PARAMETRIC EFFECT IN HTSC FILMS

V. M. Aroutiounian*, V. V. Buniatian*, S.S.Gevorkian**

*Yerevan State University Yerevan 375049 Armenia
**Chalmer University of Technology, Goeteborg, S-412 96 Sweden

1. Absrtract

Processes in a modulation amplifier on a high temperature superconducting film with nonlinear parametric inductance and active resistance are theoretically examined. The expression is obtained in this paper for the average power on the inductance. A possibility of a parametric regeneration on the modulation frequency of the surface impedance of a HTSC film offers at the irradiation of the latter by an optical radiation modulated on the intensity.

1. INTRODUCTION

Recently, great interest has been expressed in the photoresponse of high temperature superconducting (HTSC) films [1-7]. From the viewpoint of applications of superconducting electronics, the characteristics of photoresponse (including thermal and photoinducted non-equilibrum effects) must be examined and evaluated for the development of optically controlled switches [8], delay lines [9], filters [10], photodetectors [11-13] and other optical microwave systems [14-16].

When the superconducting film thickness is approximately equal to or less than the penetration depth λ_L, its surface impedance becomes a function of the superconductor film thickness [14, 15]. Let a HTSC film made of $YBa_2Cu_3O_{7-x}$ or other superconductors is grown or evaporated on a substrate like MgO, $SrTiO_3$ etc (Fig. 1).

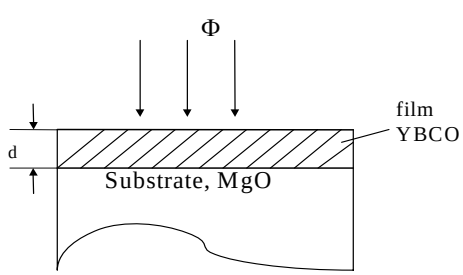

Figure 1. High temperature thin film structure.

It is known [2-4,7,15] that active and reactive components of the surface microwave impedance can be modulated under a "pump" law, when on the HTSC film an optical

signal incidents which is modulated on the intensity, $\Phi = \frac{\Phi_0}{2}\left(1+\sin\omega_s t\right)$ where Φ is the intensity of the incident radiation, ω_s is the frequency of the modulation, t is time. In these papers rather high sensitivity of superconductors and the HTSC films to the optical signal were detected. Under optical signal we understand not only a signal with energy of quanta in the visible range, but in the range of infrared wavelengths as well.

Let nonbolometric transfer of electrons through a superconducting gap (the Cooper pairs breaking) takes place at the absorption of optical radiation by the HTSC film. Let full concentration of electrons within the framework of two-liquid model [7,17-21] is kept ($n=n_s+n_N=n_{s\Phi}+n_{N\Phi}$). Then, the concentration of electrons in the superconduction condition n_s decreases, and the concentration of normal electrons n_N to be increased by the same value ΔN. Therefore expressions for the appropriate concentration of electrons at the irradiation $n_{s\Phi}$ and $n_{N\Phi}$ can be written down as:

$$n_{s\Phi}= n_s-\Delta N\,, \qquad n_{N\Phi}= n_N+\Delta N \tag{1}$$

Investigations of the influence of the excitation (laser radiation), carried out in Refs. 17-21], have shown, that the formation of additional non-paired electrons reduces the superconducting bandgap, but the superconduction condition is not destroyed down up to determined concentration of non-equilibrium electrons.

As is shown in Refs. [16-20], the concentration of non-equilibrium quasiparticles can be expressed as:

$$\Delta N \approx \frac{\eta P_s \tau_{ef}}{d\Delta\left(T,\Delta N\right)}, \tag{2}$$

where $P_1=P_S\eta$ is a modulated part of total power of the radiation (light) P_S falling per unit of the area in unit of time (J/cm^2), η is "an effective quantum yield" indicating which part of the power (energy) absorbed by the film spends on the creation of additional quasiparticles, τ_{ef} is the effective lifetime of additional quasiparticles, $\Delta(T, \Delta N)$ is the value of the superconducting bandgap, d is the thickness of the film, T is the operating temperature in K.

Using results obtained in Ref. [6,11], we can present the expression for ΔN also in the terms the concentration of photons N_Φ^0 absorbed by the HTSC film as:

$$\Delta N \approx \eta \tau'_{ef} N_{\Phi}^{0}, \tag{3}$$

where τ'_{ef} is the effective quasi-particle lifetime caused by the electron-electron and electron-phonon interactions.

2. COMPONENTS OF THE IMPEDANCE

Using Eqs. (1)-(3) and the expression for components of the surface microwave impedance of the HTSC films (see, for example, Ref. [7]), it is easy to verify (see Appendix) that for the thin film when $d<<\lambda_L$, $t_C=T/T_c<1$, where λ_L is the London penetration depth, T_C is the critical temperature,

$$R_{S\Phi}(t) \approx R_{So}(t_c)\ (1+m_1 \sin\ \omega_S t),$$

$$L_{S\Phi}(t) \approx L_{So}(t_c)\ (1+m_L \sin\ \omega_S t). \tag{4}$$

Here $R_{so}(t_c) \approx \dfrac{R_s(t_c)}{f_{10}^2} f_{20}$, $R_s(t_c) \approx (\omega\mu_0)^2 \dfrac{\lambda_L^4(t_c)\sigma_N(t_c)}{d}$,

$$L_{so}(t_c) \approx \frac{\mu_0 \lambda_L^2}{d f_{10}},\ f_{10} \approx 1 - f_1,\ f_{20} \approx 1 + f_2, \sigma_N(t_c) \approx \frac{n_N q^2 \tau_N}{m_N},\ f_1 \approx \frac{C\Phi_0}{2n_s} \approx \frac{C_1 N_{\Phi}^0}{2n_s},$$

$$f_1 \approx \frac{C\Phi_0}{2n_N} \approx \frac{C_1 N_{\Phi}^0}{2n_N},\ m_1 = m_R + 2m_L, m_R = \frac{f_2}{1+f_2},\ m_L = \frac{f_1}{1-f_1},\ (m_1 << 1, m_L << 1)$$

where m_L, and m_R are the depths of the modulation of the kinetic inductance and active resistance, accordingly, τ_N - is the relaxation time of electrons in normal condition [7].

Similar expressions (4) can be easily obtain and for a massive HTSC material.

The above-stated changes in time of the components of the impedance of the HTSC thin films are a precondition for the analysis below by us an possibility of the creation of the parametric "optically controlled" amplifier on the base of the HTSC thin film.

3. ANALYSIS OF THE CIRCUIT

Consider a circuit presented in Fig. 2. Here $U_0 \sin\omega t$ is the emf of the microwave signal generator, Φ is the generator of the optical "pump", $R_{S\Phi}(t)$ is the active component of the surface microwave impedance of the film, $L_{S\Phi}(t)$ is the kinetic inductance of the film.

Here, as opposed to known (classical) parametric converter, with frequency of the optical "pump" it is not periodically varies one of two parameters of a circuit, and both [$R_{S\Phi}(t)$, $L_{S\Phi}(t)$].

The energy saved in the inductance, as is known, is equal

$$W(t)=\psi^2(t)/2L_{S\Phi}(t),$$

where $\psi(t)$ is a magnetic flow.

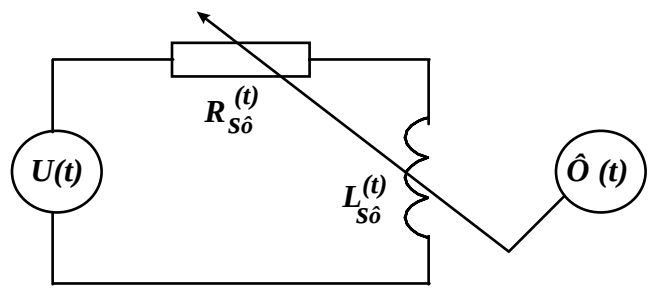

Figure 2. The parametric circuit.

It is obvious that the derivative of this energy in time is the power consumed by the inductance:

$$P(t)=\frac{dW}{dt}=\frac{\partial W}{\partial\psi}\frac{d\psi(t)}{dt}+\frac{\partial W}{\partial L_{S\Phi}(t)}\frac{dL_{S\Phi}(t)}{dt} \quad . \tag{5}$$

Taking into account that

$$\psi(t)=L_{S\Phi}(t)\cdot i(t), \qquad U_L(t)=\frac{d\psi(t)}{dt}$$

and substituting $\frac{dL_{S\Phi}(t)}{dt}$ in Eq.(5), we obtain the following expression for the power:

$$P(t)=i(t)U_L(t)-\frac{\psi^2(t)}{2L_{S\Phi}^2(t)}L_{SO}\omega_S m_L\cos\omega_s t, \tag{6}$$

where $i(t)$ is the current passing through the inductance, $U_L(t)$ is the voltage on the inductance.

First item in Eq. (6) represents the power consumed by the inductance from the electrical circuit after switching on, second item is the power provided with optical "signal". If the inductance is ideal and has not losses, the average meaning of the power consumed by the inductance on all frequencies, should be equaled to zero. The average value of each item in Eq. (6) can be different from zero. In the latter case they are equal on the value and are opposite on the sign.

With Eq. (4) we obtain the following expression for the "optical" power on the inductance:

$$P_{op}(t) = -\frac{m_L \omega_s \psi^2(t)}{2L_{SO}(t_C)} \cos \omega_s t \,. \tag{7}$$

From the differential equation of the circuit we have (Fig. 1):

$$U_L(t) + U_R(t) = U_0 \sin \omega t. \tag{8}$$

With Eq.(4) we obtain the following expression for the magnetic flow:

$$\frac{d\psi(t)}{dt} + \alpha\left(1 + m \sin \omega_S t\right)\psi(t) = U_0 \sin \omega_S t, \tag{9}$$

where $\alpha = \frac{R_s(t_c)}{L_s(t_c)}, \quad m = m_L + m_R$.

Terms of the higher orders in Eq. (9) infinitesimal, such as $m_1 m_L \sin^2 \omega_S t$ have been neglected.

At $m << \omega_s/\alpha$, we obtain from Eq. (9) the following expression for the magnetic flow

$$\psi(t) = \frac{U_0}{\sqrt{\alpha^2 + \omega^2}} \left\{ \sin(\omega t - \varphi) - \frac{\alpha m}{2} \frac{\sin\left[\left(\omega - \omega_S\right)t + \xi\right]}{\sqrt{\alpha^2 + (\omega - \omega_S)^2}} + \frac{\alpha m}{2} \frac{\sin\left[\left(\omega - \omega_S\right)t + \beta\right]}{\sqrt{\alpha^2 + (\omega - \omega_S)^2}} \right\}, \tag{10}$$

which holds up to the first order infinitesimal m, where

$$tg\varphi = \frac{\omega}{\alpha}, \quad tg\xi = \frac{\left[\alpha^2 - \omega\left(\omega - \omega_S\right)\right]}{\alpha\left(2\omega - \omega_S\right)}, \quad tg\beta = \frac{\left[\alpha^2 - \omega\left(\omega + \omega_S\right)\right]}{\alpha\left(2\omega + \omega_S\right)}.$$

Substituting Eq. (10) in Eq. (7) we obtain the following expression for average power of optical "pump" on the inductance:

$$\overline{P}_{op}(t) \approx \frac{m^2 U_0^2}{4R_{SO}(t_c)} \frac{\alpha^2 \omega_S^2 \left(\alpha^2 + \omega_S^2 - \omega^2\right)}{\left(\alpha^2 + \omega^2\right)\left[\alpha^2 + \left(\omega + \omega_S\right)^2\right]\left[\alpha^2 + \left(\omega - \omega_S\right)^2\right]}. \tag{11}$$

The oscillatory terms in the expression (11) are omitted as they do not characterize direct transfer of energy from one frequency to other one and the inductance given back [22].

From Eq. (11) the following alternatives follow:

1) At $\alpha^2 + \omega_S^2 - \omega^2 > 0$, $\overline{P}_{op} > 0$, i.e. the nonlinear (kinetic) inductance consumes the energy of optical "pump" and transforms it in the energy of combinational frequencies;

2) At $\alpha^2 + \omega_S^2 - \omega^2 < 0$, $\overline{P}_{op} < 0$, i.e. the nonlinear inductance gives back energy in the circuit on the frequency of modulation;

3) At $\alpha^2 + \omega_S^2 - \omega^2 = 0$, $\overline{P}_{op} = 0$, i.e. the nonlinear inductance represents the pure reactive resistance for the current of the optical "pump" frequency.

The most interesting is the second case. Here the analysis shows that at $m_S^* = m_N^*$, where m_S^*, m_N^* are the effective masses of electrons in the superconduction and normal conditions, accordingly, the fulfillment of the following condition is necessary for the regeneration on the frequency ω_S of the modulation:

$$m_L < \frac{\sqrt{\omega^2 - \omega_S^2}}{\omega^2 \tau_N} m_R \quad \text{or} \quad \omega_S < \omega\sqrt{1 - \delta^2 \omega^2}, \tag{12}$$

where $\delta = \dfrac{m_L \tau_N}{m_R}$.

At $\omega >> \omega_S$, from Eq. (12) the following inequality follows

$$m_L < \frac{m_R}{\omega \ \tau_N}, \tag{13}$$

which is usually fulfilled and there is in the consent with results obtained in Refs. [7,15] for optical sensitivity of the surface impedance components of the HTSC film at its irradiation by the optical radiation modulated on the intensity.

Present $\overline{P}_{op}$ as

$$\overline{P}_{op} = \frac{1}{2} \frac{U_0^2}{4R_{eq}}, \quad P_{eq} = \frac{\gamma}{2R_{SO}(t_c)},$$

where $\gamma = \dfrac{L_{SO}^2(t_c)\left(\alpha^2+\omega^2\right)\left[\alpha^2+\left(\omega+\omega_S\right)^2\right]\left[\alpha^2+\left(\omega-\omega_S\right)^2\right]}{m^2\omega_S^2\left(\alpha^2+\omega_S^2-\omega^2\right)}$, e.g. the modulation of the component of the surface microwave impedance is equivalent to the introduction of a conductivity in the circuit:

$$G_{eq} = \frac{2R_{SO}(t_c)}{\gamma}. \qquad (14)$$

At the absence of optical "pump" ($\Phi=0$) and losses in the inductance, and at the fulfillment of the second condition in the single-loop circuit (Fig. 3), we obtain the following expression for the power given back by the microwave signal generator:

$$P_\omega = \frac{1}{2}\frac{I_0^2}{4\left(G_g+G_{SO}\right)} = \frac{I_0^2}{2\cdot 4G_L} \qquad (15)$$

where I_0, G_g are the amplitudes of the current and internal conductivity of the generator, accordingly, G_{S0}. is the active conductivity of the film before irradiation.

The maximum of the power (in the absence of the amplification) is reached at $G_g = G_L$. In this case $G_L = G_g + G_{S0}$. At the connection of additional conductivity G_{eq} $(\Phi\neq 0)$, the power

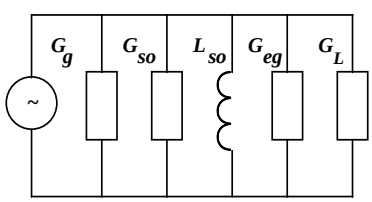

Figure 3. Equivalent single-loop circuit.

released in the conductivity G_L is equal

$$P_1 = \frac{1}{2}\cdot\frac{I_0^2}{4G_L\left(1+\dfrac{G_{eq}}{2G_L}\right)^2}.$$

We obtain the following expression for the gain in the power:

$$K_p = \frac{P_1}{P_\omega} \cdot \frac{1}{\left(1 + \frac{G_{eq}}{2G_L}\right)^2} . \tag{16}$$

Substituting the expression for G_{eq}, $R_{s0}(t_c)$, $L_{s0}(t_c)$ and γ in Eq. (16) and taking into account that $\alpha^2 + \omega_S^2 < \omega^2$, we obtain the following expression for the gain in power:

$$K_p = \left\{1 - \frac{m^2 \omega_S^2 \omega^2 \sigma_N d}{G_L \gamma_0}(1 + f_2)\right\}^{-2} , \tag{17}$$

where $\gamma_0 = \dfrac{\left(\alpha^2 + \omega^2\right)\left[\alpha^2 + \left(\omega + \omega_S\right)^2\right]\left[\alpha^2 + \left(\omega - \omega_S\right)^2\right]}{\omega^2 - (\alpha^2 + \omega_S^2)}$.

As follows from Eq. (17), the smaller γ_0 the greater is K_P. At $\alpha << \omega$,, $\alpha < \omega_s$ we have the expression for γ_0 in the form:

$$\gamma_0 \approx \omega^2\left(\omega^2 - \omega_S^2\right). \tag{18}$$

Hence, for obtaining of large meanings of K_p a proximity of frequencies ω and ω_s it is necessary, but, simultaneously, in order a condition $\overline{P}_{op} < 0$ was realized, it is necessary the fulfillment of the condition $\omega > \omega_s$. Taking into account Eq. (18) we obtain the following expression for K_p:

$$K_p = \left\{1 - \frac{A(1 + f_2)}{\left(\frac{\omega^2}{\omega_S^2} - 1\right)}\right\}^{-2} , \tag{19}$$

where $A = \dfrac{m^2 \sigma_N d}{G_L}$.

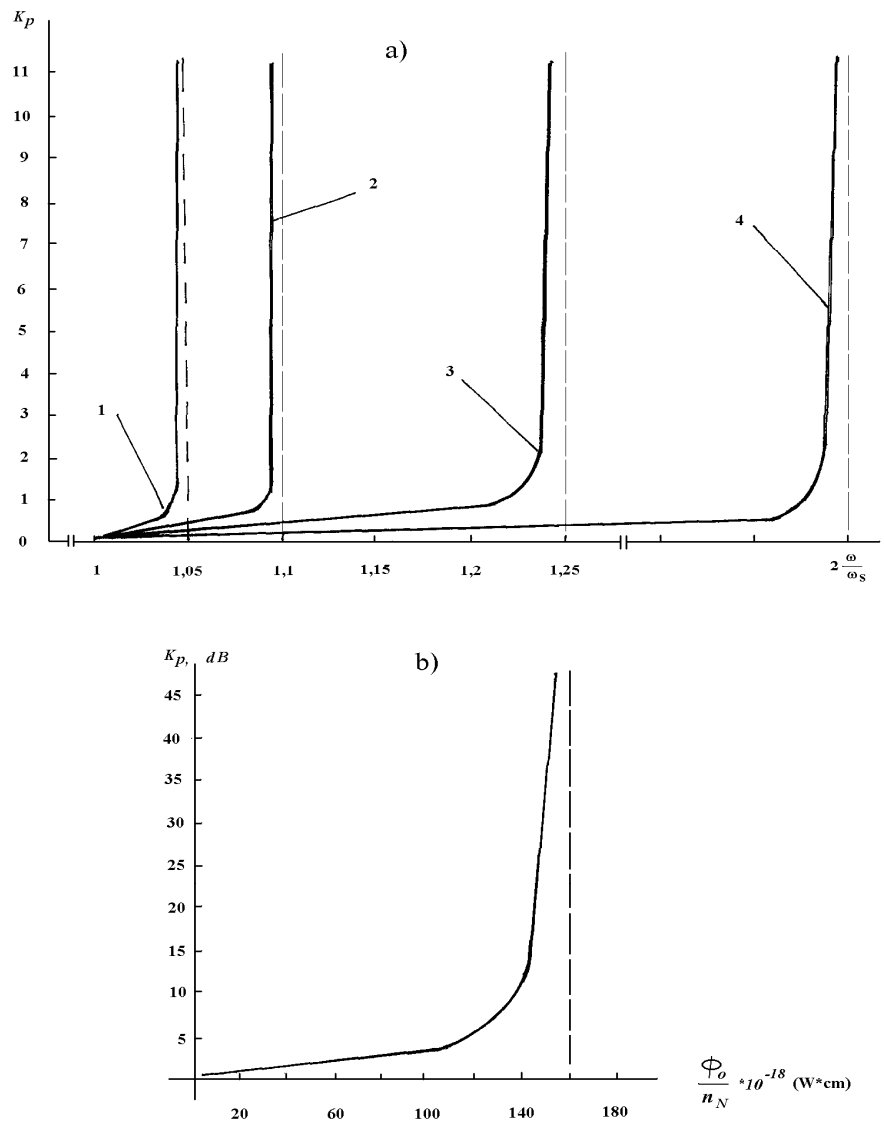

Figure 4. Calculated curves for the gain depending from ω/ω_S (a) $m_R \approx 10 m_L$, (1- $m_L \approx 0.031$, $\left.\frac{\Phi_0}{n_N}\right)_m \approx 56$, 2- $m_L \approx 0.04$, $\left.\frac{\Phi_0}{n_N}\right)_m \approx 87.5$, 3- $m_L \approx 0.057$, $\left.\frac{\Phi_0}{n_N}\right)_m \approx 168$, 4- $m_L \approx 0.083$, $\left.\frac{\Phi_0}{n_N}\right)_m \approx 630$.) and Φ_0/n_N (b) $m_L \approx 0.057$, $\omega/\omega_S = 1.25$, $G_L = 1$ Ohm

From Eq. (19) follows that at

$$\omega = \omega_S \sqrt{1 + A\left(1 + f_2\right)}\,, \tag{20}$$

K_p beyond all bounds increased. And, a selection of various values m, σ_N, d, n_N, $\Phi_0(N_\Phi^0)$, $\Delta(T,\Delta N)$, is possible (probable) at which the condition (20) is obeyed.

Table 1.

ω/ω_s	m_R	f_1	f_2	$\Phi_0/n_N.)_{max}10^{-18}$ Wcm	$\Phi_0/n_s.)_{max}10^{-18}$ Wcm
1,05	0,31	0,03	0,45	56	3,7
1,1	0,4	0,0385	0,7	87,5	4,8
1,25	0,57	0,053	1,35	168	6,6
2	0,83	0,076	5	630	9,5

For example, at σ_N~$2\cdot10^6$(Ohm*m)$^{-1}$, d~$0.3\cdot10^{-6}$m [12,14], η≈10%, τ_{ef}~10^{-8}s, τ_N~10^{11}s [11,16-19], m_L/m_R~10^{-1}, ω_s~10^8s^{-1}, G_L ~1 Ohm, α~10^6s^{-1}, $\Delta(T,\Delta N)$~0.013 eV at T=80K [16-19], the condition (20) is obeyed at meanings of the parameters, resulted in Table 1, appropriating to various values of ω/ω_s. Calculated curves for the gain depending on ω/ω_s and Φ_0/n_N are shown on Fig. 4.

As follows from Table 1 and Figs. 4, the greater ω differs from ω_s, the greater there are the modulation depths and, hence, large meanings of parameters $\Phi_0/n_N)_{max}$ and $\Phi_0/n_S)_{max}$ are required for the maintenance of the condition (20).

4. CONCLUSIONS

The possibility of the creation of the parametric amplifier on the base of the HTSC films at the modulation of its surface microwave impedance by the optical radiation modulated on the intensity is shown.

Such amplifier can operated under the optical signal in rather wide range of energy of quanta (especially at approach of the microwave field frequency to the optical modulation frequency).

The present work is carried out in the framework of the *INTAS -96-268* grant.

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APPENDIX

As follows from Eq. (2) and (3) $\Delta n_\Phi \sim \Phi$, $\Delta n_\Phi \sim N_\Phi$ and consequently in the first approximation we accept

$$2\Delta n_\Phi \cong c\Phi_0\left(1+\sin\omega_S t\right) \cong c_1 N_\Phi^0\left(1+\sin\omega_S t\right) . \tag{A1}$$

Thus

$$C \cong \frac{\eta\tau_{ef}}{d\Delta\left(T,\Delta,\ _\Phi\right)} \ , \qquad C_1 \cong \eta\tau''_{ef}$$

By expressing Δn_Φ through n_N and n_S, we have: $\Delta n_\Phi = n_N f_1 = n_S f_2$

$$2f_1 \cong c\Phi_0\left(1+\sin\omega_S t\right)n_N^{-1} \cong c_1 N_\Phi^0\left(1+\sin\omega_S t\right)n_N^{-1} = 2f_1^0\Phi_0 = 2f_{11}^0 N_\Phi^0 \ ,$$
$$2f_2 \cong c\Phi_0\left(1+\sin\omega_S t\right)n_S^{-1} \cong c_1 N_\Phi^0\left(1+\sin\omega_S t\right)n_S^{-1} = 2f_2^0\Phi_0 = 2f_{22}^0 N_\Phi^0 . \tag{A2}$$

It is known that in the HTSC films at $d << \lambda_L$, where d is the thickness of a film, and λ_L is the London penetration depth, the active and reactive parts of the surface impedance, obtained on the basis of the two-liquid model (see, for example, Ref. [14), at $t_C = T/T_C < 1$, are expressed as

$$R_S\left(t_c\right) = \left(\omega\mu_0\right)^2 \frac{\lambda_L^4\left(t_c\right)}{d}\sigma_N\left(t_c\right) , \quad \mathrm{X_S}\left(t_c\right) = \omega\mu_0 \frac{\lambda_L\left(t_c\right)}{d}, \tag{A3}$$

where

$$\lambda_L^2\left(t_c\right) = \frac{m_S}{n_S q^2 \mu_0} \ , \quad \sigma_N\left(t_c\right) = \frac{n_N q^2 \tau_N}{m_N} \ , \tag{A4}$$

T is the temperature of a sample; T_C is the critical temperature; ω is the frequency of a microwave field; μ_0 is the magnetic constant of vacuum; σ_N is the conductivity connected with a normal condition; m_S , m_N, n_S and n_N are the effective masses and concentrations of electrons in superconduction and normal (non-superconduction) states, accordingly; τ_N is the lifetime of normal electrons; q is the charge of electron.

Then, for the components of the impedance $R_{S\Phi}(t_0)$ and $X_{S\Phi}(t_0)$ we get:

$$R_{S\Phi} \approx R_S(t_C)\{1 + 2f_2 + f_1 + 2f_1 f_2\} \quad , \quad X_{S\Phi} \approx X_S(t_C)\{1 + f_2\} \tag{A5}$$

Substitution of (A1), (A2) and (A3), (A4) into (A5) yields (4).