NUMERICAL ANALYSIS OF MULTILAYERS - DIELECTRIC - MULTILAYERS FABRY-PEROT INTERFEROMETER BY USING THE METHOD OF SINGLE EXPRESSION

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Abstract

For multilayers - dielectric - multilayers (MDM) Fabry - Perot interferometer analysis an advanced exact method is applied . The main feature of the method is that a solution of the wave equation in each layer of a structure is presented in the form of a single expression. The description of the method of a single expression (MSE) is presented. An appropriate electrodynamical problem of plane wave transmission through MDM structure are successfully solved. The electrical field amplitude distribution within and outside of the structure is presented. The field localization at the resonance state is observed.

1. INTRODUCTION

Fabry-Perot interferometer is a well-known interference device. It represents itself a plane-parallel dielectric plate with metallic or multilayer dielectric coatings. In most applications multilayer dielectric coatings are preferable to metallic ones. The reason is metallic coatings possess inherent losses. Fabry-Perot interferometers constructed by multilayers - dielectric - multilayers (MDM) configuration are widely used as bandpass optical filters in optical investigations and as laser resonators [1-3]. In recent time they are widely used as narrow - band optical filters in optoelectronics for printed - circuit multichannel fiber optics communication devices [4]. The last are Fabry-Perot microinterferometers, meaning that a distance between reflective coatings is comparable both with thickness of these coatings and wavelength of electromagnetic wave.

The aim of a present paper is a numerical investigation of MDM Fabry-Perot microinterferometers by using of an advanced method of single expression. An appropriate electrodynamical problem of plane wave transmission through MDM structure is considered (Fig. 1).

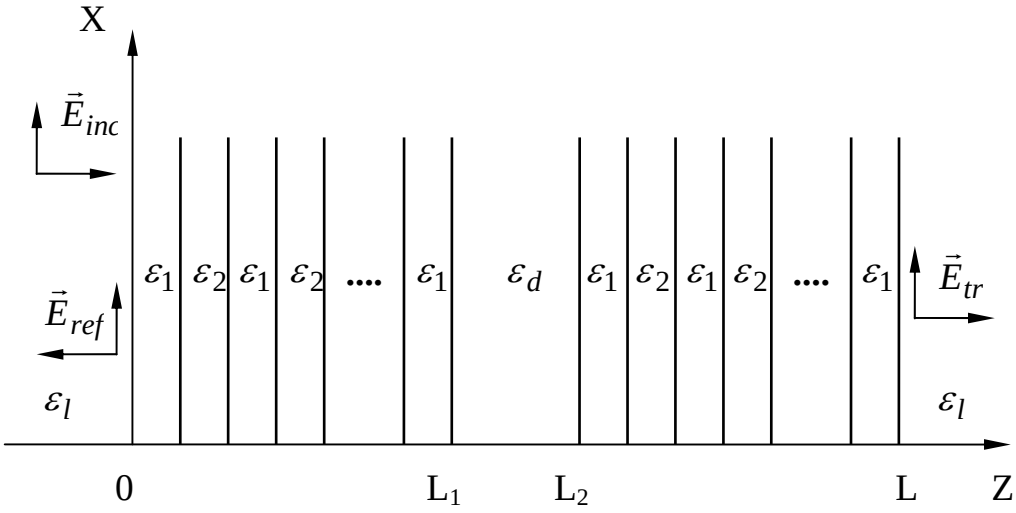

Figure1. Multilayers - dielectric - multilayers Fabry-Perot microinterferometer configuration.

All well-known traditional methods to solve this problem are based on the same approach when the solution of an appropriate wave equation is searched as a superposition of two resulting counter-propagating waves. The basic methods are: transfer matrix method [1,2], impedance method [3,5] and recurrent method [2,6]. All these methods enable to carry out investigations only for linear media and media with weak nonlinearity. In the last case one meets with complex mathematical expressions. At the same time during the last decades to solve the problem for strong nonlinear media a powerful method is being developed [7-20]. The basic distinctive feature of this method is that a solution of the wave equation in an arbitrary layer of a structure is searched in the form of a single expression, but not in the form of generally accepted counter-propagating waves. This is a new form of field presentation, that doesn't require a superposition principle fulfillment on the contrary to the traditional methods. Method of a single expression (MSE) permits to carry out investigation for media with losses (or gain) and doesn't need any preliminary assumptions concerning the form of wave equation solution (i.e. the form of traveling waves, counter-propagating waves, exponentially increasing or decreasing waves or others in the case of nonlinearity).

The MSE is very effective to investigate electromagnetic wave interaction with arbitrary multilayer media (including nonlinearity) and permits to solve numerically

diverse problems easily and comparatively rapidly by using of a well-known Runge-Kutta method of numerical integration.

Though the MSE is intended mainly for nonlinear (intensity dependent) structures numerical calculations carrying out it is obviously it well operates in linear case too. Presenting this paper we have an intention to introduce this method in detail and to exhibit its applicability in the correct multiboundary problem solving, by considering as an example a linear MDM structure.

2. METHOD OF SINGLE EXPRESSION

2.1. Principal expressions

In the present work the normal incidence of plane electromagnetic wave on MDM linear structure is considered (Fig.1).

The linear polarization of the incident electric field is assumed.

As it is known, from Maxwell's equations in the case of isotropic medium for one of the field components $\boldsymbol{E}_x$ the wave equation can be obtained:

$$\frac{\partial^2 E_x(z,t)}{\partial z^2} - \mu_0 \varepsilon_a \frac{\partial^2 E_x(z,t)}{\partial t^2} = 0, \qquad (1)$$

where $\mu_0 = 4\pi \cdot 10^{-7}$ (H/m) is the magnetic permeability of the free space, ε_a - is the permittivity of medium.

Traditionally the wave equation solution in the case of harmonic waves is represented as a sum of counter-propagating plane waves:

$$E_x(z,t) = E_0^+ \cos(\omega t - kz) + E_0^- \cos(\omega t + kz),$$

where ω - is a cyclic frequency of a steady-state plane wave, E_0^+, E_0^- - are the wave amplitudes of counter-propagating plane waves, $k = \omega\sqrt{\mu_0 \varepsilon_a}$ - the wave number of the medium.

An alternative non-traditional method presented in this paper based on the fact that solution of a wave equation (1) is searched in the form of a single expression without dividing on counter-propagating parts:

$$E_x(z,t) = U(z)\cos\big(\omega t - S(z)\big), \qquad (2)$$

where ***U(z)*** and ***S(z)*** - are the real quantities called amplitude and phase functions, respectively, and describe the resulting electromagnetic wave amplitude and phase

without the requirement of the superposition principle fulfillment. Avoiding this requirement permits to solve the problem of electromagnetic waves propagation in nonlinear media (in the media, where the permittivity depends on field intensity, i.e. there are created no new frequencies).

Such a form of solution (2) has been proposed by a number of authors [7-9] and, basically has been used by them to investigate electromagnetic wave propagation through plasma. Further this form of solution has been used to investigate electromagnetic waves propagation through nonlinear film and multilayer structures [10-20].

In the following a description of principal expressions and boundary problem solution algorithm in direct computation will be presented.

The solution (2) can be represented as a sum in complex form:

$$E_x(z,t) = \frac{\dot{E}_x(z)}{2}\exp(j\omega t) + \frac{\overset{*}{E}_x(z)}{2}\exp(-j\omega t), \tag{3}$$

where

$$\dot{E}_x(z) = U(z)\exp\big(-jS(z)\big), \tag{4}$$

and the sign $*$ shows a complex conjugation. It is preferable to use the solution in complex form, because loss (or gain) in medium can be taken into account very easily. The time dependence $\exp(j\omega t)$ will be implied but suppressed throughout the analysis.

Substituting expression (3) into the wave equation (1) gives the following Helmholtz's equation for $\dot{E}_x(z)$:

$$\frac{d^2\dot{E}_x(z)}{dz^2} + \omega^2\mu_0\tilde{\varepsilon}_a\dot{E}_x(z) = 0. \tag{5}$$

Here $\tilde{\varepsilon}_a = \varepsilon'_a + j\varepsilon''_a$ is a complex permittivity. A similar equation is obtained for the complex conjugate part $\overset{*}{E}_x(z)$.

On substituting expression (4) into equation (5) and after separation of real and imaginary parts the following set of equations is obtained:

$$\begin{cases} \dfrac{dU(z)}{d(k_0 z)} = Y(z) \\ \dfrac{dY(z)}{d(k_0 z)} = \dfrac{P^2(z)}{U^3(z)} - \varepsilon' U(z) \\ \dfrac{dP(z)}{d(k_0 z)} = \varepsilon'' U^2(z) \ . \end{cases} \tag{6}$$

where $k_0 = \omega\sqrt{\varepsilon_0 \mu_0}$ - is the free space propagation constant, $\varepsilon' = \varepsilon'_a / \varepsilon_0$, $\varepsilon'' = \varepsilon''_a / \varepsilon_0$, $\varepsilon_0 = 10^{-9} / 36\pi \ (F/m)$, ***Y(z)*** is a new variable given by the field derivative in the medium and

$$P(z) = U^2(z) \frac{dS(z)}{d(k_0 z)}, \tag{7}$$

is a quantity proportional to the power flow density along the $\boldsymbol{Z}$ -axis [8, 20].

This set of equations is completely equivalent to the Helmholtz's equation (5) and describes the spatial behaviour of electric field amplitude ***U(z),*** its derivative ***Y(z)*** and the (normalized) power flow density ***P(z)*** in the medium. In the absence of loss or gain ($\varepsilon'' = 0$) the power flow density is constant and set (6) in the presence of a cubic nonlinearity (Kerr-type nonlinearity) can be investigated by means of analytical expressions [11,14,16]. At the same time the most general approach is the direct computation of a complete set of equation (6) [12,13, 15, 17 - 20]. In this case no restrictions on nonlinearity law or on loss (or gain) are required, that allows to investigate practically important problems.

2.2. Boundary problem solution

In describing of waves outside of the investigated structure (in linear media) the traditional counter-propagating waves approach is convenient to use. Inside the structure the form of solution given by equation (2) is used. The following relations may be written:

A) In the left half-space $(z < 0)$ incident and reflected plane electromagnetic waves are expressed as follows:

$$E_{xinc} = E_{inc} \exp\left(-jk_0\sqrt{\varepsilon_{lin}}\, z\right)$$
$$E_{xref} = E_{ref} \exp\left(jk_0\sqrt{\varepsilon_{lin}}\, z\right),$$

where $\varepsilon_{lin} = \in_{lin}/\varepsilon_0$ is the permittivity of the outside linear medium. On the illuminated surface at $z = 0$ the following expressions for reflection coefficient r_{str}

$$r_{str} = \frac{E_{ref}}{E_{inc}} = \frac{U^2(0)\cdot\sqrt{\varepsilon_{lin}} - P(0) - jU(0)\cdot Y(0)}{U^2(0)\cdot\sqrt{\varepsilon_{lin}} + P(0) + jU(0)\cdot Y(0)} \tag{8}$$

and the incident field

$$E_{inc} = \left|\frac{U^2(0)\cdot\sqrt{\varepsilon_{lin}} + P(0) + jU(0)\cdot Y(0)}{2U(0)\cdot\sqrt{\varepsilon_{lin}}}\right|, \tag{9}$$

are obtained as functions of the field parameters at $z = 0$ [12], where

$$Y(0) = \left.\frac{dU(z)}{d(k_0 z)}\right|_{z=0}, \quad P(0) = U^2(0)\cdot\left.\frac{dS(z)}{d(k_0 z)}\right|_{z=0}.$$

B) For adjacent layers of the structure the ordinary boundary conditions bring to the continuity of the following values: ***U(z), Y(z), S(z),*** $dS(z)/d(k_0 z)$ and ***P(z)*** [15].

C) In the right half-space ***(z > L)*** the transmitted field is expressed as follows:

$$E_{xtrans} = E_{tr}\exp\left(-jk_0\sqrt{\varepsilon_{lin}}(z-L)\right).$$

The ordinary boundary conditions of the continuity of the electric and magnetic field tangential components at ***z = L*** in this case can be written as follows:

$$\begin{aligned} U(z=L) &= E_{tr}, \\ S(z=L) &= 0, \\ Y(z=L) &= 0, \\ P(z=L) &= U^2(L)\cdot\sqrt{\varepsilon_{lin}}. \end{aligned}$$

Starting to solve the boundary problem from the nonilluminated side of the structure ***(z = L),*** we reduce it to a Cauchy problem, i.e. to an initial value problem.

2.3. Algorithm of computation

Setting up the initial values for ***U,Y*** and ***P*** at ***z = L*** , the numerical integration of the differential equations set (6) is carried out by backward calculation, i.e. towards

the illuminated border ($z = 0$). Integration process layer by layer by using of corresponding boundary conditions for adjacent layers permits to restore E_{inc} by (9) at the end of the calculation using the obtained $U(0), Y(0), P(0)$ values and the reflection coefficient r_{str} by (8). During the integration process the spatial distributions of field amplitude $U(z)$, real $\varepsilon'(z)$ and imaginary $\varepsilon''(z)$ parts of permittivity and power flow density $P(z)$ can be obtained. The structure transmission coefficient t_{str} is calculated from: $t_{str} = E_{tr} / E_{inc}$.

3. NUMERICAL CALCULATIONS AND DISCUSSION

Numerical investigation are carried out for two configurations of multilayers - dielectric - multilayers Fabry-Perot interferometers. Multilayer reflecting coatings are constructed as $\lambda / 4$ layered structure by alternating N high-index ($n_2 = \sqrt{\varepsilon_2}$)and N+1 low-index ($n_1 = \sqrt{\varepsilon_1}$) plates (see Fig.1). Two configurations: with N=4 and N=14 are considered. The reflectivity of these layered coatings are $R = 0.0979$ and $R = 0.9469$, respectively. The other parameters of investigated structure are: $\varepsilon_1 = 2.25$; $\varepsilon_2 = 3.24$; $\varepsilon_d = 2.5$; permittivity outside of structure - $\varepsilon_l = 1$. Corresponding transmission coefficients of structures with the number of multilayer mirrors N=4 and N=14 versus normalized thickness of dielectric slab $(L_2 - L_1) / \lambda_d$ (where $\lambda_d = \lambda_0 / \sqrt{\varepsilon_d}$, λ_0 - electromagnetic wavelength in free space) are presented in Fig. 2a, 2b.

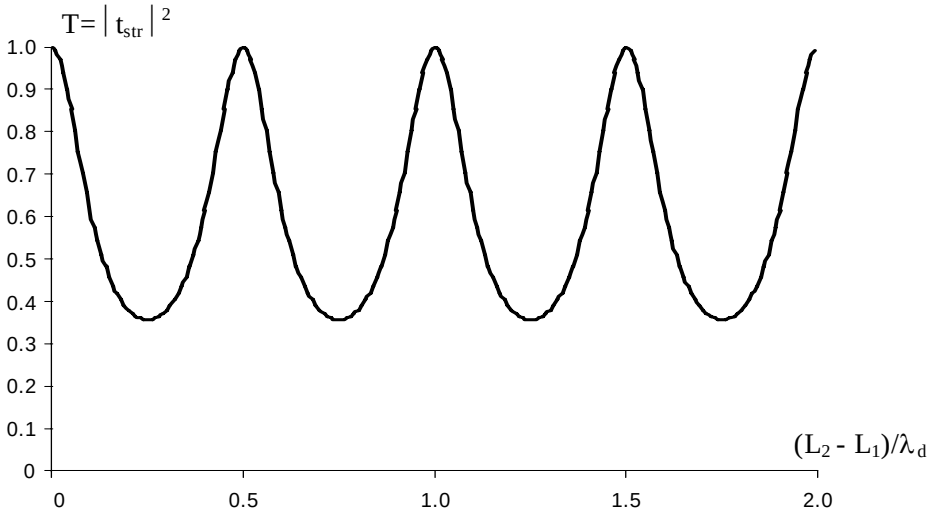

Figure 2(a). Transmission coefficient of structure with N=4 versus normalized thickness of dielectric slab.

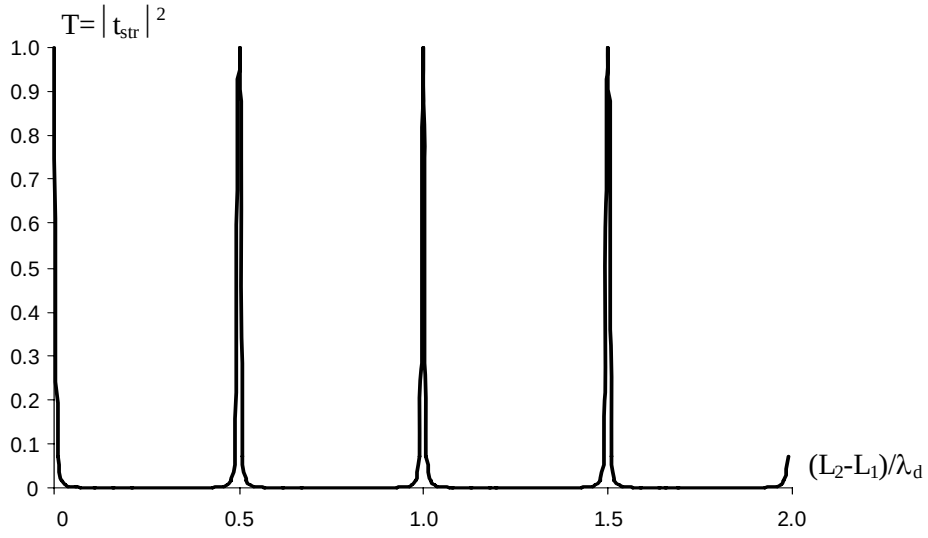

Figure 2(b). Transmission coefficient of structure with N=14 versus normalized thickness of dielectric slab.

Additionaly, through numerical investigation carried out by MSE , electrical field amplitude and permittivity distribution in structure and in near part of left half-space are obtained. In Fig. 3a, 3b corresponding dependencies are represented for structure with N=4 for minimum transmission $((L_2 - L_1)/\lambda_d = 0.25)$ and maximum $((L_2 - L_1)/\lambda_d = 0.5)$ transmission regions, accordingly.

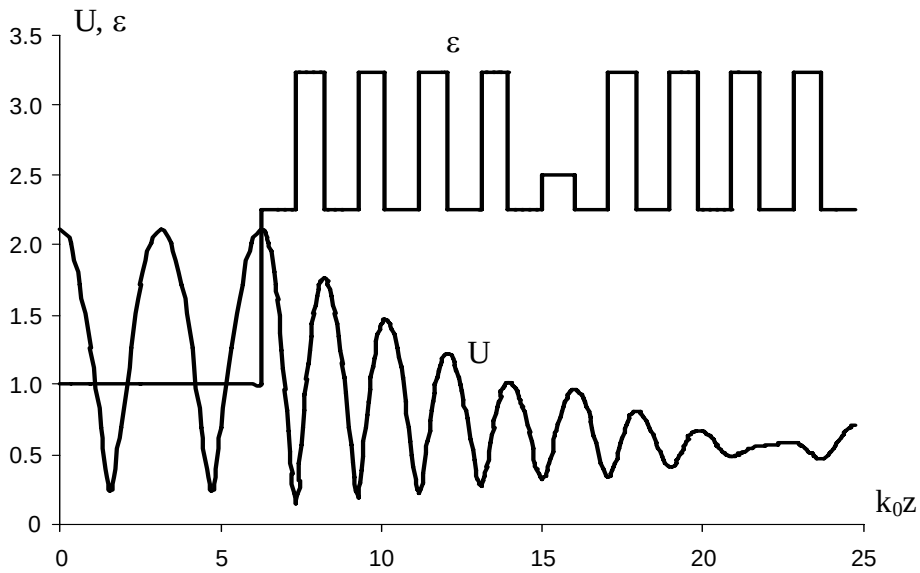

Figure 3a. Electrical field amplitude U and permittivity ε distribution in structure with N=4 for minimum transmission region.

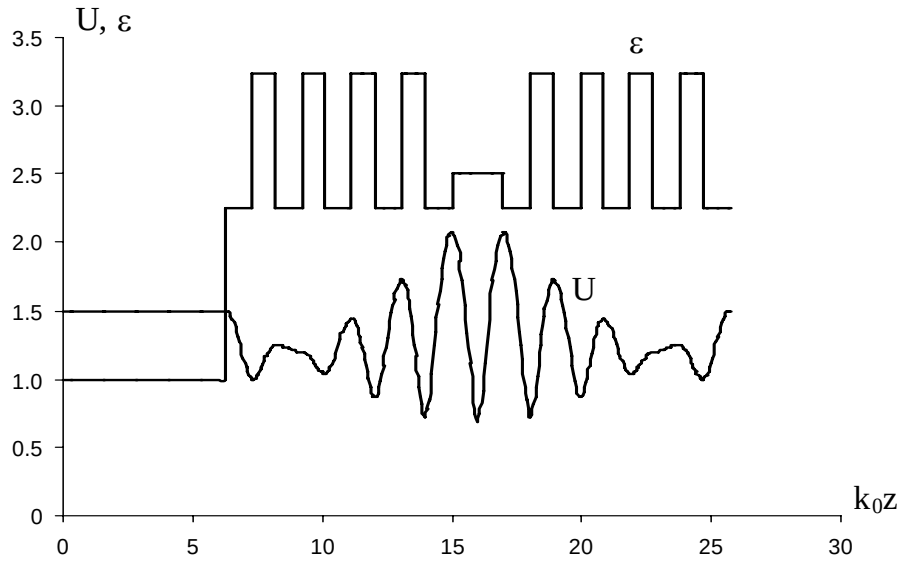

Figure 3b. Electrical field amplitude U and permittivity ε distribution in structure with N=4 for maximum transmission region.

In Fig. 4a, 4b the same dependencies are represented for structure with N=14 for minimum and maximum transmission regions, correspondingly.

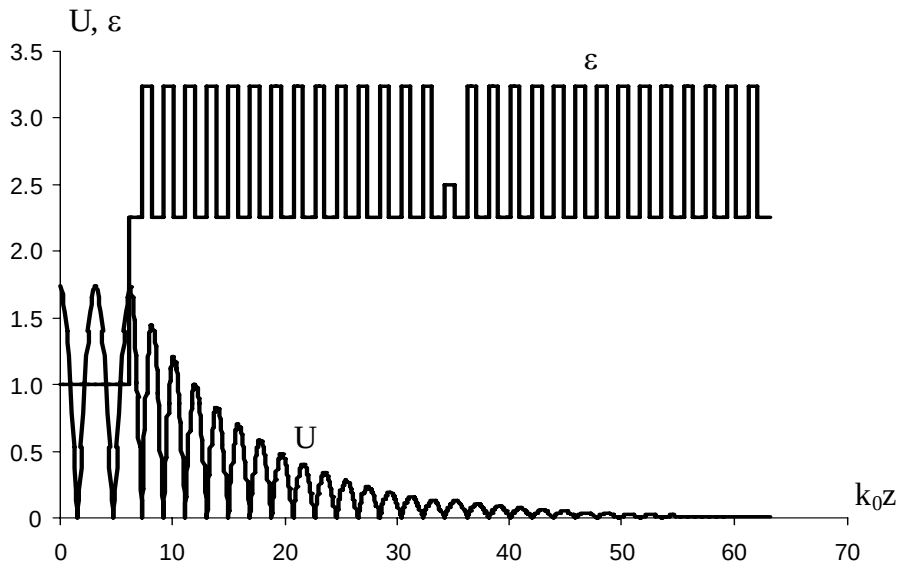

Figure 4a. Electrical field amplitude U and permittivity ε distribution in structure with N=14 for minimum transmission region.

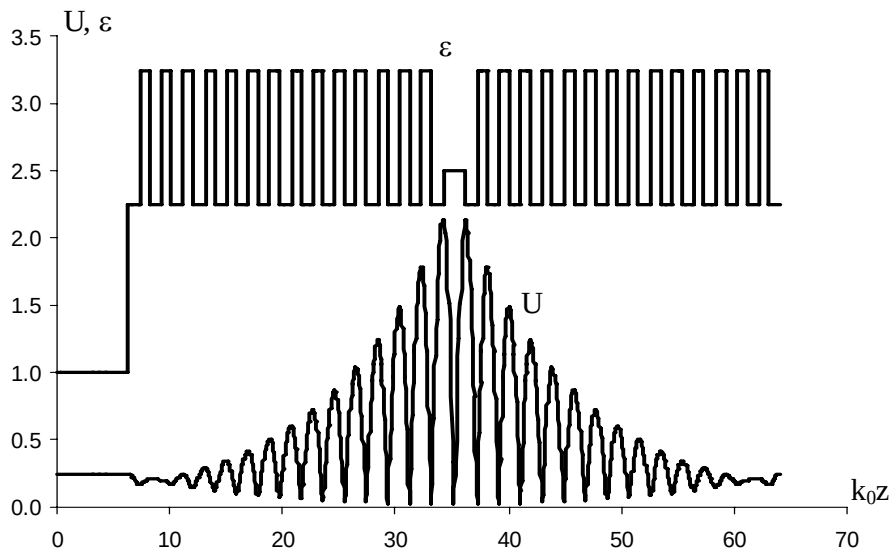

Figure 4b. Electrical field amplitude U and permittivity ε distribution in structure with N=14 for maximum transmission region.

At minimum transmission point (Fig. 3a, 4a) electrical field amplitude is localized near the illuminated side of a structure. In the left half-space there is a semi-standing wave (Fig. 3a) for structure with N=4 and a standing wave (Fig. 4a) for structure with N=14 (by the reason of high reflectivity in this case).

At maximum transmission point (Fig. 3b, 4b) the maximum of electrical field amplitude is localized in the middle part of interferometer. In the middle of multilayers - dielectric - multilayers structure the local minimum of electrical field amplitude is observed. It is interesting to point out that contrary to this at maximum transmission point in the middle of metal - dielectric - metal structure the maximum of electrical field amplitude is observed [18]. Outside of the structure there are homogeneous traveling waves with equal amplitudes.

The obtained dependencies of field localization in multilayers - dielectric - multilayers Fabry-Perot interferometer will be interesting for investigation of nonlinear properties of such type structures with nonlinear slabs.

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