RELATIVISTIC VIEWS ON PLANE ELECTROMAGNETIC WAVES IN ENGINNERING

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Abstract

Starting from Lorenz relativistic relations and considering the plane electromagnetic wave in homogeneous isotropic environment, with two co-ordinate systems one of which moving with speed V, are obtained the Poynting Vector relations for the directions, $\overline{V} = iV_x, jV_y, kV_z$ *of the electromagnetic energy and forces; are also obtained simple relations of E, H, of* $E^{'}$ *and* $H^{'}$*; there are viewed the two equivalencies: that of media and that of sources.*

1. INTRODUCTION

Plane Electromagnetic Wave (EW) and monochromatic carried out in two coordinativ system K- at rest and K^V - in uniform movement is the object of this study. As known, for considerable distances from the source the other waves in this object are reduced growing up this way the analysis limits.

The main goal of the study is to enlarge the domain of the problem for obtaing new theoretical measures with theoretical and applied values under an engineering focus, in view of sensibilisation of the scientific and engineering thoughts.

Based on that and from an applicatic point of view, one can undertake several analysis as for instance in mobile environments within electromagnetic field.
Conditions of the analysis are dielectric, homogeneous, linear, isotropic environment.
It is based on the particular conclusions of the theory of relativity starting from the Lorentz relations with regards to the electromagnetic field.

The study is lied on energeticall principal views, with some more detailed considerations of Lorentz relations, on the equivalence problems etc.

2. ENERGY CONSIDERATIONS

The Lorentz relations are welknown for two coordinativ systems K and K^V; the movement is carried out with an uniform speed $\overline{V} = k \cdot V_z$, we may write:

$$E_z^{(v)} = E_z; \quad E_x^{(v)} = \frac{E_x + \mu_a V H_y}{\sqrt{1 - \frac{V^2}{c^2}}}; \quad E_y^{(v)} = \frac{E_y - \mu_a V H_x}{\sqrt{1 - \frac{V^2}{c^2}}} \tag{1}$$

$$H_z^{(v)} = H_z; \quad H_x^{(v)} = \frac{H_x + \varepsilon_a V H_y}{\sqrt{1 - \frac{V^2}{c^2}}}; \quad H_y^{(v)} = \frac{H_y - \varepsilon_a V E_x}{\sqrt{1 - \frac{V^2}{c^2}}} \tag{2}$$

Here below the denominator will be either implied or not considered (V<<c, c- light speed)

In a vectorial form the measures 1) 2) take the compact form

$$\overline{E}^{(v)} = \overline{E} + \left[\overline{V} \cdot \overline{B}\right] \quad \overline{H}^{(v)} = \overline{H} + \left[\overline{V} \cdot \overline{D}\right] \tag{3}$$

Let consider the vector of Poynting invariant expression of which for two coordinativ systems and the considered electromagnetic field($\overline{E} = i \cdot E_x$; $\overline{H} = i \cdot H_x$) should be:

$$\overline{S} = \left[\overline{E} \cdot \overline{H}\right] = \begin{vmatrix} i & j & k \\ E_x & 0 & 0 \\ 0 & H_y & 0 \end{vmatrix} \tag{4}$$

$$\overline{S}^{(v)} = \left[\overline{E}^{(v)} \cdot \overline{H}^{(v)}\right] = \begin{vmatrix} i & j & k \\ E_x - V_z \cdot B_y & 0 & V_x \cdot B_y \\ 0 & H_x - V_z \cdot D_x & -V_y \cdot D_x \end{vmatrix} \tag{5}$$

Considering the general case, when $\overline{v} = i \cdot v_x + j \cdot v_y + k \cdot v_z$; the EW is distributed with a speed C_O following Z axis.

In this case, we can obtain:

$\overline{S}^{(v)} = i \cdot S_x^{(v)} + j \cdot S_y^{(v)} + k \cdot S_z^{(v)}$ where:

$$\left.\begin{aligned}
\overline{S}_x^{(v)} &= -\frac{V_x}{c_a}\cdot\left(1+\frac{V_z}{c_a}\right)\cdot S = k_x S \\
\overline{S}_y^{(v)} &= \frac{V_y}{c_a}\cdot\left(1-\frac{V_z}{c_a}\right)\cdot S = k_y S \\
\overline{S}_z^{(v)} &= \left(1-\frac{V_z^2}{c_a^2}\right)\cdot S = k_z S
\end{aligned}\right\} \tag{6}$$

So in the coordinativ system $K^{(v)}, \overline{S}^{(v)}$ with the above projections and the measure:

$$S^{(v)} = \sqrt{\frac{V_x^2}{c_a^2}\cdot\left(1+\frac{V_z}{c_a}\right)^2 + \frac{V_y^2}{c_a^2}\cdot\left(1-\frac{V_z}{c_a}\right)^2 + \left(1-\frac{V_z}{c_a}\right)}\cdot S \tag{7}$$

This general case gives us the possibility to consider several particular cases, presented in table I.

Based on the considered cases, we can have these conclusions:

1- The vector of Pointing $\overline{S}^{(v)}$ is different from the vector $\overline{S}$, in module and direction. It is quite influenced by the direction, module and the sense of the speed v, which represents in fact the displacement of the coordinativ system.

In all cases, in what module of v is considered, we can write:

$\overline{S}^{(v)} = k\cdot S$: where k is determined in the table; for each case we can find in it the direction and its inclination toward Z axis.

2 - In the cases 2,5,6 it is evident that $S^{(v)} > S$ and concretely:

$$S^{(v)} = \sqrt{1+\frac{V^2}{c_a^2}}\cdot S \approx \left(1+\frac{V^2}{2c_a^2}\right)S \tag{8}$$

If we compare it with the normal Z, for which:

$$S^{(v)} = \left(1-\frac{V^2}{2c_a^2}\right)S$$

we distinguish a positive increment of

$$\Delta S^{(v)} = \frac{3}{2}\frac{V^2}{c_a^2}$$

TABLE I

Case	Velocity $\overline{V}$	Projections of $\overline{S}^{(V)}$	Modul of S^V $S^V = k \cdot S$	Direction of $\overline{S}^V$	Inclination Z axis
1	$iV_x + V_y +$	$S_x^V = -\frac{V_x}{c_a}\left(1+\frac{V_z}{c_a}\right)\cdot S$ $S_y^V = \frac{V_y}{c_a}\left(1-\frac{V_z}{c_a}\right)\cdot S$ $S_z^V = \left(1-\frac{V_z^2}{c_a^2}\right)\cdot S$	$S^V = \sqrt{\frac{V_x^2}{c_a^2}\left(1+\frac{V}{c_a}\right)^2 + \frac{V_y^2}{c_a^2}\left(1-\frac{V_z}{c_a}\right)^2 +}$	on plane S_{xy} , S_z	
2	$\overline{V}_{xy} = iV_x +$	$S_x^V = -\frac{V_x}{c_a}\cdot S$ $S_y^V = \frac{V_y}{c_a}\cdot S$ $S_z^V = S$	$S^V = \sqrt{\left(1+\frac{V_{xy}^2}{c_a^2}\right)}\cdot S$	on plane V_{-xy} , Z	$\varphi = arctg\frac{V_{xy}}{c_a}$
3	$\overline{V}_{xz} = iV_x +$	$S_x^V = -\frac{V_x}{c_a}\left(1+\frac{V_z}{c_a}\right)\cdot S$ $S_y^V = 0$ $S_z^V = \left(1-\frac{V_z^2}{c_a^2}\right)\cdot S$	$S^V = \sqrt{\frac{V_x^2}{c_a^2}\left[\left(1+\frac{V_z}{c_a}\right)^2 + \left(1-\frac{V_z^2}{c_a^2}\right)\right]}\cdot S$	on plane xoy	$\varphi = \mathrm{arctg}\frac{-V_x}{c_a\left(1-\frac{V_z}{c_a}\right)}$
4	$\overline{V}_{yz} = jV_y$	$S_x^V = 0$ $S_y^V = \frac{V_y}{c_a}\left(1-\frac{V_z}{c_a}\right)\cdot S$ $S_z^V = \left(1-\frac{V_z^2}{c_a^2}\right)\cdot S$	$S^V = \sqrt{\left(1-\frac{V_z}{c_a}\right)^2\left[\frac{V_y^2}{c_a} + \left(1+\frac{V_z}{c_a}\right)^2\right]}\cdot S$	on plane yoz	$\varphi = \mathrm{arctg}\frac{V_y}{c_a\left(1-\frac{V_z}{c_a}\right)}$
5	iV_x	$S_x^V = -\frac{V_x}{c_a}\cdot S$ $S_y^V = 0$ $S_z^V = S$	$S^V = \sqrt{1+\frac{V^2}{c_a}}\cdot S$	on plane -xoz	$\varphi = arctg\frac{-V_x}{c_a}$
6	jV_y	$S_x^V = 0$ $S_y^V = \frac{V_y}{c_a}\cdot S$ $S_z^V = S$	$S^V = \sqrt{1+\frac{V^2}{c_a}}\cdot S$	on plane yoz	$\varphi = arctg\frac{V_y}{c_a}$
		$S_x^V = 0$			

At the figure 1, the representation of the function is done from the quality point of view.

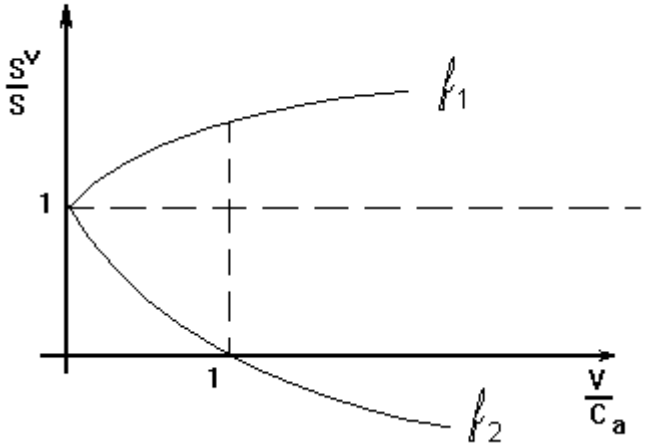

Figure 1.

$$\frac{S^{(v)}}{S} \approx 1 + \frac{V^2}{2c_a^2} = f_1\left(\frac{v}{c_a}\right) \tag{9}$$

and
$$\frac{S^{(v)}}{S^v} = 1 - \frac{V^2}{c_a^2} = f_2\left(\frac{v}{c_a}\right) \tag{10}$$

Consequently, it is quite clear that the EW is more powerful in the case of k relation then in the Z relation one.

Other cases should be discussed concretely in relation with the projections of the vector $\overline{v}$

3 - In the cylindrical coordinativ system (r, Θ, z), when electrical machines can be potentially analysed, we may obtain the same final results. So f. i., the above particular cases would led to:

$$S^{(v)} = \sqrt{1 + \frac{V_\Theta^2}{c_a^2}} \cdot S \approx \left(1 + \frac{V^2}{2c_a^2}\right) S$$

and $$S^{(v)} = \left(1 - \frac{V_\Theta^2}{2c_a^2}\right) S$$

We will obtain the same thing for the spherical coordinativ system in total accordance with the relativity theory conclusion according to which "The nature laws have to be conveyed in such a mode as to remain unchangeable when passing from a certain co-ordinates system to another (L-2)"

4 - All these conclusions are valid even for the energy and force indications:

$$W_{em}^{(v)} = k \cdot W_{em} \qquad f_{em}^{(v)} = k \cdot f_{em}$$

$$W_{e}^{(v)} = k \cdot W_{e} \qquad f_{e}^{(v)} = k \cdot f_{e}$$

$$W_{m}^{(v)} = k \cdot W_{m} \qquad f_{m}^{(v)} = k \cdot f_{m}$$

The coefficient k will eventually take on the same values indicated in the chart.

3. MORE DETAILED REPORTS ON VECTORS $\overline{E}$, $\overline{H}$

In Lorentz reports, the 31 dynamic components of vectors $\overline{E}$, $\overline{H}$, in the general case where, $\overline{V} = iV_x + jV_y + kV_z$ take the values

$$\overline{E}' = \left[\overline{V} \cdot \overline{B}\right] = \begin{vmatrix} i & j & k \\ V_x & V_y & V_z \\ 0 & B_y & 0 \end{vmatrix} = -iV_z B_y + kV_x B_y \tag{11}$$

or occurs on xoz domain and has the module:

$$E' = \frac{\sqrt{V_x^2 + V_y^2}}{c_a} E = \frac{V_{xy}}{c_a} E \tag{12}$$

while H in this case, is:

$$\overline{H}' = \left[\overline{V} \cdot \overline{D}\right] = \begin{vmatrix} i & j & k \\ V_x & V_y & V_z \\ D_x & 0 & 0 \end{vmatrix} = jV_z D_x - kV_y B_x \tag{13}$$

lies on yoz domain and has the module:

$$H' = \frac{\sqrt{V_y^2 + V_z^2}}{c_a} H = \frac{V_{yz}}{c_a} H \tag{14}$$

In this general case, for the plane electromagnetic wave, the Lorentz relations should be:

$$\overline{E}^{(v)} = i\left(1 - \frac{V_z}{c_a}\right) E_x + k \frac{V_x}{c_a} E_x \tag{15}$$

$$\overline{H}^{(v)} = j\left(1+\frac{V_z}{c_a}\right)H_y - k\frac{V_y}{c_a}H_y \tag{16}$$

Hence in the coordinates system K^v, $E^{(v)}$ occurs on xoz domain and H^v on yoz domain, temporarily with the modules:

$$\overline{E}^{(v)} = \sqrt{\left(1-\frac{V_z}{c_a}\right)^2 + \frac{V_x^2}{c_a^2}} \cdot E \tag{17}$$

$$\overline{H}^{(v)} = \sqrt{\left(1+\frac{V_z}{c_a}\right)^2 + \frac{V_y^2}{c_a^2}} \cdot H \tag{18}$$

The vectorial product of 15 with 16 ' procreates $\overline{S}^v$, formerly generated.

We noticed that in the system of X^N co-ordinates the electromagnetic plane remains plane and its directions is not after Z, but after $\overline{S}^{(v)}$ previously discussed.

From the above analysis we draw attention to the following:

1. The supplement $\overline{E}'$ and consequently $\overline{E}^{(v)}$ do not depend on the components of the velocity V_x; the supplement H' and consequently H^v do not depend on the component V_y.

2. When $\overline{V} = i \cdot V_x + j \cdot V_y$ simultaneously $\overline{E}^{(v)} > \overline{E}$ and $\overline{H}^{(v)} > \overline{H}$ hence and both domain are perceived as more consolidated in the coordinates system Kv; in this case Poynting's vector S^v>S.

3. In all cases $\overline{E}^{(v)}$ lies on domain xyz, whereas H^v domain yoz.

4. The cases 1, 3, and 2, 5, as well as 4, 7 have the same magnitude of $\overline{E}^v$; whereas the cases 1, 4 and 2, 6 as well as 3, 7 have the same magnitude of $\overline{H}^{(v)}$

5. In the coordinates system K_v the plane wave its diffusion direction; it's not that on the coordinates system K. As the result, there is procreated another configuration of electromagnetic domains, of force lines and semipotential lines subordinate to the values of E and H, thus subordinate to the velocity of the shift V.

4. FIRST EQUIVALENCE - ENVIRONMENTS EQUIVALENCE

The electromagnetic plane wave, in the coordinates system K_v , may be examined in a system K at rest, assuming a fictive equivalent environment with the parameters $\mu_a^{(v)}\varepsilon_a^{(v)}$.

The velocity of the wave ca diffusion according to z, with the velocity of the coordinates system V shift, procreate the velocity of the wave $\bar{c}_a^v$ in the coordinates system K^v.

$\bar{c}_a^v = \bar{c}_a + \bar{v}$

Henceforth we could generate

$$\left(\mu_a^v \cdot \varepsilon_a^v\right) = \frac{\mu_a \cdot \varepsilon_a}{\sqrt{1 \mp 2V_z\mu_a\varepsilon_a + \mu_a^2 Z_a^2 V^2}} \tag{19}$$

The magnitudes $\mu_a^v \cdot \varepsilon_a^v$ take different values but different velocity.

For instance, when $V=kV_Z$ we could have

$$\left(\mu_a^v \cdot \varepsilon_a^v\right)_z = \frac{\mu_a \cdot \varepsilon_a}{\sqrt{1 \mp 2V_z\mu_a Z_a}}$$

When $\bar{V} = iV_x$

$$\left(\mu_a^v \cdot \varepsilon_a^v\right)_x = \frac{\mu_a \cdot \varepsilon_a}{\sqrt{1 + \mu_a^2\varepsilon_a^2 V_x^2}}$$

When $\bar{V} = jV_y$

$$\left(\mu_a^v \cdot \varepsilon_a^v\right)_y = \frac{\mu_a \cdot \varepsilon_a}{\sqrt{1 + \mu_a^2\varepsilon_a^2 V_y^2}}$$

Hence, as it can be noted, the parameters of equivalent environment, are in conformity with velocity, with her components. The problem, put in forward, leads to the anisotropy treatment in which, as we know, the analysis is rendered more difficult especially in the general case when velocity is of any sort.

5. SECOND EQUIVALENCE - SOURCES EQUIVALENCE

The electromagnetic plane wave in the coordinates system K^v in motion may be scrutinised in the coordinates system K at rest, as if it were created by definite equivalent sources.

The principle of fields assumptions leads to the principle of supplements' action independence i.e. stopping by the two Maxwell equations, we'd have

$$rot\,\overline{H}' = rot\left[\overline{V}\,\overline{D}\right] = \overline{\delta}' \tag{20}$$

$$rot\overline{E}' = rot\left[\overline{V}\,\overline{B}\right] = -\frac{\partial B'}{\partial t} = e' \tag{21}$$

where

$\overline{\delta}' = \overline{\delta}^{v} - \overline{\delta}'; \quad \overline{e}' = \overline{e}^{v} - \overline{e}$

We can write regarding the electromagnetic plane wave being scrutinised and the velocity of the shift V in the general case.

- Regarding the supplements e'

$$\overline{\delta}' = rot\,\overline{H}' = rot\left[\overline{V}\,\overline{D}\right] = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & V_z D_x & -V_y D_x \end{vmatrix}$$

Henceforth, there is procreated

$$\overline{\delta}' = -iV_z \frac{\partial}{\partial z} D_x = -i\frac{V_z}{c_a}\frac{\partial}{\partial z}\overline{H} \tag{22}$$

and taking into account that regarding the electromagnetic wave

$$\overline{\delta}' = rot\,\overline{H}' = -i\frac{\partial}{\partial z}\overline{H} \tag{23}$$

there is generated

$$\overline{\delta}' = \frac{V_z}{c_a}\overline{\delta} \tag{24}$$

- Regarding the supplement e'

In the same way, we obtain

$$rot\,\overline{E}' = rot\left[\overline{V}\,\overline{B}\right] = -j\frac{V_z}{c_a}\frac{\partial}{\partial z}E \tag{25}$$

and

$$rot\,\overline{E} = j\frac{\partial}{\partial z}E$$

Henceforth

$$\overline{e}' = -\frac{V_z}{c_a}\overline{e} \tag{26}$$

Consequently, the equivalent sources are determined by the relations

$$\bar{\delta}_e = \bar{\delta}^v = \bar{\delta} + \bar{\delta}' = \left(1 + \frac{V_z}{c_a}\right)\bar{\delta} \tag{27}$$

and

$$\bar{e}_e = \bar{e}^v = \bar{e} + \bar{e}' = \left(1 + \frac{V_z}{c_a}\right)\bar{e} \tag{28}$$

The expressions 27), 28) that depend just upon V_Z; consequently, in all other cases when V_Z= V, the supplements $\bar{\delta}'$, $\bar{e}'$ are zero.

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