PENETRATION OF ELECTROMAGNETIC WAVE INTO PLASMA EXPANDING IN A WAVEGUIDE

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Abstract

An evolution of an electromagnetic signal in a half-restricted plasma that is formed at zero moment of time and then is expanded in a waveguide is considered. The electromagnetic signal propagates from the empty side of the waveguide. A signal source works from the certain moment of time and is located at some distance from the point where the plasma boundary was at zero moment. The problem is solved by an evolutionary approach based on the integral Volterra equation. The structure of the field penetrated into plasma is derived. The feature determined by the availability of a double dispersion, a waveguide and plasma dispersion, is investigated.

1. INTRODUCTION

Investigation of an electromagnetic wave interaction with a moving plasma have been carried out for a long time since the first papers of Landecker [1] and Lampert [2] and it is of interest up to now [3-11]. It is known, the electromagnetic wave exhibits simultaneously amplitude and frequency change and duration compression in this interaction. In the preceding works, it was shown that amplitude and a frequency of an electromagnetic wave reflected from a moving medium boundary can enhance considerably. The efficiency of this change is characterized by the ratio of a boundary velocity to a wave group velocity that is of prime importance in a waveguide where a double dispersion mechanism is in existence [9,10,12]. A power characteristics is considered usually, but if the plasma comes into being and its movement begins at a finite moment of time then a wave interaction with the plasma will be accompanied by electromagnetic transients. A spectral structure of the secondary electromagnetic waves is of interest even in the case of a stationary uniform movement of medium boundary [8]. If the movement is not stationary, one ought to consider the evolution of the electromagnetic wave. This problem is considered in this paper, in which an evolution of an electromagnetic signal in a half-restricted plasma that comes into being at zero moment of time and then is expanded in a waveguide is considered.

2. MAIN RELATIONS

The homogeneous cold plasma is restricted at one side by a plane that is normal to the waveguide axis x and moves with the velocity u. The plasma is created at zero moment of time in the waveguide half-space and has a plasma frequency ω_e, Figure1.

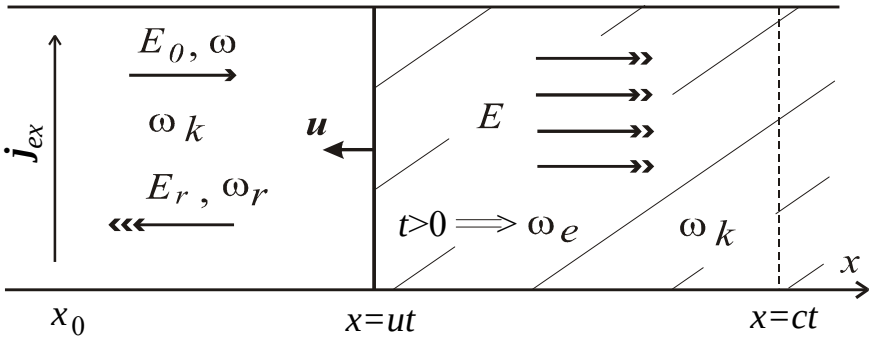

Figure 1. The geometry of the problem

A signal source

$$\boldsymbol{j}_{ex} = \sum_k \boldsymbol{b}_k \delta(x - x_0)\theta(t - t_0)\, j_k(t) \tag{1}$$

works from the point in time t_0 and is located at a waveguide cross-section x_0, where x_0 is a distance from the plasma boundary at a moment when it comes into being. In (1) $\boldsymbol{b}_k$ is a waveguide vector eigen-function and k denotes a waveguide mode with a waveguide critical frequency ω_k. With considered geometry of the problem, there is not a mode transformation.

An electromagnetic signal generated by the current $\boldsymbol{j}_{ex}$ propagates from the empty side of the waveguide and is described by the expression

$$E_0(t,x) = -\frac{2\pi}{c}\sum_k \left[\Gamma_k(t - t_0, x - x_0)\, j_k(t_0) + \int_{t_0}^{\infty} \Gamma_k(t - t', x - x_0) \frac{\partial\, j_k(t')}{\partial t'} dt' \right], \tag{2}$$

where,

$$\Gamma_k = J_0\left(\omega_k \sqrt{(t - t')^2 - \frac{(x - x')^2}{c^2}} \right) \theta\left(t - t' - \frac{|x - x'|}{c} \right), \tag{3}$$

J_0 is the Bessel function, θ is the Heaviside unit function, c is the light velocity in vacuum.

The electromagnetic field penetrated into the half-bounded plasma after zero moment of time satisfies an evolutionary integral Volterra equation [13]:

$$E = F - \frac{2\pi}{c}\int_0^{\infty} dt' \int_0^{\infty} dx' \frac{\partial^2}{\partial t^2} \Gamma_k(t - t', x - x')\theta(x' - ut')P(t', x') \tag{4}$$

Here, $F = \theta(x-ut)E_0(t,x)$ is a free term of the integral equation and a constitutive relation of cold plasma is given by

$$P = \frac{\omega_e{}^2}{4\pi}\int_0^t (t-t')\theta(x-ut)E(t',x')dt' \tag{5}$$

From this point the mode subscript for the field is dropped out.

The solution of the equation (4) can be found by the resolvent method

$$E = F(t,x) + \int_0^\infty dt' \int_0^\infty \left[R_1(t,t',x,x') + R_2(t,t',x,x')\right]F(t',x')dx' \tag{6}$$

where the first term is the resolvent for the problem in the unbounded plasma

$$R_1(t,t',x,x') = \int_{-i\infty}^{i\infty} \frac{dp}{2\pi i}\frac{1}{2c\psi(p)} e^{p(t-t')-\psi(p)\frac{|x-x'|}{c}}\theta(x'-ut') \tag{7a}$$

where $\psi(p) = \sqrt{p^2+\omega_{ke}{}^2}$, $\omega_{ke}{}^2 = \omega_k{}^2 + \omega_e{}^2$,

and the second term is caused by the moving plasma boundary

$$R_2(t,t',x,x') =$$

$$= \int_{-i\infty}^{i\infty}\frac{dp}{2\pi i}\int_{-\infty}^{\infty}\frac{ds}{2\pi}R(q)\frac{\exp\left[p(t-t') + is\left(x - \frac{\beta}{1-\beta^2}t'\right) - \frac{\varphi_{ke}(q)-\beta p}{c(1-\beta^2)}(x'-ut')\right]}{2\varphi_{ke}(q)\left[\varphi_{ke}(q)+\beta p + ics\right]}\theta(x'-ut'), \tag{7b}$$

where, $R(q) = \dfrac{\varphi_k(q)-\varphi_{ke}(q)}{\varphi_k(q)+\varphi_{ke}(q)}$, $\varphi_{ke}(q) = \sqrt{q^2+(1-\beta^2)\omega_{ke}{}^2}$, $\operatorname{Re}\varphi_{ke} > 0$,

$\varphi_k(q) = \sqrt{q^2+(1-\beta^2)\omega_k{}^2}$, $\operatorname{Re}\varphi_k > 0$, $q = p + ius$, $\beta = u/c$

3. THE HARMONIC SOURCE

The primary signal (2) created by the harmonic source $j_k = e^{i\omega t}$ has the form for considered waveguide mode

$$E_0(t,x) = E_0\theta(t - t_0 - \frac{|x - x_0|}{c})\left\{ J_0(\omega_k\sqrt{(t-t_0)^2 - \frac{(x-x_0)^2}{c^2}}) + \right.$$

$$\left. + i\omega \int_{\frac{|x-x_0|}{c}}^{t-t_0} J_0(\omega_k\sqrt{\tau^2 - \frac{(x-x_0)^2}{c^2}})e^{i\omega(t-\tau)}d\tau \right\} \tag{8}$$

If the source spatial arrangement and the point in time of its turning on are matched by the condition $t_0 - \frac{x_0}{c} \approx 0$, then the incident wave of considered mode might be presented approximately in the form of a plane wave:

$$E_0(t,x) = E_0\theta(t - \frac{x}{c})e^{i[\omega t - k_0(t,x)x]}, \tag{9}$$

where $k_0(t,x) = \frac{\omega}{c}$, if $x \approx ct$ and $k_0(t,x) = \frac{1}{c}\sqrt{\omega^2 - \omega_k{}^2}$, if $x << ct$ and $t \to \infty$.

It is followed from (6) and (7) that a field penetrated into plasma is different from zero in the region that is arranged between the plasma boundary and the plane $x = ct$. In this region the field consists of four terms:

$$E(t,x) = E_1 + E_2 + E_3 + E_{tr} \tag{10}$$

1) The first term

$$E_1(t,x) = F(t,x) - \frac{\omega_e{}^2}{\omega_{ke}{}^2 + c^2 k_0{}^2 - \omega^2} e^{i(\omega t - k_0 x)} \tag{11}$$

is a relic of the incident wave in the penetrated field. Near the signal front ($x \approx ct$) this wave frequency and wave number are as the incident wave has, and the amplitude changes in the ratio of $\omega_k^2 / \omega_{ke}^2$. This relic disappears with a distance from the signal front. In this case, the wavenumber $k_0 \to \frac{1}{c}\sqrt{\omega^2 - \omega_k{}^2}$ and the two items in (11) are canceled. In an open space ($\omega_k = 0$) this relic is lacking. In the empty waveguide ($\omega_e = 0$) this relic is the whole incident wave as the other items in (10) are equal to zero.

2) The second term in (10) is the stationary wave as in the case when the plasma is in existence from the infinite past in time:

$$E_2 = \frac{\omega_e{}^2(\varphi_k{}^* - \beta\omega + ck_0)}{(\varphi_k{}^* + \varphi_{ke}{}^*)(\omega_{ke}{}^2 + c^2 k_0{}^2 - \omega^2)} e^{i(\Omega_2 t - k_2 x)}. \tag{12}$$

Here, $\varphi_k{}^* = \sqrt{\Omega_1{}^2 - (1-\beta^2)\omega_k{}^2}$, $\varphi_{ke}{}^* = \sqrt{\Omega_1{}^2 - (1-\beta^2)\omega_{ke}{}^2}$,

$$\Omega_2 = \frac{\Omega_1 + \beta\varphi_{ke}{}^*}{1-\beta^2}, \; k_2 = \frac{\beta\Omega_1 + \varphi_{ke}{}^*}{c(1-\beta^2)}, \; \Omega_1 = \omega - uk_0. \tag{13}$$

The frequency Ω_2 satisfies the condition of equality of incident and penetrated wave phases on the moving plasma boundary

$$(\omega t - k_0 x)|_{x=ut} = (\Omega_2 t - \sqrt{\Omega_2{}^2 + \omega_{ke}{}^2}\,\frac{x}{c})|_{x=ut}. \tag{14}$$

The frequency Ω_1 satisfies the condition of equality of the incident wave phase on the moving plasma boundary and the penetrated wave phase

$$(\omega t - k_0 x)|_{x=ut} = \left(1-\beta^2\right)^{-1/2}(\Omega_1 t' - \varphi_{ke}{}^* \frac{x'}{c})|_{x=ut}. \tag{15}$$

if the penetrated wave phase is converted to the rest frame with respect to the moving boundary

$$t' = \left(1-\beta^2\right)^{-1/2}\left(t - xu/c^2\right), \; x' = \left(1-\beta^2\right)^{-1/2}\left(x - ut\right) \tag{16}$$

In the open space ($\omega_k = 0$) the wave E_2 represents the whole penetrated field. It vanishes in the absence of the plasma ($\omega_e = 0$). Far away from the signal front ($x << ct$) this wave amplitude becomes equal

$$E_2 = \frac{2(ck_0 - \beta\omega)}{(\varphi_{ke}{}^* + ck_0 - \beta\omega)} \tag{17}$$

and coincides with the corresponding one in the case of the open space ($\omega_k = 0$).

3) The third term in (10) is absent near the signal front ($x \approx ct$), and far away from it ($x << ct$) this term fulfills the function of the relic

$$E_3 = -4(1-\beta^2)\Omega_4{}^2\omega_e{}^2\Big[(1-\beta)\omega_{ke}{}^2 - (1+\beta)\Omega_4{}^2 + \\ + \sqrt{\left[(1+\beta)\Omega_4{}^2 + (1-\beta)\omega_{ke}{}^2\right]^2 - 4(1-\beta^2)\omega_{ke}{}^2\Omega_4{}^2}\Big]^{-2} e^{i(\Omega_3 t - k_3 x)}, \tag{18}$$

where

$$\Omega_3 = \frac{(1-\beta)^2\omega_{ke}{}^2 + (1+\beta)^2\Omega_4{}^2}{2\Omega_4(1-\beta^2)}, \; k_3 = \frac{(1+\beta)^2\Omega_4{}^2 - (1-\beta)^2\omega_{ke}{}^2}{2c\Omega_4(1-\beta^2)},$$

$$\Omega_4 = \omega - ck_0 \tag{19}$$

This wave amplitude turns to zero with $\omega_k = 0$ as well as with $\omega_e = 0$, that is this wave exists only with the availability of a double dispersion mechanism, namely, a waveguide and plasma dispersion. When the plasma boundary moves with a relativistic velocity ($\beta \to -1$) this wave also turns to zero.

4) The fourth term in (10) is determined by the following from (6, 7) integrals over branch cuts in the complex plain

$$I_1 = -\omega_e{}^2(1-\beta)\int_{-i\infty}^{i\infty}\frac{dp}{2\pi i}\frac{\exp(pt-\psi x/c)}{2\psi(p-\beta\psi-i\Omega_1)(p-\psi-i\Omega_2)},$$

$$I_2 = \omega_e{}^2(1-\beta^2)\int_{-i\infty}^{i\infty}\frac{dq}{2\pi i}\frac{(\varphi_k-\varphi_{ke})\exp(\frac{q+\beta\varphi_{ke}}{1-\beta^2}t-\frac{\beta q+\varphi_{ke}}{1-\beta^2}\frac{x}{c})}{2\varphi_{ke}(\varphi_k+\varphi_{ke})(q-i\Omega_1)(q+\varphi_{ke}-i\Omega_2(1+\beta))} \tag{20}$$

The branch cuts are drawn such that the conditions $\mathrm{Re}\,\psi > 0$, $\mathrm{Re}\,\varphi_k > 0$, $\mathrm{Re}\,\varphi_{ke} > 0$ are fulfilled. The integrals (20) yield the continuous wave spectrum that describes the transition process. This field is absent near the signal front as the integrals (20) turn out to zero when $x = ct$. The transient is damped out when $t \to \infty$. The distribution of the secondary waves in the interval $(x = ut,\ x = ct)$ that is between the plasma boundary and the signal front is shown in Figure 2.

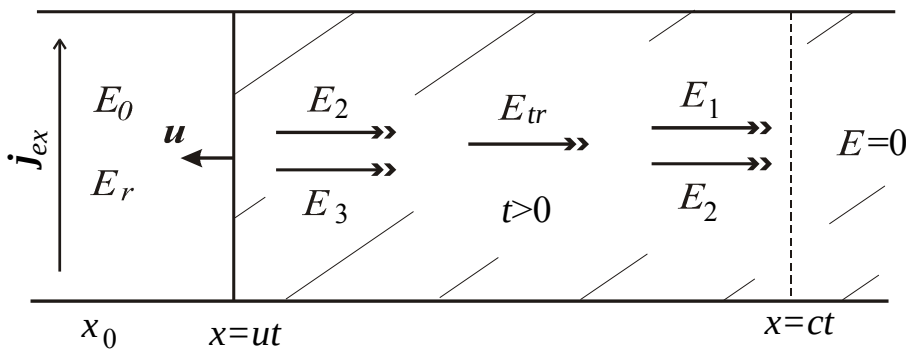

Figure 2. The distribution of the secondary waves.

4. CONCLUSION

The penetration of the electromagnetic signal restricted by the front through the moving plasma boundary excites the complex field in the plasma. Near the signal front this field consists of two waves - the relic of the incident wave and the stationary wave that is analogue of the wave for the case when the movement of the plasma boundary begins at the infinite remote past. In an open space, the relic disappears and the

stationary wave represents the whole penetrated field. When the plasma is absent the stationary wave disappears and the relic represents the whole penetrated field.

At the region far from the signal front, the field consists of two waves also. The new wave acts the function of the relic but this wave exists only with the double dispersion mechanism - the waveguide and the plasma dispersion. The second wave is the stationary wave that converts to the incident wave if the plasma is absent.

The whole interval between the plasma boundary and the signal front is filled by the transient field that is asymptotically damped out in time.

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