[1]SCATTERING OF RADIATION BY AN OBJECT LOCATED NEAR THE INTERFACE OF A NONSTATIONARY MEDIUM

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Abstract

The interaction of an electromagnetic field with a scatterer placed near a nonstationary composite medium using time domain techniques is examined. The composite medium is modeled by a nonstationary object located near a plane surface. The surface appears at a zero moment of time and breaks down homogeneous medium into two half- spaces in one of which the permittivity and conductivity changes abruptly at this moment of time. The problem is solved by a Volterra integral equation that is equivalent to an initial and boundary value problem for the Maxwell's electromagnetic equations. The integral equation is derived by means of a spatial-temporal Green's function, obtained for a piecewise homogeneous and time independent medium.

1. INTRODUCTION

Recently there is a strong interest in analyzing electromagnetic phenomena using time domain techniques. Such phenomena are important in geophysical subsurface probing, remote sensing, stealth technologies, measurement of electrical properties of dielectric substrates, hyperthermia used in cancer therapy, subsurface communication, etc. [1-10]. Other important applications of electromagnetic wave propagation and scattering in media having time-varying parameters are related to ionosphere and other media, control of dielectric-resonator antenna by parameter variation, time-varying surface conductivity that can affect the fields radiated and scattered from objects. Furthermore, with the continuing and increasing interest in semiconductor and optical integrated devices, more-accurate and realistic simulations of these ones are needed [11]. In all these structures electromagnetic problems concerning medium with time-varying parameters (permittivity, permeability, and conductivity) are encountered.

In this paper, a time domain analysis is pursued to treat the electromagnetic field in a nonstationary composite medium which has time dependent properties. The composite medium is modeled by a nonstationary object located near a plane surface. The surface appears at zero moment of time and breaks down a homogeneous medium into two half- spaces in one of which the permittivity and conductivity abruptly changes at this moment. The problem is solved by using a Volterra integral equation. This is equivalent to an initial and boundary value problem for the Maxwell's

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electromagnetic equations. The integral equation is derived by means of a spatial- time dependent Green's function corresponding to a piecewise homogeneous and time independentt medium.

It is shown that the specific form of the integral expression depends on the object location and does not depend on the half-space medium parameters variations. In case when the object is within the nonstationary half- space the equation for the field is the Volterra integral equation of the second kind. In the other half space the electromagnetic field is derived by integrating the internal field over the object region.

It is also shown that after the medium characteristics change and the surface creation a free term appears in the integral equation. This term is caused by remainder phenomena due to a medium "memory" that results from a time delay because of the limitation of the light velocity, and determines a field interaction with a medium until changing of permittivity and conductivity in one of the half-spaces.

2. STATEMENT OF THE PROBLEM

The problem being studied is stated as follows: let in a background medium which is described by a continuous vector functions of electrical as $\boldsymbol{P}_{ex}$ and magnetic $\boldsymbol{M}_{ex}$ polarizations there is a region *V(t)* and a subregion *U(t)* that contain different material media, as shown in Fig. 1. Consequently, the medium parameters have discontinuities on the boundaries of *V(t)* and *U(t).*

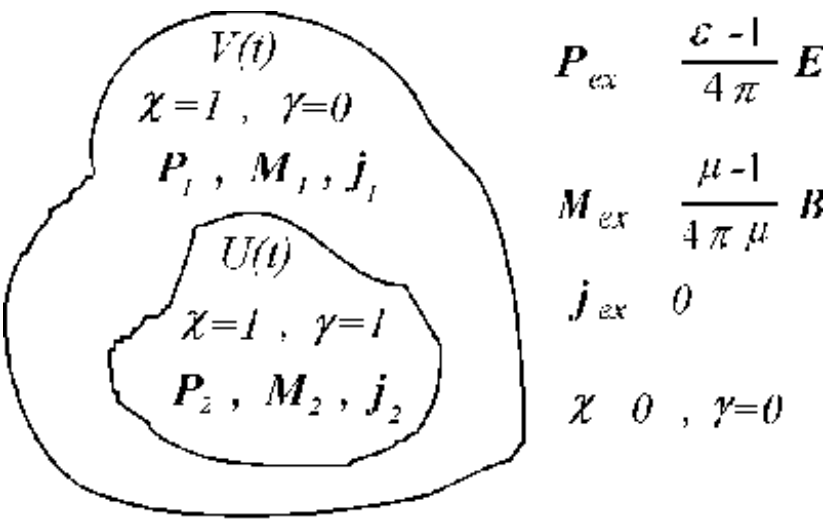

Figure 1. The problem geometry in general case.

An electromagnetic field in the whole space satisfies the Maxwell’s equations [12]

$$\left.\begin{aligned}&\nabla\times\boldsymbol{B}-\varepsilon_0\mu_0\frac{\partial\boldsymbol{E}}{\partial t}=\varepsilon_0\mu_0\frac{\partial\boldsymbol{P}}{\partial t}+\mu_0\nabla\times\boldsymbol{M}+\mu_0\boldsymbol{j},\\&\nabla\times\boldsymbol{E}+\frac{\partial\boldsymbol{B}}{\partial t}=0.\end{aligned}\right\}\tag{1}$$

where $\boldsymbol{P}$ and $\boldsymbol{M}$ are generalized electrical and magnetic polarization vectors of a medium, $\boldsymbol{j}$ is a generalized current density, ε_0 and μ_0 are the electrical and magnetic permittivities of vacuum. The Maxwell equations in (1) can be written in a generalized form after ref. [13] as follows:

$$\frac{\partial\boldsymbol{a}}{\partial t}=\left\{\frac{\partial\boldsymbol{a}}{\partial t}\right\}-\left[\boldsymbol{a}\right]_s v_n\delta(S(t)),$$
$$\nabla\times\boldsymbol{a}=\left\{\nabla\times\boldsymbol{a}\right\}+\hat{\boldsymbol{n}}\times\left[\boldsymbol{a}\right]_s\delta(S(t))$$

where $\{\ \}$ denotes classical derivative at that point where it has sense, $\delta(S)$ is a surface delta-function, *S(t)* is the discontinuity surface of the field, v_n is the normal (transverse) component of the velocity of a given surface region, $\hat{\boldsymbol{n}}$ is a normal vector to a surface *S(t)*, $\left[\boldsymbol{a}\right]_s$ is a jump of the discontinuous vector function $\boldsymbol{a}$ on a surface *S(t)*.

The equations in (1) involve the boundary conditions for fields at all discontinuity surfaces, so generalized solutions of these equations meet these conditions [14].

Assume that the background is described by following polarization vectors.

$$\boldsymbol{P}_{ex}=\varepsilon_0(\varepsilon-1)\boldsymbol{E},\ \boldsymbol{M}_{ex}=\frac{1}{\mu_0}\left(1-\frac{1}{\mu}\right)\boldsymbol{B},\tag{2}$$

and the corresponding polarization vectors at the subregions *V(t)\U(t)* and *U(t)* are described by the vectors $\boldsymbol{P}_1,\boldsymbol{M}_1,\boldsymbol{j}_1$ and $\boldsymbol{P}_2,\boldsymbol{M}_2,\boldsymbol{j}_2$ respectively. Then the generalized functions describing the medium influense on the fields in the whole space can be written as follows:

$$\begin{aligned}&\boldsymbol{P}=\gamma(\boldsymbol{P}_2-\boldsymbol{P}_1)+\chi(\boldsymbol{P}_1-\boldsymbol{P}_{\text{ex}})+\boldsymbol{P}_{\text{ex}},\\&\boldsymbol{M}=\gamma(\boldsymbol{M}_2-\boldsymbol{M}_1)+\chi(\boldsymbol{M}_1-\boldsymbol{M}_{\text{ex}})+\boldsymbol{M}_{\text{ex}},\\&\boldsymbol{j}=\gamma(\boldsymbol{j}_2-\boldsymbol{j}_1)+\chi\boldsymbol{j}_1+\boldsymbol{j}_{\text{extr}}\end{aligned}\tag{3}$$

where χ and γ are defined in Fig.1, $\boldsymbol{j}_{extr}$ is the external source current.

From Eqs. (1) and (3) it follows that an electromagnetic field satisfies the generalized inhomogeneous wave equation

$$\begin{aligned}&\nabla\times(\nabla\times\boldsymbol{E})+\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\varepsilon\mu\boldsymbol{E}+\frac{1}{c^2\varepsilon_0}\frac{\partial^2}{\partial t^2}\mu\chi\left(\boldsymbol{P}_1-\boldsymbol{P}_{ex}\right)+\frac{1}{c^2\varepsilon_0}\frac{\partial}{\partial t}\mu\nabla\times\chi\left(\boldsymbol{M}_1-\boldsymbol{M}_{ex}\right)+\\&+\frac{1}{c^2\varepsilon_0}\frac{\partial}{\partial t}\chi\mu\boldsymbol{j}_1=\boldsymbol{F}_2-\frac{1}{c^2\varepsilon_0}\frac{\partial}{\partial t}\mu\boldsymbol{j}_{extr}\end{aligned}\tag{4}$$

where $c=1/\sqrt{\varepsilon_0\mu_0}$ is the light velocity in vacuum. The left-hand side of this equation takes into account also the boundary conditions on the boundary of *V(t)*. The right-hand side term

$$\boldsymbol{F}_2 = -\frac{4\pi}{c^2}\frac{\partial^2}{\partial t^2}\mu\gamma\left(\boldsymbol{P}_2 - \boldsymbol{P}_1\right) - \frac{4\pi}{c}\frac{\partial}{\partial t}\mu\, rot\,\gamma\left(\boldsymbol{M}_2 - \boldsymbol{M}_1\right) - \frac{4\pi}{c^2}\frac{\partial}{\partial t}\mu\gamma\left(\boldsymbol{j}_2 - \boldsymbol{j}_1\right) \tag{5}$$

has also a discontinuity on the boundary of *U(t)*.

The differential equation (4) is transformed into an integral equation by means of the Green's function *G* [13]

$$\boldsymbol{E} = \boldsymbol{E}_0' + G \otimes \left(\boldsymbol{F}_2 - \frac{1}{c^2\varepsilon_0}\frac{\partial}{\partial t}\boldsymbol{j}_{extr}\right). \tag{6}$$

Here, the sign $\otimes$ denotes a convolution in the four-dimensional space-time, $\boldsymbol{a} \otimes \boldsymbol{b} = \int_{\infty} d\boldsymbol{x}'\, \boldsymbol{a}(\boldsymbol{x} - \boldsymbol{x}')\boldsymbol{b}(\boldsymbol{x}')$, where $\boldsymbol{x} = (t, \boldsymbol{r})$. The term $\boldsymbol{E}_0'$ is the solution of (4) with zero right-hand side, that is the field in a medium whose parameters have discontinuity on the boundary of *V(t)* only. Therefore the field $\boldsymbol{E}_0'$ satisfies the boundary conditions on the boundary, and presents an electric field in the piecewise homogeneous medium without an object *U(t)*.

Then equation (6) can be rewritten as follows

$$\boldsymbol{E} = \boldsymbol{E}_0 + G \otimes \boldsymbol{F}_2, \tag{7}$$

Where $\boldsymbol{E}_0 = \boldsymbol{E}_0' - \frac{1}{c^2\varepsilon_0}\mathrm{G}\otimes\frac{\partial}{\partial t}\boldsymbol{j}_{extr}$, and the second item in the right-hand side is a field determined by an external source current $\boldsymbol{j}_{extr}$. To write an explicit form of equation (7) one needs the Green's function corresponding to Eq. (4).

3. GREEN'S FUNCTION FOR A PROBLEM WITH PLANE INTERFACE THAT HAS COME INTO BEING

It is known, that in general terms the Green's function in Eq (4) pocesses an ambiguity since an arbitrary solution of the homogeneous equation can be added to it. This solution is presented in (6) by the item $\boldsymbol{E}_0'$, so in the following only the singularity part of the Green's function is needed. This part satisfies the equation

$$\begin{aligned}&\nabla\times(\nabla\times G) + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\varepsilon\mu G + \frac{1}{c^2\varepsilon_0}\frac{\partial^2}{\partial t^2}\mu\chi_1\left(\hat{P}_1 - \hat{P}_{ex}\right)G + \frac{1}{c^2\varepsilon_0}\frac{\partial}{\partial t}\mu\nabla\times\chi_1\left(\hat{M}_1 - \hat{M}_{ex}\right)G + \\ &+\frac{1}{c^2\varepsilon_0}\frac{\partial}{\partial t}\chi_1\,\mu\,\hat{j}_1 G = \hat{I}\delta\left(\boldsymbol{x} - \boldsymbol{x}'\right)\end{aligned} \tag{8}$$

Here, $\hat{P}, \hat{M}, \hat{j}$ are operators corresponding to functions $\boldsymbol{P}$, $\boldsymbol{M}$ and $\boldsymbol{j}$, $\hat{I}$ is an identical operator, $\delta(x)$ - the Dirac delta-function.

As equation (8) takes into account the boundary conditions at the surface of the region *V(t)*, the *G* function beeing also a solution of it, also satisfies these conditions.

In solving equation (8) it is transformed into an integral equation whose solution will be achieved by virtue of a resolvent method. To do end the terms with discontinuity coefficients, are rearranged to the right-hand side of the equation [15]:

$$\nabla \times (\nabla \times G) + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\varepsilon\mu G = F_1, \tag{9}$$

where

$$F_1 = -\frac{\mu}{c^2\varepsilon_0}\left[\frac{\partial^2}{\partial t^2}\chi\left(\hat{P}_1 - \hat{P}_{ex}\right) + \frac{\partial}{\partial t}\nabla \times \chi\left(\hat{M}_1 - \hat{M}_{ex}\right) + \frac{\partial}{\partial t}\chi\,\hat{j}_1\right]G + \hat{I}\delta(\boldsymbol{x} - \boldsymbol{x}') \tag{10}$$

Then from (9) the function G can be computed as a convolution $G = G_0 * F_1$, where G_0 is a Green's function related to equation (9). As differentiation with respect to time can be transferred to G_0, this convolution takes the form

$$G = G_0 - \frac{\mu}{c^2\varepsilon_0}\frac{\partial^2 G_0}{\partial t^2} * \chi\left(\hat{P}_1 - \hat{P}_{ex}\right)G - \frac{\mu}{c^2\varepsilon_0}\frac{\partial G_0}{\partial t} * \left[\nabla \times \chi\left(\hat{M}_1 - \hat{M}_{ex}\right) + \chi\,\hat{j}_1\right]G. \tag{11}$$

From here on, the case when the region $V(t)$ originates with a zero moment of time is considered. This means that, $\hat{P}_1 = \hat{P}_{ex}$, $\hat{M}_1 = \hat{M}_{ex}$ $\hat{j}_1 = 0$ for $t<0$. Then equation (11) can be converted into an integral equation for $t \geq 0$:

$$G_1 = G_0 + \hat{K}G_1, \tag{12}$$

where the Green's function for $t \geq 0$ is denoted as G_1, so $G = \theta(-t)G_0 + \theta(t)G_1$. The operator in (12) is

$$\hat{K} = -\frac{\mu}{c^2\varepsilon_0}\int d\boldsymbol{x}'\left\{\frac{\partial^2 G_0(\boldsymbol{x} - \boldsymbol{x}')}{\partial t^2}\chi(\boldsymbol{x}')\left(\hat{P}_1 - \hat{P}_{ex}\right) + \right.$$
$$\left. + \frac{\partial G_0(\boldsymbol{x} - \boldsymbol{x}')}{\partial t}\left[\nabla' \times \chi(\boldsymbol{x}')\left(\hat{M}_1 - \hat{M}_{ex}\right) + \chi(\boldsymbol{x}')\,\hat{j}_1\right]\right\}. \tag{13}$$

In above eguation, the integral $\int d\boldsymbol{x}'$ is over the four-dimensional half-space $t \geq 0$.

The $\hat{K}$ operator properties are determined by properties of the function G_0 which is the Green's function of the wave equation (9) assotiated with the background,

$$\nabla \times (\nabla \times G) + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\varepsilon\mu G = \hat{I}\delta(\boldsymbol{x} - \boldsymbol{x}'). \tag{14}$$

Then one can easily take the singular part of the Green's function of equation (14) because a regular part of this function gives a solution corresponding a homogeneous equation for the field, that is a field in the background. However this field is presented by the item $\boldsymbol{E}'_0$ in (6) and one does not need to be repeat it. With these remarks we can take the known function:

$$G_0 = -\frac{v^2}{4\pi}\left(\nabla\nabla - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right)\frac{1}{|\boldsymbol{r} - \boldsymbol{r}'|}\left(t - t' - \frac{|\boldsymbol{r} - \boldsymbol{r}'|}{v}\right)\theta\left(t - t' - \frac{|\boldsymbol{r} - \boldsymbol{r}'|}{v}\right), \tag{15}$$

where $v = \frac{c}{\sqrt{\varepsilon\mu}}$ and $\theta(t)$ is the Heaviside unit function.

The carrier of the function G_0 is the light cone for t'<t with the vertex at the observation point $(t,\boldsymbol{r})$. As this function has an integrable singularity in the four-dimensional space, the operator $\hat{K}$ is a Volterra integral operator. The specific form of the $\hat{R}$ operator is obtained by substituting Eq. (15) into Eq. (13)

$$\hat{K} = \frac{1}{4\pi}\int d\boldsymbol{x}'\left[(\nabla\nabla - \frac{1}{v^2}\frac{\partial^2}{\partial t^2})\frac{1}{\varepsilon\varepsilon_0}\left\{\frac{\partial}{\partial t}\chi(\boldsymbol{x}')\left(\hat{P}_1 - \hat{P}_{ex}\right) + \chi(\boldsymbol{x}')\hat{j}_1\right\} - \right.$$
$$\left. -\mu\mu_0\frac{\partial^2}{\partial t^2}\nabla\times\chi(\boldsymbol{x}')\left(\hat{M}_1 - \hat{M}_{ex}\right)\right] f(\boldsymbol{x}-\boldsymbol{x}') \tag{16}$$

where $f(\boldsymbol{x}-\boldsymbol{x}') = \frac{1}{|\boldsymbol{r}-\boldsymbol{r}'|}\theta(t - t' - \frac{|\boldsymbol{r}-\boldsymbol{r}'|}{v})$.

The meaning of the expressions (7, 12) essentially depends on the observation point $\boldsymbol{r}$ according to the general theory of the integral equation method as in Ref. [16]. Inside the region *V(t)*, (χ=1), the expression (12) is the Volterra integral equation of the second kind and its solution can be obtained by the resolvent $\hat{R}$ [14]: $G_1^{(in)} = \chi G_0 + \chi\hat{R}\chi G_0$. Outside this region, (1-χ=1), the expression (12) is a quadrature: $G_1^{(ex)} = (1-\chi)G_0 + \hat{K}^{(ex)}\left(\hat{I} + \chi\hat{R}\right)\chi G_0$, where superscript (*ex*) on the operator $\hat{K}$ denotes that the argument $\boldsymbol{r}$ is checked outside the region *V(t)*. So, the expression for the desired Green's function G_1 can be defined by a formula

$$G_1 = G_0 + \chi\hat{R}\chi G_0 + (1-\chi)\hat{K}^{(ex)}\left(\hat{I} + \chi\hat{R}\right)\chi G_0. \tag{17}$$

In the following analysis specific construction of the Green's function is carried out when *V(t)* is a half-space bounded with a plane, that is $\chi = \theta(x)$, as shown in Fig. 2. The medium in this half-space is a lossy dielectric. This implies that for *t>0*

$$\hat{P}_1 = \varepsilon_0(\varepsilon_1 - 1),\ \hat{M}_1 = \hat{M}_{ex} = 0,\ \hat{j}_1 = \sigma_1 \tag{18}$$

The medium permeability in the whole space is supposed to be μ=1.

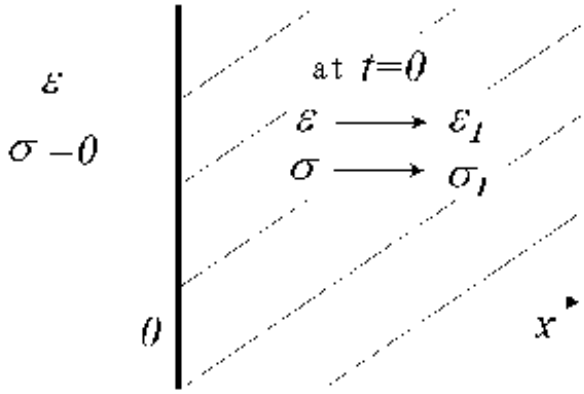

Figure 2. Time jumps of medium parameters in one half-space.

The operators in Eq. (17) are integral operators: $\hat{A} = \int d\boldsymbol{x}' \left\langle \boldsymbol{x} \middle| \hat{A} \middle| \boldsymbol{x}' \right\rangle$. Hereafter, the kernels of these operators are represented as inverse Fourier-Laplace transformations

$\left\langle \boldsymbol{x} \middle| \hat{A} \middle| \boldsymbol{x}' \right\rangle = \int d\boldsymbol{p} e^{pt + i\boldsymbol{kr}} \left\langle \boldsymbol{p} \middle| \hat{A} \middle| \boldsymbol{x}' \right\rangle$, where $\boldsymbol{p} = (p, \boldsymbol{k})$, $\boldsymbol{k} = (k_1, k_2, k_3)$, and

$$\int d\boldsymbol{p} = \int_{-i\infty}^{+i\infty} \frac{dp}{2\pi i} \int_{-\infty}^{\infty} \frac{d\boldsymbol{k}}{(2\pi)^3}, \quad Re\, p > 0.$$

Then from (16) it follows that the kernel of the operator $\hat{K}$ is equal to:

$$\left\langle \boldsymbol{p} \middle| \hat{K} \middle| \boldsymbol{x}' \right\rangle = \frac{v_1^2 p - v^2 (p + \overline{\sigma})}{v_1^2 p} \frac{v^2 Q + p^2 I}{p^2 + v^2 k^2} e^{-pt' - i\boldsymbol{kr}'} \theta(x'), \tag{19}$$

where $Q = \begin{pmatrix} k_1^2 & k_1 \boldsymbol{k}_\perp \\ k_1 \boldsymbol{k}_\perp^* & \boldsymbol{k}_\perp^* \boldsymbol{k}_\perp \end{pmatrix}$, $\boldsymbol{k}_\perp = (k_2, k_3)$, $\boldsymbol{k}_\perp^* = \begin{pmatrix} k_2 \\ k_3 \end{pmatrix}$, I - the unit matrix,

$v_1 = \dfrac{c}{\sqrt{\varepsilon_1 \varepsilon_0 \mu_0}}$, $\overline{\sigma} = \dfrac{\sigma_1}{\varepsilon_1 \varepsilon_0}$.

The resolvent operator $\hat{R}$ for a lossy dielectric half-space is constructed in [6, 17] and its kernel has the form:

$$\left\langle \boldsymbol{p} \middle| \hat{R} \middle| \boldsymbol{x}' \right\rangle = \frac{v_1^2 p - v^2 (p + \overline{\sigma})}{v^2 (p + \overline{\sigma})} \theta(x) \left\{ \frac{v_1^2 Q + p(p + \overline{\sigma}) I}{\varphi_1^2 + v_1^2 k_1^2} e^{-ik_1 x'} - \right.$$
$$\left. - \frac{1}{2\varphi_1} \left[\frac{v_1^2 Y_1 + p(p + \overline{\sigma}) I_\perp}{\varphi_1 - i v_1 k_1} + \frac{X}{\varphi_1 + i v_1 k_1} \right] e^{-\frac{\varphi_1}{v_1} x'} \right\} e^{-pt' - i\boldsymbol{k}_\perp \boldsymbol{r}_\perp'} \theta(x'), \tag{20}$$

where $Y_1 = \begin{pmatrix} -k_\perp^2 & -i \dfrac{\varphi_1}{v_1} \boldsymbol{k}_\perp \\ -i \dfrac{\varphi_1}{v_1} \boldsymbol{k}_\perp^* & \boldsymbol{k}_\perp^* \boldsymbol{k}_\perp \end{pmatrix}$, $I_\perp = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$,

$$X = -v_1^2 V_m \begin{pmatrix} k_\perp^2 & i\frac{\varphi_1}{v_1}\boldsymbol{k}_\perp \\ -i\frac{\varphi_1}{v_1}\boldsymbol{k}_\perp^* & \boldsymbol{k}_\perp^*\boldsymbol{k}_\perp \end{pmatrix} + 2vv_1 V_{em}\begin{pmatrix} 0 & 0 \\ 0 & \boldsymbol{k}_\perp^*\boldsymbol{k}_\perp \end{pmatrix} + p(p+\overline{\sigma})V_e I_\perp, \tag{21}$$

$$V_m = \frac{v\varphi(p+\overline{\sigma}) - v_1\varphi_1 p}{v\varphi(p+\overline{\sigma}) + v_1\varphi_1 p}, \quad V_{em} = (p+\overline{\sigma})\frac{v_1\varphi - v\varphi_1}{v\varphi(p+\overline{\sigma}) + v_1\varphi_1 p}, \quad V_e = \frac{v_1\varphi - v\varphi_1}{v_1\varphi + v\varphi_1},$$

$$\varphi = \sqrt{p^2 + v^2 k_\perp^2}, \quad \varphi_1 = \sqrt{p(p+2\overline{\sigma}) + v_1^2 k_\perp^2}, \quad Re\,\varphi, \varphi_1 > 0, \quad \boldsymbol{r}_\perp = (y,z).$$

After lenghthy operations and some rearrangements we have for $t>0$:

$$\begin{aligned} G_1 = \theta(x)\{[G_{11}\theta(x-x') + G_{12}\theta(x'-x) - S_1]\theta(x') + W_1\theta(-x')\} + \\ +\theta(-x)\{W_2\theta(x') + [S_2 + G_{01}\theta(x-x') + G_{02}\theta(x'-x)]\theta(-x')\} \end{aligned} \tag{22}$$

Here,

$$G_{11}(x-x') = \hat{L}\frac{v_1}{2\varphi_1 p(p+\overline{\sigma})}\left(v_1^2\begin{pmatrix} -k_\perp^2 & i\frac{\varphi_1}{v_1}\boldsymbol{k}_\perp \\ i\frac{\varphi_1}{v_1}\boldsymbol{k}_\perp^* & \boldsymbol{k}_\perp^*\boldsymbol{k}_\perp \end{pmatrix} + p(p+\overline{\sigma})I_\perp\right)e^{-\frac{\varphi_1}{v_1}(x-x')},$$

$$G_{12}(x-x') = \hat{L}\frac{v_1}{2\varphi_1 p(p+\overline{\sigma})}\left(v_1^2\begin{pmatrix} -k_\perp^2 & -i\frac{\varphi_1}{v_1}\boldsymbol{k}_\perp \\ -i\frac{\varphi_1}{v_1}\boldsymbol{k}_\perp^* & \boldsymbol{k}_\perp^*\boldsymbol{k}_\perp \end{pmatrix} + p(p+\overline{\sigma})I_\perp\right)e^{\frac{\varphi_1}{v_1}(x-x')},$$

$$S_1(x+x') = \hat{L}\frac{v_1}{2\varphi_1 p(p+\overline{\sigma})}X\, e^{-\frac{\varphi_1}{v_1}(x+x')},$$

$$W_1(x,x') = \hat{L}\frac{v}{2\varphi p^2}\left[vv_1 T_m\begin{pmatrix} -k_\perp^2 & i\frac{\varphi}{v}\boldsymbol{k}_\perp \\ i\frac{\varphi_1}{v_1}\boldsymbol{k}_\perp^* & \boldsymbol{k}_\perp^*\boldsymbol{k}_\perp \end{pmatrix} + p^2 T_e I_\perp\right]e^{-\frac{\varphi_1}{v_1}x + \frac{\varphi}{v}x'},$$

$$W_2(x,x') = \hat{L}\frac{v}{2\varphi p^2}\left[vv_1 T_m\begin{pmatrix} -k_\perp^2 & -i\frac{\varphi_1}{v_1}\boldsymbol{k}_\perp \\ -i\frac{\varphi}{v}\boldsymbol{k}_\perp^* & \boldsymbol{k}_\perp^*\boldsymbol{k}_\perp \end{pmatrix} + p^2 T_e I_\perp\right]e^{\frac{\varphi}{v}x - \frac{\varphi_1}{v_1}x'}, \tag{23}$$

$$S_2(x+x') = \hat{L}\frac{v}{2\varphi p^2}X^{(ex)}e^{\frac{\varphi}{v}(x+x')},$$

where $\hat{L} = \int d\boldsymbol{p}_\perp e^{p(t-t') + i\boldsymbol{k}_\perp(\boldsymbol{r}_\perp - \boldsymbol{r}'_\perp)} = \int_0^\infty \frac{dk_\perp}{2\pi}k_\perp J_0(k_\perp|\boldsymbol{r}_\perp - \boldsymbol{r}'_\perp|\int_{-i\infty}^{i\infty}\frac{dp}{2\pi i}e^{p(t-t')}$, $\boldsymbol{p}_\perp = (p, \boldsymbol{k}_\perp)$,

$$X^{(ex)} = -v^2 V_m\begin{pmatrix} k_\perp^2 & -i\frac{\varphi}{v}\boldsymbol{k}_\perp \\ i\frac{\varphi}{v}\boldsymbol{k}_\perp^* & \boldsymbol{k}_\perp^*\boldsymbol{k}_\perp \end{pmatrix} + 2vv_1\frac{p}{p+\overline{\sigma}}V_{em}\begin{pmatrix} 0 & 0 \\ 0 & \boldsymbol{k}_\perp^*\boldsymbol{k}_\perp \end{pmatrix} + p(p+\overline{\sigma})V_e I_\perp,$$

$$T_m = \frac{2v_1\varphi p}{v\varphi(p+\overline{\sigma}) + v_1\varphi_1 p}, \quad T_e = \frac{2v_1\varphi}{v_1\varphi + v\varphi_1}. \tag{24}$$

The background Green's function can be presented as two items also

$$G_0 = G_{01}\,\theta(x-x') + G_{02}\,\theta(x'-x),$$

where

$$G_{01}(x-x') = \hat{L}\frac{v}{2\varphi p(p+\overline{\sigma})}\left(v^2\begin{pmatrix} -k_\perp{}^2 & i\frac{\varphi}{v}\boldsymbol{k}_\perp \\ i\frac{\varphi}{v}\boldsymbol{k}_\perp^* & \boldsymbol{k}_\perp^*\boldsymbol{k}_\perp \end{pmatrix} + p(p+\overline{\sigma})I_\perp\right)e^{-\frac{\varphi}{v}(x-x')},$$

$$G_{02}(x-x') = \hat{L}\frac{v}{2\varphi p(p+\overline{\sigma})}\left(v^2\begin{pmatrix} -k_\perp{}^2 & -i\frac{\varphi}{v}\boldsymbol{k}_\perp \\ -i\frac{\varphi}{v}\boldsymbol{k}_\perp^* & \boldsymbol{k}_\perp^*\boldsymbol{k}_\perp \end{pmatrix} + p(p+\overline{\sigma})I_\perp\right)e^{\frac{\varphi}{v}(x-x')}, \tag{25}$$

The arrangement of Green's function G_1 items of on (x,x') plane is shown in Fig. 3.

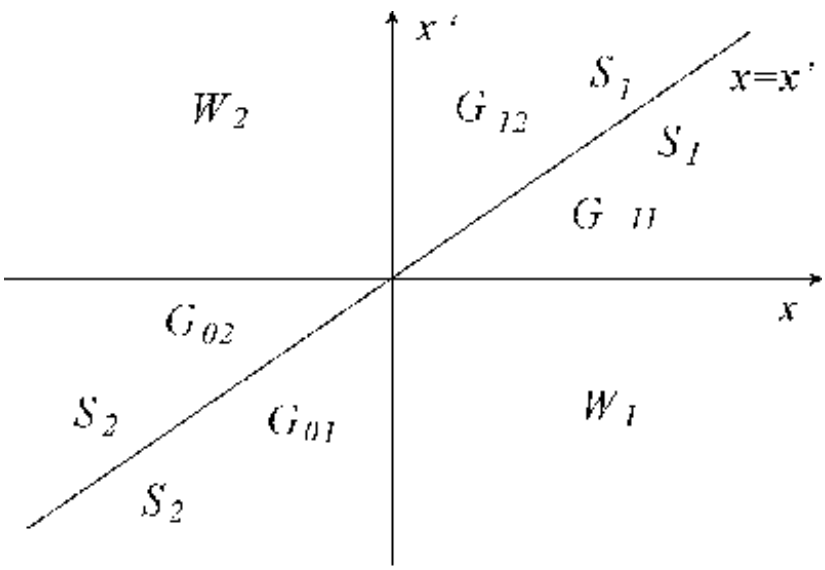

Figure 3. The arrangement of Green's function terms on the plane (x,x').

All terms of the function G_1 equal to zero for t-$t'<0$ because of their analytic properties. For $v_1 \to v$ the function G_1 transfers to G_0 as a limit case which gives: $S_1 = S_2 = 0, \quad G_{11} = W_1 = G_{01}, \quad G_{12} = W_2 = G_{02}$.

From Eqs. (23) and (25) the properties of the Green's functions G_1 on discontinuity lines are observed. On the line x=x' the functions jump is:

$$[G_0] = G_{01} - G_{02} = \hat{L}\frac{iv^2}{p(p+\overline{\sigma})}\begin{pmatrix} 0 & \boldsymbol{k}_\perp \\ \boldsymbol{k}_\perp^* & \hat{0} \end{pmatrix}, \; [G_1] = G_{11} - G_{12} = \hat{L}\frac{iv_1{}^2}{p(p+\overline{\sigma})}\begin{pmatrix} 0 & \boldsymbol{k}_\perp \\ \boldsymbol{k}_\perp^* & \hat{0} \end{pmatrix} \tag{26}$$

On line x'=0 the G_1 jump is equal:

for x>0

$$[G_1] = G_{11} + S_1 - W_1 = \hat{L}\frac{[v^2(p+\overline{\sigma}) - v_1{}^2 p]v_1}{2\varphi p(p+\overline{\sigma})} T_m \begin{pmatrix} k_\perp^2 & 0 \\ -i\dfrac{\varphi_1}{v_1}\boldsymbol{k}_\perp^* & \hat{0} \end{pmatrix} \tag{27}$$

for x<0

$$[G_1] = W_2 - G_{02} - S_2 = \hat{L}\frac{[v^2(p+\overline{\sigma}) - v_1{}^2 p]v^2}{2\varphi p^3 v_1} T_m \begin{pmatrix} k_\perp^2 & 0 \\ i\dfrac{\varphi}{v}\boldsymbol{k}_\perp^* & \hat{0} \end{pmatrix} \tag{28}$$

On line x=0 the G_1 jump is equal:

for x'>0

$$[G_1] = G_{12} + S_1 - W_2 = \hat{L}\frac{(v^2 - v_1{}^2)v_1}{2\varphi p^2} T_m \begin{pmatrix} k_\perp^2 & i\dfrac{\varphi_1}{v_1}\boldsymbol{k}_\perp \\ 0 & \hat{0} \end{pmatrix} \tag{29}$$

for x'<0

$$[G_1] = W_1 - G_{01} - S_2 = \hat{L}\frac{(v^2 - v_1{}^2)v^2}{2\varphi p^2 v_1} T_m \begin{pmatrix} k_\perp^2 & -i\dfrac{\varphi}{v}\boldsymbol{k}_\perp \\ 0 & \hat{0} \end{pmatrix} \tag{30}$$

From the properties of the obtained Green's function on the discontinuity lines it is follows: that the action of the Green's function on any vector gives a vector that has discontinuity on the line $x'=0$, but this discontinuity is caused by an initial vector component that is normal to an interface between media. On the line $x=0$ only a resulting vector component that is normal to the boundary has a discontinuity.

4. INTEGRAL EQUATIONS

As it follows from the equations (5) and (7) in the case of nonmagnetic medium $(\mu = 1)$, the electric field satisfies to the equation

$$\boldsymbol{E} = \boldsymbol{E}_0 - \frac{1}{c^2\varepsilon_0} G \otimes \left\{ \frac{\partial^2}{\partial t^2}\gamma(\boldsymbol{P}_2 - \boldsymbol{P}_1) + \frac{\partial}{\partial t}\gamma(\boldsymbol{j}_2 - \boldsymbol{j}_1) \right\}. \tag{31}$$

where $G = \theta(-t)G_0 + \theta(t)G_1$.

The obtained Green's function allows us to write out in an explicit form the expression that governs the electric field in the case when the object $U(t)$ is situated in the lossy dielectric medium, that from zero moment of time is divided by a plane boundary into two half-spaces. In one of them the permittivity and conductivity change in time abruptly. The permittivity in both half-spaces was the same and equals ε until $t=0$. At $t=0$ it changes by jump to ε_1 in the half-space $x>0$. The conductivity appears by jump from zero at $t=0$ in this half-space and its value is σ_1. Then the general integral equation for the electric field is followed from (31):

$$\boldsymbol{E} = \boldsymbol{E}_0 -$$
$$-\theta(-t)\frac{1}{c^2\varepsilon_0}\frac{\partial}{\partial t}G_0 \otimes \left\{\frac{\partial}{\partial t}\gamma(\boldsymbol{P}_2 - \boldsymbol{P}_{ex}) + \gamma\boldsymbol{j}_2\right\} - \theta(t)\frac{1}{c^2\varepsilon_0}\frac{\partial}{\partial t}G_1 \otimes \theta(-t)\left\{\frac{\partial}{\partial t}\gamma(\boldsymbol{P}_2 - \boldsymbol{P}_{ex}) + \gamma\boldsymbol{j}_2\right\} - \quad (32)$$
$$-\theta(t)\frac{1}{c^2\varepsilon_0}\frac{\partial}{\partial t}G_1 \otimes \theta(t)\left\{\frac{\partial}{\partial t}\gamma(\boldsymbol{P}_2 - \boldsymbol{P}_1) + \gamma(\boldsymbol{j}_2 - \boldsymbol{j}_1)\right\}.$$

It is worth noting that the second and third items in this expression are determined by vectors $(\boldsymbol{P}_2 - \boldsymbol{P}_{ex})$ and $\boldsymbol{j}_2$ for $t<0$.

As it follows from Eq. (32) the field for $t<0$ satisfies the integral equation

$$\boldsymbol{E} = \boldsymbol{E}_0 - \theta(-t)\frac{1}{c^2\varepsilon_0}\int_{-\infty}^{t} dt' \int_{\infty} d\boldsymbol{r}' \frac{\partial}{\partial t}G_0\left[\frac{\partial}{\partial t'}\gamma\left(\boldsymbol{P}_2 - \boldsymbol{P}_{ex}\right) + \gamma\boldsymbol{j}_2\right] \quad (33)$$

This is a Volterra integral equation that describes a problem for a nonstationary object $U(t)$ in an unbounded stationary space. The field obtained from this equation will be determined the third item in (32).

After the surface appearence, for $t\geq 0$ in the integral equation appears a free term been due to remainder phenomena. These phenomena are caused by a medium "memory" that results from a time delay because of the limitation of the light velocity, and determines a field interaction with a medium until changing of parameters in the nonstationary half-space. Consequently, this item can be included in the free item of equation (32) which for $t>0$ takes the form

$$\boldsymbol{E} = \boldsymbol{F} -$$
$$-\theta(t)\frac{1}{c^2\varepsilon_0}\frac{\partial G_1}{\partial t} \otimes \theta(t)\left\{\frac{\partial}{\partial t}\gamma(\boldsymbol{P}_2 - \boldsymbol{P}_1) + \gamma(\boldsymbol{j}_2 - \boldsymbol{j}_1)\right\}, \quad (34)$$

where

$$\boldsymbol{F} = \boldsymbol{E}_0 - \theta(t)\frac{1}{c^2\varepsilon_0}\int_{-\infty}^{0} dt' \int_{\infty} d\boldsymbol{r}' \frac{\partial G_1}{\partial t}\left[\frac{\partial}{\partial t'}\gamma\left(\boldsymbol{P}_2 - \boldsymbol{P}_{ex}\right) + \gamma\boldsymbol{j}_2\right] \quad (35)$$

If the object $U(t)$ appears after the moment $t=0$, that is $\boldsymbol{P}_2 = \boldsymbol{P}_{ex}$ when $t<0$, then the free item in (34) is equal to an undisturbed field in the background, $\boldsymbol{F} = \boldsymbol{E}_0$.

The form of the free term and the equations essentially depend on in what half-space a field is considered and the object $U(t$ is situated.

Case A. The scatterer object *U(t)* is placed inside the nonstationary half-space.

The placement of the object $U(t)$ is shown in Fig. 4. The integral equation at the nonstationary half-space has the form

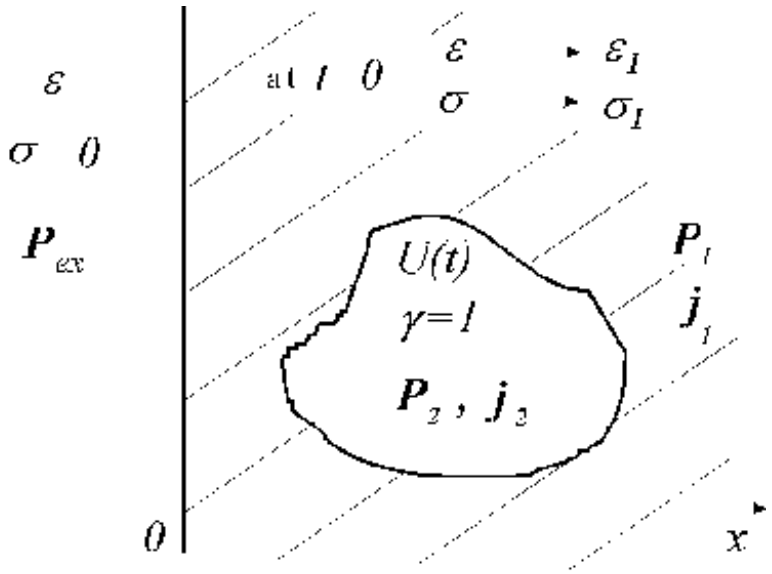

Figure 4. The placement of the object *U(t)* inside the nonstationary half-space.

$$\boldsymbol{E} = \boldsymbol{F} - \theta(x)\frac{1}{c^2\varepsilon_0}\int_0^t dt' \int_{U(t')} d\boldsymbol{r}' \left\{\frac{\partial G_{11}}{\partial t} + \frac{\partial G_{12}}{\partial t} - \frac{\partial S_1}{\partial t}\right\}\left[\frac{\partial}{\partial t'}\gamma\left(\boldsymbol{P}_2 - \boldsymbol{P}_1\right) + \gamma(\boldsymbol{j}_2 - \boldsymbol{j}_1)\right] \tag{36}$$

If the observation point is located in the region *U(t)* this expression is a Volterra integral equation of the second kind. If the observation point is located outside of the region *U(t)*, this formula allows to determine the field in the half-space $x>0$ through integrating of the internal field over *U(t)*.

In the other half-space, $x<0$, the scattered field is determined by the formula via integrating of the internal field over *U(t)* with the item W_2 of the Green's function

$$\boldsymbol{E} = \boldsymbol{F} - \theta(-x)\frac{1}{c^2\varepsilon_0}\int_0^t dt' \int_{U(t')} d\boldsymbol{r}' \frac{\partial W_2}{\partial t}\left[\frac{\partial}{\partial t'}\gamma\left(\boldsymbol{P}_2 - \boldsymbol{P}_1\right) + \gamma(\boldsymbol{j}_2 - \boldsymbol{j}_1)\right] \tag{37}$$

Case B. The scatterer object *U(t)* is placed outside of the nonstationary half-space.

The placement of the object *U(t)* is shown in Fig. 5. If the observation point is located in the nonstationary half-space $x>0$ the scattered field is determined by the formula

$$\boldsymbol{E} = \boldsymbol{F} - \theta(x)\frac{1}{c^2\varepsilon_0}\int_0^t dt' \int_{U(t')} d\boldsymbol{r}' \frac{\partial W_1}{\partial t}\left[\frac{\partial}{\partial t'}\gamma\left(\boldsymbol{P}_2 - \boldsymbol{P}_1\right) + \gamma(\boldsymbol{j}_2 - \boldsymbol{j}_1)\right] \tag{38}$$

This equation is analogues to the one (37) for the half-space $x<0$ in the previous case.

In the half-space $x<0$ the scattered field is determined by the formula that is analogues to (36)

$$\boldsymbol{E} = \boldsymbol{F} - \theta(-x)\frac{1}{c^2\varepsilon_0}\int_0^t dt' \int_{U(t')} d\boldsymbol{r}' \left\{\frac{\partial G_{01}}{\partial t} + \frac{\partial G_{02}}{\partial t} + \frac{\partial S_2}{\partial t}\right\}\left[\frac{\partial}{\partial t'}\gamma\left(\boldsymbol{P}_2 - \boldsymbol{P}_1\right) + \gamma(\boldsymbol{j}_2 - \boldsymbol{j}_1)\right] \tag{39}$$

The field in the half-space *x<0* is described by the equation that is a Volterra integral equation of the second kind if the observation point is located in the region *U(t)*. If the observation point is located

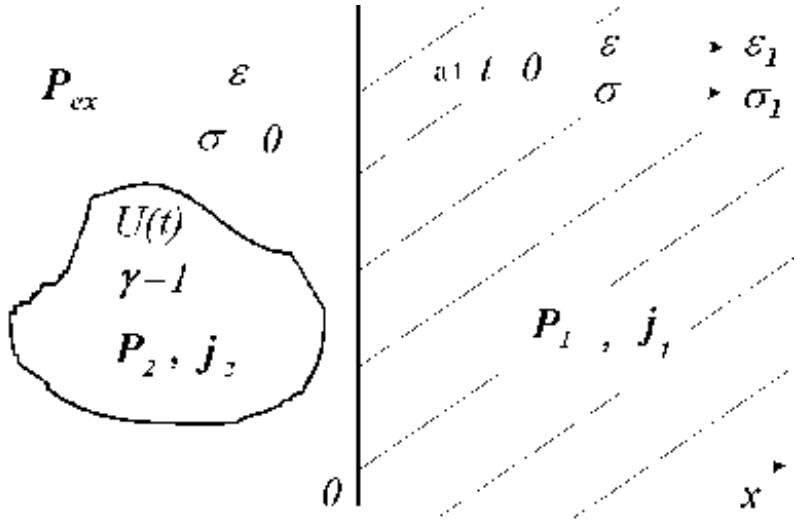

Figure 5. The placement of the object *U(t)* outside the nonstationary half-space.

outside of the region *U(t)* this formula allows to determine the field through integrating of the internal field over *U(t)*. As this is compared with the previous case it is seen that the specific form of the expression depends on which half-space the region *U(t)* is placed rather than where a permittivity jump has occurred.

CONCLUSION

The Green's function for the Maxwell's equations in a medium consisting of two semi-infinitive media is derived. In one of these semi-infinitive media the permittivity and conductivity change abruptly in time. The obtained function allows the formulation of an integral expressions for the electric field scattered by an object that is located in either half-space. It is shown that the specific form of the expression depends on the half-space that the object is placed rather than where the permittivity and conductivity jump has occurred.

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