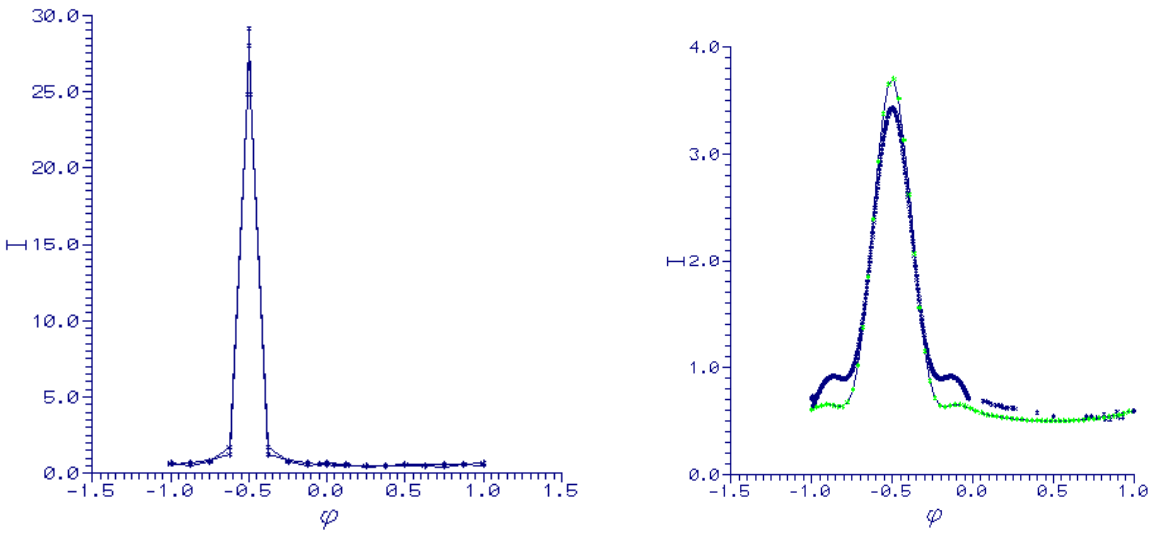
$$I(y,t) = I_0 \sum_{n=-\infty}^{\infty} i^n \exp[in(y+\varphi)] J_n(nt - n^3 t^3 \varepsilon), \tag{10}$$

where J_n is the Bessel function of n-th order, $y=\kappa x$ and $t=z\Psi_0\kappa^2$ are nondimensional coordinates. Notice that the plane t=1 is the plane where the gigantic peculiarities are concentrated in geometrical approximation. The most important feature of the last expression is the parameter

$$\varepsilon = 1/(24k^2\psi_0^2).$$

It characterizes a correlation between two contrary mechanisms: focusing and diffraction. We can remember the analogy of waves intensity and particles density behavior and then call this parameter the ”rays temperature”. The smaller ε is, the larger the intensity peculiarities are.

One can see it comparing the intensity fields with different ε. Since parameters y and φ possess equal rights in (10), we can examine the dependence of I on random phase φ in the fixed point (y=0, t=1). The choised point is interesting because of it is the region where strong focusing of the wave takes place in geometrical approximation, so $I(\varphi)$ is infinite under some φ. Here we study the influence of the “ray’s temperature” if ε=0 and ε=0.01.



(a) ε=0 (b) ε=0.01

Figure 2. Intensity field I (y=0,t=1) versus phase φ. Parameter I_0 =1. Exact data (dots) are plotted for comparison.

On Fig.2,*a* the intensity field in this point is plotted under the condition ε=0. The high-intensity spike on the curve corresponds to the strong focusing of rays in geometrical approximation. The influence of ε is shown on the Fig. 2,*b*. Clearly, Fig.2,*b* confirms our conclusion that the ε growth corresponds to diffraction destroys the spikes before the focusing begins.

To check our results the second curve is plotted on each figure. On the Fig.2,*a* it is the exact geometric optical solution

$$I_{geom}(y,t) = I_0 \sum_m \frac{1}{|1 - t\sin(b_m + y)|},$$

where b_m are the solutions of the following equation

$$y - b - t\cos(b + y) = 0.$$

Obviously it isn't possible to distinguish these curves. On the other hand the convenience of our screen model is it allows to find the exact intensity field from (1)

$$I_{exact}(y,t) = \left| J_0(1/\sqrt{24\varepsilon}) + \sum_{n=1}^{\infty} \exp\left[-in^2 t/(2\sqrt{24\varepsilon})\right] J_n(1/\sqrt{24\varepsilon})\left[\exp(iny) + (-1)^n \exp(-iny)\right]\right|.$$

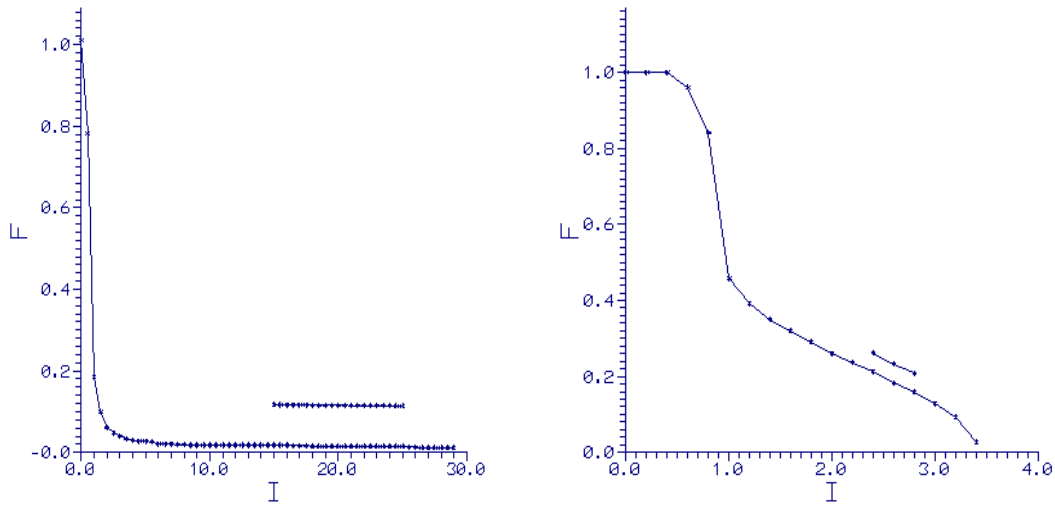
So, the exact field is also plotted on Fig.2,*b* and an important observation from this plot is that exact and our solutions coinside very closely. It means the adequacy of our method is good enough.

We can go now to study the probabilistic characteristics of the intensity. Notice that the most simple way is to calculate the integral probability function. Obviously it is equal to the relative amount of values $I(\varphi)$ for which the inequality $I(\varphi)>I$ is satisfied.

It can be shown [10] in geometrical optics approximation in the fixed space point the integral probability function $F(I)$ is inversely proportional to I:

$$F(I) \sim 1/I$$

and we have found numerically that the same result our method gives if $\varepsilon=0$ (Fig.3,*a*).



(a) ε=0, n=1 (b) ε=0.01, n=2

Figure 3. The integral probability function $F(I)$. The upper lines are approximating curves $F_{appr} \sim I^{-n}$.

Just this result means all higher statistical moments of intensity are unlimited. When the ε grows the diffraction influence is stronger and then faster distribution functions decreases. For example, using the data for intensity field $I(\varphi)$ when ε=0.01 we have computed that $F(I) \sim I^{-2}$ in a region of large I. This result is shown in Fig. 3,*b* (the removing lines on Fig. 3 exactly are approximating curves with powers -1 and -2 respectively). Hence, the larger ε is, the stronger statistical moments are limited. For example, the "warm rays" method helps to calculate the dispersion of intensity fluctuations. The hypothesis of statistical splitting of diffractional and geometrical averages [11] makes it possible to obtain the dispersion of intensity fluctuations σ_I^2 for Gaussian law of phase distribution on the screen in the form

$$\sigma_I^2(t) = \frac{1}{2\pi t}\int_{-\infty}^{\infty} dx d\mu \exp(ix\mu / t)\left\{\exp\left[-\mu^2 g_1(x) - \mu^4 \varepsilon g_2(x)\right] - 1\right\},$$

where

$$g_1(x) = 1 - \exp\left(-x^2/2\right), \qquad g_2(x) = 1 - \left(1 - x^2\right)\exp\left(-x^2/2\right).$$

One can verify this integral converges in every point t, it being found that if ε=0.01 then $(\sigma_I^2)_{max} = \sigma_I^2(t=1)=2.8$. Furthermore, the increase of ”ray’s temperature” ε causes the decrease of the intensity fluctuations.

Notice that all these results are obtained in the assumption λ is much more smaller than all typical dimensions on the screen so we only touch up the simplest geometrical approximation. Nevertheless the use of the ”effective temperature” allows to calculate the intensity behind a screen as well as to find the dispersion of intensity fluctuations and asimptotic law for the probability density function.

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