ELECTROMAGNETIC SIGNAL EVOLUTION IN A NONSTATIONARY PLASMA WITH A MAGNETIC FIELD JUMP

Alexander G. Nerukh*, Konstantin M. Yemelyanov**

* Kharkov Technical University of Radio Electronics
14, Lenin Ave., Kharkov, 310726, Ukraine
e–mail: Nerukh@ddan.kharkov.ua

** Kharkov State University
4, Svobody Sq., Kharkov, 310077, Ukraine
e–mail: K.Yemelyanov@univer.kharkov.ua

Abstract

Electromagnetic field transformation caused by a time variation in external magnetic field in plasma is considered. The time variations of magnetic field and plasma density and are approximated by a sequence of step functions. Different initial wave orientations have been analyzed. Numerical illustrations are presented. Detailed analysis for a single jump of the external magnetic field is performed.

1. INTRODUCTION

In a series of papers, see for example [1-6], the effects in a magnetized plasma whose density changes in time have been examined. It is well known that the switching on of an external magnetic field in the plasma changes the type of electromagnetic oscillations in it, i.e. changes its properties. The transformation of electromagnetic waves in the plasma with a slow time-varying external magnetic field has been considered in [7] in the adiabatic approximation. However, the adiabatic conditions are also held in the case of an abrupt magnetic field jump, when the period of oscillations is much longer than the duration of the jump. For instance, in the free electron lasers [8], the electronic beam passes through the domain of the magnetic field polarity change within 10^{-8} – 10^{-10} sec. Therefore, in the microwave band, the condition of the abrupt jump is fulfilled. This condition is $\omega\Delta t << 1$ where ω is the characteristic oscillation frequency, Δt is the jump duration.

It should be noted that, in general, the external magnetic field becomes inhomogeneous when it changes in time. Indeed, if the dimension of the considered domain is a, then the duration of the transient magnetic field is $\Delta t \approx a/u$, where u is the propagation velocity of the magnetic field disturbance. Therefore, the condition of the abrupt jump is $a << u/\omega$. In its turn, the magnetic field can be considered homogenous if $a\kappa << 1$, where κ is the spatial scale of the field. As $\kappa > \omega_m/u$,

where ω_m is the frequency of the magnetic field oscillations, we obtain the following conditions for the time change of a homogenous magnetic field to be abrupt:

$$a << \frac{u}{\omega}, \qquad a << \frac{u}{\omega_m}.$$

The fulfillment of these conditions is determined by both the type of considered electromagnetic oscillations and the way of the external magnetic field orientation that can be directed along one of the numerous branches of the plasma oscillations [9]. These conditions can be satisfied at least locally.

The paper is organized as follows. The formulation of the problem is presented in the second section. In the third section, the expression for the medium operator is obtained. In the fourth section, the solution for a single jump in every parameter is obtained exactly by means of the resolvent of the Volterra integral equation for electromagnetic field in a magnetized plasma. In the fifth section it is shown that the resolvent obtained for a single jump in parameters can be used for a sequence of jumps, i.e. an arbitrary time dependence of parameters can be approximated. The evolution of a plane electromagnetic wave and plasma oscillations in the case of a time jump in external magnetic field has been considered in the sixth section. It is shown that the initial wave is transformed to three pairs of waves with different frequencies, the waves in each pair remain transverse and propagate in the opposite directions provided that the magnetic field is parallel to the initial wave. In the case of a normal magnetic field orientation, the waves have both transverse and longitudinal components. At last, the seventh section is concerned with the investigation of the transformation of the plasma oscillations.

2. BASIC EQUATIONS

Let us consider an electromagnetic field in plasma when the external magnetic field and the plasma density vary in time starting from the zero moment. In this case the field is described by the following operator equation [10]

$$\boldsymbol{E} = \boldsymbol{F} + \hat{K}\boldsymbol{E}, \tag{1}$$

where $\hat{K} = \hat{\Gamma}\hat{V}$ is the product of the operators $\hat{\Gamma}$ and $\hat{V}$. By means of the Dirac bracket notations [11], the propagation operator $\hat{\Gamma}$ is defined as

$$\hat{\Gamma} = \int_0^t dt' \int_\infty d\boldsymbol{r}' \left\langle \boldsymbol{x} \middle| \hat{\Gamma} \middle| \boldsymbol{x}' \right\rangle,$$

where $\left\langle \boldsymbol{x}\middle|\hat{\Gamma}\middle|\boldsymbol{x}'\right\rangle = \left(\nabla\nabla - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\frac{\delta\left(t-t'-\frac{1}{c}|\boldsymbol{r}-\boldsymbol{r}'|\right)}{|\boldsymbol{r}-\boldsymbol{r}'|}$, and $\boldsymbol{x} = (t,\boldsymbol{r})$ denotes the time-spatial four-vector. In these expressions, c is the speed of light, $\delta(t)$ is a Dirac delta function.

The medium operator $\hat{V}$ is defined by the constitutive equations as $\hat{V}\boldsymbol{E} = \boldsymbol{P}(\boldsymbol{E})$ where $\boldsymbol{P}$ is the polarization vector. The free term in the (1) consists of two items

$$\boldsymbol{F}(t,\boldsymbol{r}) = \boldsymbol{F}^{(0)}(t,\boldsymbol{r}) + \int_0^\infty dt' \int_\infty d\boldsymbol{r}' \left\langle \boldsymbol{x}\middle|\hat{\Gamma}\middle|\boldsymbol{x}'\right\rangle \boldsymbol{P}_r(t',\boldsymbol{r}') \ , \tag{2}$$

where the first item

$$\boldsymbol{F}^{(0)}(t,\boldsymbol{r}) = \int_{-\infty}^{0} dt' \int_\infty d\boldsymbol{r}' \left\langle \boldsymbol{x}\middle|\hat{\Gamma}\middle|\boldsymbol{x}'\right\rangle \frac{\omega_e{}^2}{4\pi} \int_{-\infty}^{t'} dt''(t'-t'')\boldsymbol{E}(t'',\boldsymbol{r}') \tag{3}$$

takes into account only the prehistory of the electromagnetic field interaction with the plasma until the beginning of the change in the magnetic field, and $\boldsymbol{P}_r$ is the residual polarization.

A solution to (1) can be obtained by the resolvent method in the following form $\boldsymbol{E} = \boldsymbol{F} + \hat{R}\boldsymbol{F}$. We find the resolvent $\hat{R}$ as a solution to the corresponding operator equation An integral equation for the matrix element of the resolvent operator is given by:

$$\left\langle \boldsymbol{x}\middle|\hat{R}\middle|\boldsymbol{x}'\right\rangle_{ij} = \left\langle \boldsymbol{x}\middle|\hat{K}\middle|\boldsymbol{x}'\right\rangle_{ij} + \int_0^\infty dt'' \int_\infty d\boldsymbol{r}'' \left\langle \boldsymbol{x}\middle|\hat{K}\middle|\boldsymbol{x}''\right\rangle_{in} \left\langle \boldsymbol{x}''\middle|\hat{R}\middle|\boldsymbol{x}'\right\rangle_{nj} \ , \tag{4}$$

where $\left\langle \boldsymbol{x}\middle|\hat{K}\middle|\boldsymbol{x}'\right\rangle_{ij} = \int_0^\infty dt'' \int_\infty d\boldsymbol{r}'' \left\langle \boldsymbol{x}\middle|\hat{\Gamma}\middle|\boldsymbol{x}''\right\rangle_{in} \left\langle \boldsymbol{x}''\middle|\hat{V}\middle|\boldsymbol{x}'\right\rangle_{nj}$,

$$\left\langle \boldsymbol{x}\middle|\hat{\Gamma}\middle|\boldsymbol{x}'\right\rangle_{in} = \left(\frac{\partial^2}{\partial x_i \partial x_n} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\delta_{in}\right)\frac{\delta\left(t-t'-\frac{1}{c}|\boldsymbol{r}-\boldsymbol{r}'|\right)}{|\boldsymbol{r}-\boldsymbol{r}'|},$$

δ_{ij} is the Kroneker index.

3. CONSTITUTIVE RELATIONS

For obtaining a solution to (4), we have to define the medium operator in addition to the propagation operator, i.e. an expression for the polarization vector $\boldsymbol{P}$ should be

found. Let us consider a cold plasma in the approximation of isolated particles. As defined in [9], it is possible to write $\boldsymbol{P}$ in the following form:

$$\boldsymbol{P} = \int_{-\infty}^{t} \boldsymbol{j}\, dt' = \int_{-\infty}^{t} ne\boldsymbol{v}\, dt', \tag{5}$$

where n is the electron number density in plasma, $\boldsymbol{v}$ is the electron velocity. The electron velocity obeys the linearized equation of motion:

$$\frac{d\boldsymbol{v}}{dt} = \frac{e}{m}\boldsymbol{E} + \frac{e}{mc}[\boldsymbol{v}, \boldsymbol{B}_0]\theta(t), \tag{6}$$

where $\boldsymbol{B_0}$ is the external magnetic field, $\theta(t)$ is the Heaviside step function that takes into account the switching of the magnetic field at the zero moment. The solution

$$\boldsymbol{v}_-(t) = \frac{e}{m}\int_{-\infty}^{t} \boldsymbol{E}(t')dt' \tag{7}$$

to (6) till the zero moment determines the plasma polarization in the stationary state as described by the well known expression

$$\boldsymbol{P}_1 = \frac{\omega_e{}^2}{4\pi}\int_{-\infty}^{t} (t-t')\boldsymbol{E}(t')dt', \tag{8}$$

where $\omega_e{}^2 = 4\pi e^2 n/m$ is the plasma frequency.

After the zero moment, the solution to (6) is obtained by the Laplace transform technique and is given by

$$\begin{aligned}\boldsymbol{v}_+ &= \boldsymbol{v}_0\cos\Omega t + \frac{e}{m}\int_0^t \cos\Omega(t-t')\boldsymbol{E}(t')dt' - \\ &-\frac{1}{B_0}\left\{[\boldsymbol{B}_0, \boldsymbol{v}_0]\sin\Omega\, t + \boldsymbol{B}_0\frac{(\boldsymbol{B}_0, \boldsymbol{v}_0)}{B_0}(\cos\Omega\, t - 1)\right\} - \\ &-\frac{e}{mB_0}\left\{\int_0^t \sin\Omega\,(t-t')[\boldsymbol{B}_0, \boldsymbol{E}(t')]\, dt' + \frac{1}{B_0}\int_0^t [\cos\Omega(t-t') - \theta(t-t')]\boldsymbol{B}_0(\boldsymbol{B}_0, \boldsymbol{E}(t'))dt'\right\},\end{aligned} \tag{9}$$

where $\boldsymbol{v}_0 = \boldsymbol{v}_-(0)$, $\Omega = eB_0/mc$ is the Larmor frequency.

For the positive time intervals, after substituting (7) and (9) into (5), we find that the expression for the polarization consists of the two terms: $\boldsymbol{P}_2 = \boldsymbol{P}_r + \boldsymbol{P}$. The first term is the residual polarization caused by the prehistory of the electromagnetic field

till the switching on of a constant magnetic field

$$\boldsymbol{P}_r(t) = \boldsymbol{P}_1(0,\boldsymbol{r}) + \hat{\alpha}(t)\frac{d\boldsymbol{P}_1(0,\boldsymbol{r})}{dt}, \tag{10}$$

and the second term $\boldsymbol{P}$ defines the polarization of the plasma in the "new" state:

$$\boldsymbol{P}(t,\boldsymbol{r}) = \frac{\omega_e^2}{4\pi}\int_0^t \hat{\alpha}(t-t')\boldsymbol{E}(t',\boldsymbol{r})dt'. \tag{11}$$

In these equations, the susceptibility tensor $\hat{\alpha}$ represents the well-known expression for the polarization in a magnitoactive plasma [9]:

$$\alpha_{ij}(t) = \frac{1}{\Omega}\sin\Omega t\,\delta_{ij} + \frac{1}{\Omega}(\cos\Omega t - 1)e_{ikj}b_k + \left(1 - \frac{1}{\Omega}\sin\Omega t\right)b_i b_j, \tag{12}$$

where $\boldsymbol{b}$ is the unit vector in the direction of the external magnetic field after its jump, e_{ijk} is the anti-symmetric tensor of the third rank.

Now it is possible to write the expression for the medium operator. From (11) we obtain:

$$\hat{V} = \frac{\omega_e^2}{4\pi}\int_0^t dt'\,\hat{\alpha}(t-t'). \tag{13}$$

In the case of the magnetic field $\boldsymbol{B}_0$ directed along the z axis, the expression for the $\hat{\alpha}$ tensor (12) can be simplified, namely:

$$\hat{\alpha}(t) = \frac{1}{\Omega}\begin{pmatrix} \sin\Omega t & 1-\cos\Omega t & 0 \\ -1+\cos\Omega t & \sin\Omega t & 0 \\ 0 & 0 & \Omega t \end{pmatrix}. \tag{14}$$

4. THE RESOLVENT OF THE INTEGRAL EQUATION

The solution to (4) for the resolvent is convenient to obtain by using the Fourier–Laplace transform technique:

$$\left\langle \boldsymbol{p}\middle|\hat{R}\middle|\boldsymbol{p}\right\rangle_{ij} = \int_0^\infty dt\int_0^\infty dt'\int_\infty d\boldsymbol{r}\int_\infty d\boldsymbol{r}' e^{-pt-i\boldsymbol{k}\boldsymbol{r}}\left\langle \boldsymbol{x}\middle|\hat{R}\middle|\boldsymbol{x}'\right\rangle_{ij} e^{p't'+i\boldsymbol{k}'\boldsymbol{r}'}, \tag{15}$$

where $\boldsymbol{p} = (p,\boldsymbol{k})$, p is a complex variable, and $\boldsymbol{k}$ is a vector. In this representation, the matrix elements of the operators have the following form:

$$\left\langle \boldsymbol{p}\middle|\hat{\Gamma}\middle|\boldsymbol{p}'\right\rangle_{ij} = -4\pi\frac{c^2k_ik_j + p^2\delta_{ij}}{p^2 + c^2k^2}\Delta(\boldsymbol{p}-\boldsymbol{p}'), \tag{16}$$

$$\left\langle \boldsymbol{p}\middle|\hat{V}\middle|\boldsymbol{p}'\right\rangle_{ij} = \frac{\omega_e^2}{4\pi} V_{ij}\Delta(\boldsymbol{p}-\boldsymbol{p}'),$$

where $\Delta(\boldsymbol{p}-\boldsymbol{p}') = (2\pi)^3 \dfrac{\delta(\boldsymbol{k}-\boldsymbol{k}')}{p-p'}$, $V_{ij} = \tilde{\alpha}_{ij}$, $\tilde{\alpha}_{ij}$ are the Laplace transformation of the elements of the matrix $\hat{\alpha}$ (see (12)).

In the case of the external magnetic field directed along the z-axis: $\boldsymbol{b} = \{0,0,1\}$, the $\hat{V}$ matrix becomes

$$\hat{V} = \frac{1}{p^2(p^2+\Omega^2)}\begin{pmatrix} p^2 & p\Omega & 0 \\ -p\Omega & p^2 & 0 \\ 0 & 0 & p^2+\Omega^2 \end{pmatrix}. \tag{17}$$

The transformations made above enable us to obtain the resolvent of (7) as

$$\left\langle \boldsymbol{x}\middle|R^{(1)}\middle|\boldsymbol{x}'\right\rangle = \int_{-i\infty}^{i\infty}\frac{dp}{2\pi i}\int_{\infty}\frac{d\boldsymbol{k}}{(2\pi)^3}T(p,\boldsymbol{k})e^{p(t-t')+i\boldsymbol{k}(\boldsymbol{r}-\boldsymbol{r}')}, \tag{18}$$

where the T matrix has the following form:

$$T(p,\boldsymbol{k}) = \frac{-\omega_e{}^2c^2}{G(p,\boldsymbol{k})}\varphi_e{}^2\left(p^2T_1 + p\,\Omega\,T_2 + \frac{p^2}{c^2}T_3 + \frac{p^2}{c^2}(p^2+\omega_e{}^2)\,\hat{I}\right). \tag{19}$$

Here $\varphi_e^2 = p^2 + c^2k^2 + \omega_e{}^2$, and $\hat{I}$ is the identity matrix.

The T_j matrices are equal to:

$$T_1 = \begin{pmatrix} k_1{}^2 & k_1k_2 & \left(1+\dfrac{\Omega^2\varphi^2}{p^2\varphi_e{}^2}\right)k_1k_3 \\ k_2k_1 & k_2{}^2 & \left(1+\dfrac{\Omega^2\varphi^2}{p^2\varphi_e{}^2}\right)k_2k_3 \\ k_3k_1 & k_3k_2 & \left(1+\dfrac{\Omega^2\varphi^2}{p^2\varphi_e{}^2}\right)k_3{}^2 \end{pmatrix}, \tag{20}$$

$$T_2 = \begin{pmatrix} -k_1k_2 & k_2{}^2 + \dfrac{\omega_e{}^2}{\varphi_e{}^2}k_3{}^2 & -\dfrac{\omega_e{}^2}{\varphi_e{}^2}k_2k_3 \\ -k_2{}^2 - \dfrac{\omega_e{}^2}{\varphi_e{}^2}k_3{}^2 & k_2{}^2 & \dfrac{\omega_e{}^2}{\varphi_e{}^2}k_1k_3 \\ -\dfrac{\varphi^2}{\varphi_e{}^2}k_3k_2 & \dfrac{\varphi^2}{\varphi_e{}^2}k_3k_1 & 0 \end{pmatrix},$$

$$T_3 = \begin{pmatrix} 0 & p\Omega & 0 \\ -p\Omega & 0 & 0 \\ 0 & 0 & \Omega^2 \varphi^2 / \varphi_e^2 \end{pmatrix}$$

where $\varphi^2 = p^2 + c^2 k^2$.

The gyromagnetic polynomial in (19) has the following representation:

$$G(p, \boldsymbol{k}) = p^2 \varphi_e{}^4 (p^2 + \omega_e{}^2) + \Omega^2 \varphi^2 (p^2 \varphi_e{}^4 + \omega_e{}^2 c^2 k_3{}^2) . \tag{21}$$

5. THE CASE OF THE ARBITRARY TIME–VARYING MAGNETIC FIELD APPROXIMATION

An arbitrary time-varying magnetic field can be approximated by a sequence of magnetic field jumps. In this case there is a need to advance the initial point of time sequentially from step to step. After each step, the transformed electromagnetic field is determined by the constructed resolvent in which the value Ω should be replaced by a new one corresponding to a new value of the external magnetic field, i.e. Ω_{n-1} should be replaced by Ω_n when passing from *(n-1)*-th step to the *n*-th one. Clearly, there is a need to take into account the change in the equation free term to which after each step the integral over the previous time interval should be added. Let us consider that the external magnetic field is absent till the zero moment and from this moment it varies with time step by step being constant between jumps. Then, as it follows from the equation of motion for the cold plasma, after the *n*-th jump in the external magnetic field, the polarization vector $\boldsymbol{P}^{(n)}$ is determined by the following relations:

$$\boldsymbol{P}^{(n)}(t) = \boldsymbol{P}_r{}^{(n)}(t) + \frac{\omega_e{}^2}{4\pi} \int_{t_n}^{t} \hat{\alpha}^{(n)}(t - t') \boldsymbol{E}^{(n)}(t') dt' , \tag{22}$$

$$\boldsymbol{P}_r{}^{(n)}(t) = \boldsymbol{P}^{(n-1)}(t_n) + \hat{\alpha}^{(n)}(t - t_n) \frac{d\boldsymbol{P}^{(n-1)}(t_n)}{dt} , \qquad \boldsymbol{P}_r{}^{(0)} = 0 , \tag{23}$$

where t_n is the moment of the *n*-th jump in the magnetic field, $\hat{\alpha}^{(n)}$ is the corresponding susceptibility tensor.

Taking into account the expressions for the electric polarization (22), (23), the equation for the electric intensity of the electromagnetic field, after the *n*-th jump in the external magnetic field, can be represented as the Volterra integral equation of the second kind:

$$\boldsymbol{E}^{(n)}(t,\boldsymbol{r}) = \boldsymbol{F}^{(n)}(t,\boldsymbol{r}) + \int_{t_n}^{t} dt' \int_{\infty} d\boldsymbol{r}' \left\langle \boldsymbol{x} \middle| \hat{\Gamma} \middle| \boldsymbol{x}' \right\rangle \frac{\omega_e{}^2}{4\pi} \int_{t_n}^{t'} \hat{\alpha}^{(n)}(t'-t'')\boldsymbol{E}^{(n)}(t'',\boldsymbol{r}')dt'' . \quad (24)$$

The free term of this equation is equal to

$$\boldsymbol{F}^{(n)}(t,\boldsymbol{r}) = \boldsymbol{F}^{(0)}(t,\boldsymbol{r}) + \\ + \sum_{k=1}^{n-1} \int_{t_k}^{t_{k+1}} dt' \int_{\infty} d\boldsymbol{r}' \left\langle \boldsymbol{x} \middle| \hat{\Gamma} \middle| \boldsymbol{x}' \right\rangle \boldsymbol{P}_r^{(k)}(t',\boldsymbol{r}') + \int_{t_n}^{t} dt' \int_{\infty} d\boldsymbol{r}' \left\langle \boldsymbol{x} \middle| \hat{\Gamma} \middle| \boldsymbol{x}' \right\rangle \boldsymbol{P}_r^{(n)}(t',\boldsymbol{r}') . \quad (25)$$

The first term in (25) takes into account only the prehistory of the electromagnetic field interaction with the plasma until the beginning of the change in the magnetic field and is determined by (3), the other terms describe after-effects caused by the external magnetic field jumps.

To solve (24), for $\boldsymbol{E}^{(n)}$, the resolvent method is used. With the aid of the resolvent, the formula for the field intensity in the magnitoactive plasma after the *n*th jump in the magnetic field can be expressed as

$$\boldsymbol{E}^{(n)}(t,\boldsymbol{r}) = \boldsymbol{F}^{(n)}(t,\boldsymbol{r}) + \int_{0}^{t} dt' \int_{\infty} d\boldsymbol{r}' \left\langle \boldsymbol{x} \middle| R^{(n)} \middle| \boldsymbol{x}' \right\rangle \boldsymbol{F}^{(n)}(t',\boldsymbol{r}') . \quad (26)$$

As it has been mentioned above, the expression for the resolvent for the sequence of jumps is the same as the resolvent for a single jump in the external magnetic field if the Larmor frequency is changed from step to step.

6. THE TRANSFORMATION OF A PLANE WAVE

Consider the initial field as that of an eigenwave of the plasma, i.e., a plane wave: $\boldsymbol{E}_0(t,\boldsymbol{r}) = \boldsymbol{E}_0 \exp[i(\omega t - \boldsymbol{sr})]$, where $s^2 = c^2(\omega^2 - \omega_e{}^2)$. Suppose that it exists till the external magnetic field in the plasma is switched on. Let us consider that the transformations of the wave are made for a single jump in the magnetic field. Substituting the expression for the initial field in (22), (23) and then substituting the obtained expressions for $\boldsymbol{P}_r$ and $\boldsymbol{P}$ in (25) yields the free term as follows:

$$\boldsymbol{F}^{(1)}(t,\boldsymbol{r}) = \int_{-i\infty}^{i\infty} \frac{dp}{2\pi i} \Phi(p) e^{pt - i\boldsymbol{sr}} , \quad (27)$$

where

$$\Phi(p) = \frac{i}{\omega\,(p^2 + c^2 s^2)} \left[(c^2 s^2 - i\omega p)\boldsymbol{E}_0 + p^2 \omega_e{}^2 \hat{V} \boldsymbol{E}_0 + c^2 \omega_e{}^2 \boldsymbol{s}(\boldsymbol{s}\hat{V}\boldsymbol{E}_0) \right],$$

and $\hat{V}$ is the well-known gyromagnetic matrix (see (17)).

Substituting (27) into (26) yields the expression for the transformed electric field when the external magnetic field is arbitrary orientated:

$$\boldsymbol{E}^{(1)}(t,\boldsymbol{r}) = \int_{-i\infty}^{i\infty} \frac{dp}{2\pi i}\left(\hat{I} + T(p,\boldsymbol{s})\right)\Phi(p)e^{pt-i\boldsymbol{sr}} \ . \tag{28}$$

Formula (28) is a general expression which determines the transformed electromagnetic field for an arbitrary orientation of the external magnetic field. It can be simplified if the orientation of the external magnetic field is properly specified. Let us investigate two special cases of the mutual orientation of the magnetic field $\boldsymbol{B}_0$ and the wave vector $\boldsymbol{s}$.

1. The external magnetic field $\boldsymbol{B}_0$ is orientated along the propagation direction of the primary wave, i.e. $\boldsymbol{b} \parallel \boldsymbol{s} = \{0,0,s\}$, $\boldsymbol{E}_0 = \{E_0,0,0\}$. In such a case, the gyromagnetic polynomial can be simplified:

$$G(p,\boldsymbol{s}) = (p^2 + \omega_e{}^2)H(p)\,, \tag{29}$$

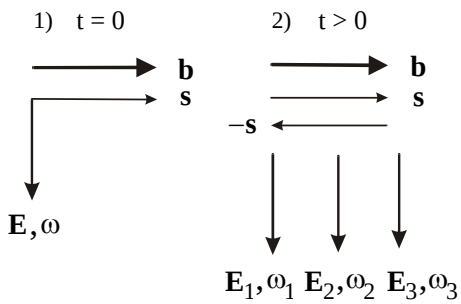
where $H(p) = p^2(p^2+\omega^2)^2 + \Omega^2(p^2+c^2s^2)^2$.

The electromagnetic field is defined by the following expression:

$$\boldsymbol{E}^{(1)}(t,\boldsymbol{r}) = \int_{-i\infty}^{i\infty} \frac{dp}{2\pi i}\frac{e^{pt-i\boldsymbol{sr}}}{\omega H(p)}\begin{Bmatrix} \omega(p+i\omega)Q^{(1)}(p) - i\omega_e{}^2\Omega^2(p^2+c^2s^2) \\ -i\omega_e{}^2\Omega\, p^2(p+i\omega) \\ 0 \end{Bmatrix}, \tag{30}$$

where $Q^{(1)}(p) = p^4 + (\omega^2+\Omega^2)p^2 + \Omega^2c^2s^2$.

The analysis of (30) gives us a possibility to conclude that the transformed electric field consists of three pairs of waves with the frequencies p_k, which are the roots of the polynomial $H(p)$ (see Figure 1). Waves in each pair remain transversal, preserve the wave vector ***s,*** and propagate in the opposite directions (see Figure 2).



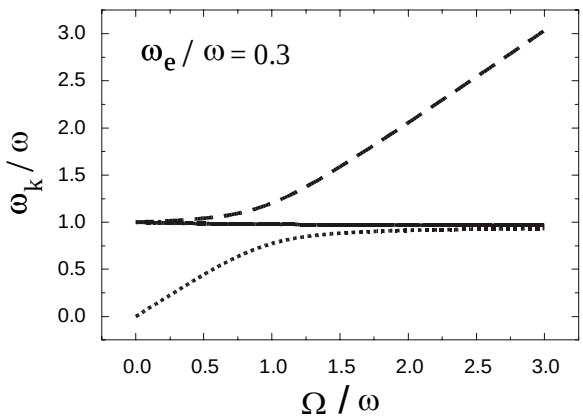


Figure 1. New frequencies in the case when the external magnetic field is orientated along the propagation direction of the primary wave ($\boldsymbol{b} \parallel \boldsymbol{s}$).

Figure 2. Transformation of electromagnetic waves when $\boldsymbol{b} \parallel \boldsymbol{s}$.

2. The external magnetic field is switched on normally to the wave vector, i.e. $\boldsymbol{b} \perp \boldsymbol{s} = \{s,0,0\}$, $\boldsymbol{E}_0 = \{0, E_2, E_3\}$. Then, the transformed electric field will be determined by the expression

$$\boldsymbol{E}^{(1)}(t,\boldsymbol{r}) = \boldsymbol{E}_{\parallel}^{(1)} \exp[i(\omega t - \boldsymbol{sr})] + \sum_{m=0}^{1}\sum_{k=1}^{2} \boldsymbol{E}_{\perp mk}^{(1)} \frac{\exp((-1)^m i\, p_k t - i\boldsymbol{sr})}{\omega Q^{(2)}((-1)^m i\, p_k)}, \qquad (31)$$

where $\boldsymbol{E}_{\parallel}^{(1)} = \{0,\ 0,\ E_3\}$ is directed along the magnetic field and is the same as the primary wave (see Figure 3),

$$\boldsymbol{E}_{\perp mk}^{(1)} = \begin{Bmatrix} -\Omega\omega_e{}^2((-1)^m p_k + \omega) \\ i\omega\ ((-1)^m p_k + \omega)(-p_k{}^2 + \omega_e{}^2 + \Omega^2) - i\Omega^2\omega_e{}^2 \\ 0 \end{Bmatrix} E_2,$$

$$Q^{(2)}(p) = p^4 + p^2(\omega^2 + \omega_e{}^2 + \Omega^2) + \omega^2\omega_e{}^2 + \Omega^2 c^2 s^2.$$

The new frequencies are defined as

$$\omega_k{}^2 = \frac{1}{2}\left(\omega^2 + \omega_e{}^2 + \Omega^2 + (-1)^{k-1}\sqrt{(\omega^2 - \omega_e{}^2 - \Omega^2)^2 + 4\omega_e{}^2\Omega^2} \right), \qquad (32)$$

and presented in Figure 4.

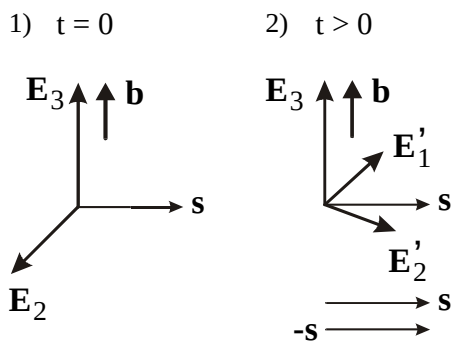


Figure 3. Transformation of electromagnetic waves when $\boldsymbol{b} \perp \boldsymbol{s}$.

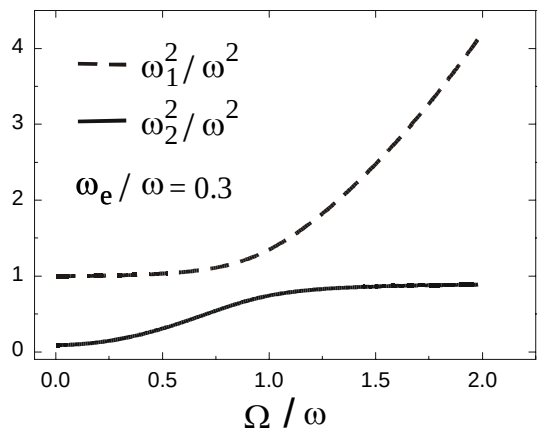


Figure 4. New frequencies in the case when the external magnetic field is normal to the direction of the primary wave $\boldsymbol{b} \perp \boldsymbol{s}$.

In this case the component of the primary electric vector parallel to $\boldsymbol{b}$ is not changed. The component normal to $\boldsymbol{b}$ forms two pairs of waves with the frequencies $\pm\omega_k$ (see Figure 3). These waves have both longitudinal and transversal components and correspond to the fast and the slow extraordinary waves, respectively [12].

It should be pointed out that in all the cases an abrupt change in the external magnetic field transfers a line spectrum of a primary electromagnetic field to the line spectrum of the transformed field.

7. THE TRANSFORMATION OF THE PLASMA OSCILLATIONS

By substituting $\boldsymbol{s} = 0$ and $\omega = \omega_e$ into (30), we obtain the transformation for the plasma oscillations. Substituting $\omega = \omega_e$ in (30) yields the transformation of plasma oscillations when the external magnetic field is switched on at the zero moment and its vector $\boldsymbol{b}$ is normal to the electric field in the plasma oscillations. The latter transforms into two elliptically polarized oscillations:

$$\boldsymbol{E}^{(1)}(t) = \sum_{k=1}^{2} \frac{E_0}{2(\omega_e{}^2 - \omega_k{}^2) + \Omega^2} \left\{ (\omega_e{}^2 - \omega_k{}^2 + \Omega^2)\cos\omega_k t, \quad \frac{\omega_e{}^2\Omega}{\omega_k}\sin\omega_k t, \quad 0 \right\}, \tag{33}$$

where $\omega_k{}^2 = \frac{1}{2}\left(2\omega_e{}^2 + \Omega^2 + (-1)^{k-1}\Omega\sqrt{4\omega_e{}^2 + \Omega^2}\right)$. The time dependence of the plasma oscillations field on the external magnetic field is shown in Figure 5. The new frequencies of the plasma oscillations are presented in Figure 6.

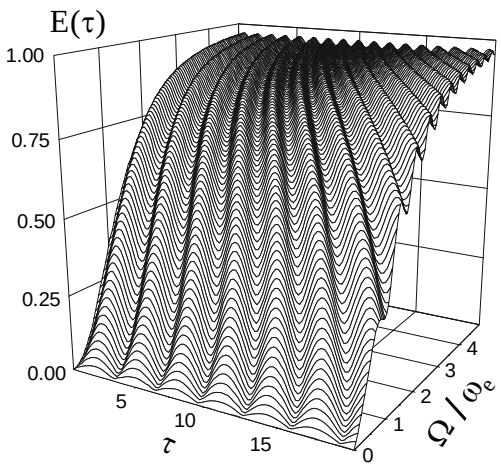


Figure 5. The time dependence of the plasma oscillations field when the external magnetic field is normal to the direction of the primary wave $\boldsymbol{b} \perp \boldsymbol{s}$.

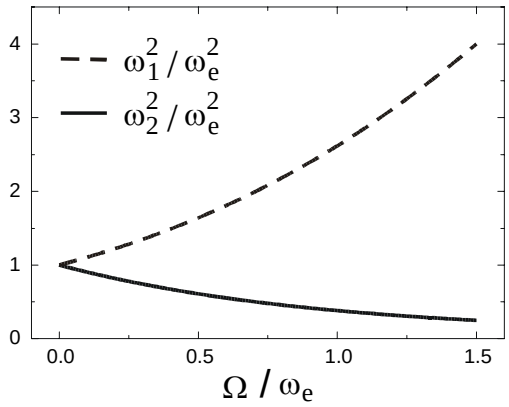


Figure 6. New frequencies of the plasma oscillations when the external magnetic field is orientated along the propagation direction of the primary wave ($\boldsymbol{b} \parallel \boldsymbol{s}$).

In the case of a weak magnetic field: $\Omega << \omega_e$, the both types of oscillations are almost circularly polarized and have the frequencies $\omega_k{}^2 \approx \omega_e{}^2 \pm \omega_k \Omega$, which are slightly different from each other.

$$\boldsymbol{E}^{(1)}(t) = \sum_{k=1}^{2} \frac{E_0\, \omega_e}{\Omega - (-1)^{k-1} 2\omega_e} \times \\ \times \left\{ \left[\frac{\Omega}{\omega_e} - (-1)^{k-1} \right] \cos \omega_k t, \quad \left[1 - (-1)^{k-1} \frac{\Omega}{2\omega_e} \right] \sin \omega_k t, \quad 0 \right\}. \tag{34}$$

In the case of a strong magnetic field: $\Omega >> \omega_e$, the oscillation with the frequency $\omega_1 \approx \Omega$ will be a circularly polarized cyclotron oscillation with a small amplitude:

$$\boldsymbol{E}_1(t) = -E_0 \frac{\omega_e{}^2}{\Omega^2} \{ \cos \Omega t, \quad \sin \Omega t, \quad 0 \}. \tag{35}$$

The other oscillation will have the frequency $\omega_2{}^2 \approx \omega_e{}^4 / \Omega^2$ and almost linear polarization:

$$\boldsymbol{E}_2(t) = -E_0 \{ \cos \omega_2 t, \quad \sin \omega_2 t, \quad 0 \}. \tag{36}$$

When the external magnetic field is parallel to the electric field of the plasma oscillations, i.e. $\boldsymbol{b} \parallel \boldsymbol{E}_0 = \{E_0, 0, 0\}$, the latter are not changed as it follows from (31), (32).

8. CONCLUSION

In this paper, a solution to the problem of electromagnetic wave propagation in a plasma with a time-varying magnetic field has been obtained. The method which is used allows to consider the simultaneous changes in the external magnetic field as well as in the plasma density. The problem is formulated as the Volterra integral equations and its solution is made by the resolvent method.

The transformation of the arbitrary orientated external magnetic field with respect to the primary plane wave has been considered. A general expression which determines the transformed electromagnetic field has been obtained. When the external magnetic field is orientated along the propagation direction of the primary wave, the transformed electric field consists of three pairs of waves. Waves in each pair remain transversal propagate in the opposite directions. In the case when the external magnetic field is switched on normally to the wave vector, the component of the primary electric vector parallel to the external magnetic field is not changed, and the component normal to the external magnetic field forms two pairs of waves which have both longitudinal and transversal components.

It should be noted that the sequential transfer of the zero moment gives a possibility to investigate the transformation of oscillations in the case of a sequence of arbitrary magnetic field jumps. In any case, the field transformed under the influence of magnetization has a discrete spectrum.

It is shown that the switching of the magnetic field that is normal to the electric field in plasma oscillations transforms the oscillations into two elliptically polarized oscillations with different frequencies. In the case of a strong magnetic field, the first of these oscillations has almost a circular polarization and frequency close to the cyclotron one, and the second has almost linear polarization and frequency close to the zero frequency. In the case of a weak magnetic field, the oscillations have the frequencies which are slightly different, and the both are almost circularly polarized. The switching on of the magnetic field parallel to the electric field does not affect the latter.

REFERENCES

[1] D.K. Kalluri, *Electromagnetics of Complex Media (Frequency Shifting by a Transient Magnetoplasma Medium)*, CRC Press, Boca Raton, 1998.

[2] T.T. Huang, J.H. Lee, D.K. Kalluri, and K.M. Groves, "Wave propagation in transient plasma: Development of Green's function", *IEEE Trans. Plasma Science*, Vol. 26, No 1, pp.19-25, 1998.

[3] D.K. Kalluri, V.R Goteti, "Frequency upshifting with power intensification of a whistler wave by a collapsing plasma medium", *J. Appl. Phys.*, Vol. 79, No 8, pp. 3875-3899, 1996.

[4] D.K. Kalluri, "Frequency shifting using magnetoplasma medium: flash ionization", *IEEE Trans. Plasma Science*, Vol. 21, No 1, pp. 77-81, 1993.

[5] B.V. Stanic, I.N. Draganovich, "Electromagnetic waves in a suddenly created magnetized Lorentz medium (transverse propagation)", *IEEE Trans. Antennas Propagation*, Vol. 44, No 10, pp. 1394-1398, 1996.

[6] M.I. Bakunov, S.N. Zhukov, "Trapping of an electromagnetic wave by the boundary of a suddenly created plasma half-space", *Proc. Int. Conf. on Mathematical Methods in Electromagnetic Theory (MMET'98)*, Vol. 1, Kharkov, pp. 340-342, 1998.

[7] N.S. Stepanov, "About adiabatic transform of electromagnetic waves spectrum in plasma", *Sov. Phys. JETP*, Vol. 53, No 6(1), pp. 2186-2193, 1967 (in Russian).

[8] T. Marshall, *Free-Electron Lasers*, Macmillan Publ., London, 1985.

[9] V.L. Ginsburg, A.A. Rukhadze, *Waves in magnitoactive plasma,* Nauka Publ., Moscow, 1975 (in Russian).

[10] A.G. Nerukh and N.A. Khizhnyak, *Modern Problems of Transient Macroscopic Electrodynamics*, Test-Radio Publ., Kharkov, 1991 (in Russian).

[11] R Balescu, *Statistical Mechanics of Charged Particles*, Interscience Publishers, London, 1963.

[12] A.I Akhiezer, et al., *Plasma Electrodynamics*, Nauka Publ., Moscow, 1974 (in Russian).