A GENERALIZED THREE DIMENSIONAL GLOBAL ANALYSIS OF MMIC STRUCTURES

R. Makri , M. Gargalakos , and N.K. Uzunoglu

Institute of Communications and Computer Systems
National Technical University of Athens
9, Heroon Polytechniou Str. Zografou Athens 15773, Greece
e-mail: rodia@esd.ece.ntua.gr

Abstract

A generalized three dimensional global analysis technique is presented for Monolithic Microwave Integrated Circuit (MMIC) structures based on a full wave approach using integral equation techniques. The MMIC structures are taken to have an arbitrary geometry consisting of orthogonal parallelepiped blocks having specific electrical characteristics. Then an integral equation method is employed to obtain a rigorous formulation. The use of Green's function theoretic approaches provides a generalized formulation allowing modeling of arbitrary geometries. The numerical solution of the corresponding integral equations is obtained by using a Galerkin technique leading into a highly numerical solution. The modeling of two port MMIC is achieved by considering the connection of the three dimensional circuit structure into input – output microstrip lines. A novel approach to treat the incident – reflected and transmitted waves at the input and output microstrip lines is proposed based on singular integral theory. Numerical results are computed for a specific structure and presented.

1. INTRODUCTION

The increasingly use of higher frequencies at millimeter wavelengths poses the necessity of developing Computer Aided Design (CAD) tools based on accurate numerical techniques. At the frequencies exceeding 30 GHz the possibility of using traditional circuit theory based on lumped or even distributed parameter element modeling has limited applicability [1], [2], [3].

In the present paper a generalized approach to treat three-dimensional integrated circuit structures is proposed. In Fig. 1 a reference structure is shown. The whole structure is assumed to be placed on a two layer substrate which is grounded at the z=0 plane. Above the double layer substrate, a circuit structure is present which is driven by two microstrip lines consisting of infinite conductivity – diminishingly thin strips. The circuit structure is assumed to be made of orthogonal parallelepiped brick shape "cells". The electromagnetic properties of each unit cell is characterized by the

homogeneous complex relative permittivity value $\dot{\varepsilon}_{ri} = \varepsilon_{ri} - j\frac{\sigma_i}{\omega\varepsilon_o}$ (ε_{ri} and σ_i being the real permittivity and conductivity value of the i-th cell while ω is the angular radiation frequency and $\varepsilon_o = \frac{10^{-9}}{36\pi}\frac{F}{m}$ free space permittivity). The substrate layers are characterized with the corresponding complex relative permittivities $\dot{\varepsilon}_{rs1}$ and $\dot{\varepsilon}_{rs2}$ (see Fig.1). The whole space is assumed to have the same magnetic permeability $\mu = \mu_0 = 4\pi \cdot 10^{-7} (H/m)$ while the medium above the substrate and outside of the circuit structure is taken to be free space.

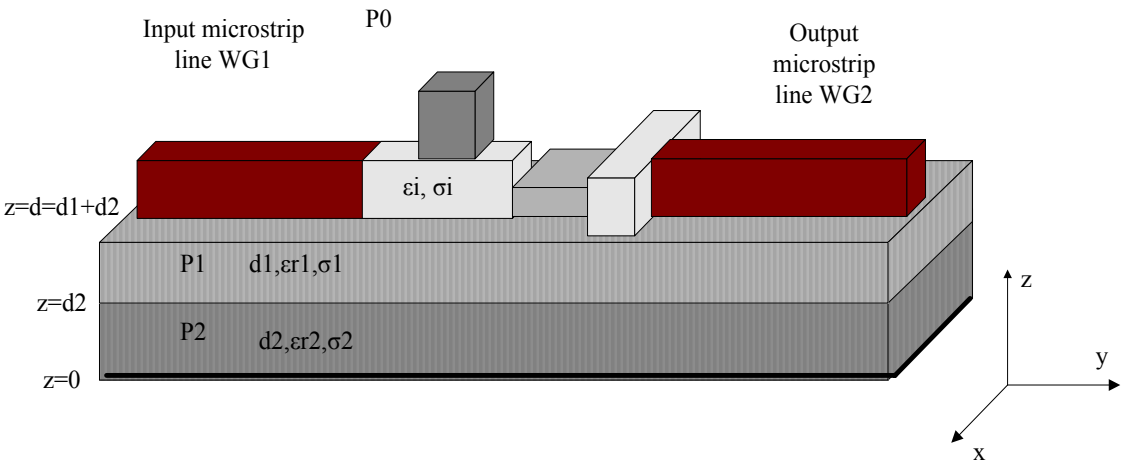

Figure 1. General circuit structure

The proposed analysis is persued in the following aims, to compute the response of the structure when an input wave is incident at one end of the microstrip line connected to the three dimensional circuit. To this end an incident wave at the microstrip line in the left hand of the circuit of Fig. 1 is assumed which has an electric field:

$$\overline{E}_i = \overline{e}_o(x,z) \cdot e^{-j\beta_o y} \tag{1}$$

where $\overline{e}_o(x,z)$ and β_o are the incident wave amplitude and propagation constant respectively. The incident wave propagation (phase) constant β_o is taken to be a real

number and the incident wave ia assumed to be free of losses. The last approximation is a reasonable assumption and will be justified in the following.

The excitation of the three dimensional circuit structure by the incident wave given in eq. (1) results into two "response" waves travelling into opposite directions along the y-axis of the input and output microstrip lines. The complex amplitudes of those two waves – measured at sufficiently large distance from the circuit structure – characterizes the scattering parameters of the structure under investigation. Indeed the computation of reflection (R) and transmission (T) coefficients of the two travelling waves are of fundamental importance to model MMIC structures despite the fact that the field at each "cell" element is computed as will be described later.

In the following analysis an $e^{+j\omega t}$ time dependence is assumed and is suppressed throughout the analysis.

2. INTEGRAL EQUATION FORMULATION

In order to develop a formulation that will allow the efficient analysis of the posed generalized structure shown in Fig.1 an integral equation approach is implemented. In this context consider the case neither when no circuit structure and microstrip lines are present above the double layers grounded substrate. Then the electromagnetic response of this infinite dimensional structure can be computed based on dyadic Green's function theory. Indeed if an elementary source excitation is assumed to be placed at the position defined by the position vector $\bar{r}'$ the response field at an arbitrary point $\bar{r}$ is computed by applying the well known Sommerfeld technique based on Spectral Domain integral representations. In essence the method leads into an analytic solution describing the field quantities in terms of Fourier type integrals defined in the literature as "Sommerfeld integrals" [4], [5].

The corresponding dyadic Green's function is obtained by solving the Helmholtz equation:

$$(\nabla^2 + k_0^2)\overline{\overline{G}}(\bar{r} \mid \bar{r}') = -\overline{\overline{I}}\delta(\bar{r} - \bar{r}') \qquad (2)$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$, $k_o = \frac{2\pi}{\lambda}$ is the free space wave number (propagation constant), $\delta(\bar{r} - \bar{r}')$ delta function, $\bar{\bar{I}}$ is the unit dyadic and satisfying the boundary conditions on the ground plane and the substrate interface layers.

Following a lengthy algebra the dyadic Green's function is obtained as a superposition of two terms in the following format:

$$\bar{\bar{G}}(\bar{r} \mid \bar{r}') = \bar{\bar{G}}_o(\bar{r} \mid \bar{r}') + \bar{\bar{G}}_r(\bar{r} \mid \bar{r}') \tag{3}$$

where $\bar{\bar{G}}_o(\bar{r} \mid \bar{r}')$ is the infinite space singular Green's function associated with the substrate medium where the "source" point $\bar{r}'$ is in the air medium since the "cell" elements belongs to this region and $\bar{\bar{G}}_r(\bar{r} \mid \bar{r}')$ accounts for the response of the grounded double layer medium.

It is well known that,

$$\bar{\bar{G}}_o(\bar{r} \mid \bar{r}') = (\bar{\bar{I}} + k_o^{-2}\nabla\nabla)\frac{e^{-jk_o|\bar{r}-\bar{r}'|}}{4\pi|\bar{r}-\bar{r}'|} \tag{4}$$

and

$$\bar{\bar{G}}_r(\bar{r} \mid \bar{r}') = \int_{-\infty}^{+\infty} dk_x \int_{-\infty}^{+\infty} dk_y e^{jk_x(x-x')+jk_y(y-y')}(F_-(k_x,k_y)e^{-jg(k_x,k_y)z} + F_+(k_x,k_y)e^{+jg(k_x,k_y)z}) \tag{5}$$

where $g(k_x, k_y) = \sqrt{k_o^2\dot{\varepsilon}_{rs} - (k_x^2 + k_y^2)}$ and F_-, F_+ are layer dependent functions.

Notice that eq. (3), (5) are valid for the air medium above the double layer substrate.

Assuming the dyadic Green's function is known then the electric field at an arbitrary point $\bar{r}$ can be computed making use of the following integral equation:

$$\bar{E}(\bar{r}) = \sum_{\substack{\text{circuit}\\ \text{cells}\\ i=1,\ldots N}} \iiint_{V_i} k_o^2 \delta\dot{\varepsilon}_i \bar{\bar{G}}(\bar{r}\,|\,\bar{r}') \cdot \bar{E}(\bar{r}')d\bar{r}' - j\omega\mu_o \iint_{\substack{\text{Input}\\ \text{M/str.}\\ \text{Line}}} \bar{\bar{G}}(\bar{r}\,|\,\bar{r}') \cdot \vec{\bar{J}}(\bar{r}')ds' - j\omega\mu_o \iint_{\substack{\text{Output}\\ \text{M/str.}\\ \text{Line}}} \bar{\bar{G}}(\bar{r}\,|\,\bar{r}') \cdot \overleftarrow{\bar{J}}(\bar{r}')ds' \tag{6}$$

where $\delta\dot{\varepsilon}_i = \dot{\varepsilon}_{ri} - 1$, V_i the volume corresponding to the i-th circuit cell, $\vec{\bar{J}}(\bar{r}')$ the current distribution on the input line and $\overleftarrow{\bar{J}}(\bar{r}')$ the current distribution on the output line.

Notice that in eq. (6) the first term in the right hand side is associated with the "Polarization" currents of the circuit cell elements while the second and third terms are related to input and output lines respectively.

The input current consists of two terms: (a) incident and (b) reflected waves while the outgoing current consists of only transmitted current. Describing these currents mathematically the following expressions are obtained:

$$\vec{\bar{J}}(\bar{r}') = \bar{w}(x)e^{-j\dot{\beta}_0 y} + R\bar{w}(x)e^{+j\dot{\beta}_o y} \tag{7}$$

$$\overleftarrow{\bar{J}}(\bar{r}') = T\bar{w}(x)e^{\ j\dot{\beta}_0 y} \tag{8}$$

where R and T are the reflection and transmission coefficients respectively, $\bar{w}(x)$ is a standard current distribution profile taking into account the edge conditions at the microstrip line edges while $\dot{\beta}_o$ is the complex propagation constant of the quasi-TEM mode and $\beta_o = \mathrm{Re}(\dot{\beta}_o)$.

The electric field $\overline{E}(\overline{r})$ induced inside parallelepiped cells (see eq.6) is described in terms of exponential waves satisfying the corresponding wave equation. According to this the following describing equations are employed within the i-th cell:

$$\overline{E}_i(\overline{r}) = \iint d\hat{k} e^{jk_o\sqrt{\varepsilon_{ri}}\hat{k}\cdot\overline{r}}\overline{c}_i(\hat{k}) \tag{9}$$

where $\overline{c}_i(\hat{k})$ is the unknown spectral expansion function. Notice that since the $\psi(\hat{k}) = e^{jk_o\sqrt{\varepsilon_{ri}}\hat{k}\cdot\overline{r}}$ function satisfies the wave equation

$$(\nabla^2 + k_o^2\varepsilon_{ri})\psi(\hat{k}) = 0 \tag{10}$$

the unit vector $\hat{k}$ represents the propagation direction of the plane wave function used as describing function [6].

In order to pursue the Galerkin technique [7], [8], [9], [10] the representations given in eqs. (9), (7), and (8) are substituted into the fundamental equation (6) and then the inner products of the corresponding testing functions are computed in a Galerkin manner. Following an extremely lengthy algebra and analysis, the fundamental system is reduced into a linear system of equations of the following form

$$\begin{bmatrix} \overline{\overline{F}}_{1111} & \cdots & \overline{\overline{F}}_{1kli} & \cdots & \overline{\overline{F}}_{1klN} & \overline{\overline{H}}_{1klWG1} & \overline{\overline{H}}_{1klWG1} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \overline{\overline{F}}_{jkl1} & \cdots & \overline{\overline{F}}_{jkli} & \cdots & \overline{\overline{F}}_{jklN} & \overline{\overline{H}}_{jklWG1} & \overline{\overline{H}}_{jklWG1} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \overline{\overline{F}}_{Nkl1} & \cdots & \overline{\overline{F}}_{Nkli} & \cdots & \overline{\overline{F}}_{NklN} & \overline{\overline{H}}_{NklWG1} & \overline{\overline{H}}_{NklWG1} \\ \overline{\overline{V}}_{WG1kl1} & \cdots & \overline{\overline{V}}_{WG1kli} & \cdots & \overline{\overline{V}}_{WG1klN} & \overline{\overline{W}}_{WG1WG1} & \overline{\overline{W}}_{WG1WG2} \\ \overline{\overline{P}}_{WG2kl1} & \cdots & \overline{\overline{P}}_{WG2kli} & \cdots & \overline{\overline{P}}_{WG2klN} & \overline{\overline{Q}}_{WG2WG1} & \overline{\overline{Q}}_{WG2WG2} \end{bmatrix} \cdot \begin{bmatrix} \overline{c}_1 \\ \vdots \\ \overline{c}_i \\ \vdots \\ \overline{c}_L \\ R \\ T \end{bmatrix} = \begin{bmatrix} B_1 \\ \vdots \\ B_i \\ \vdots \\ B_N \\ B_R \\ B_T \end{bmatrix}$$

Where the "dyadic" coupling elements $\overline{\overline{F}}_{ij}$ are associated with the interaction between the parallelepiped cell circuit elements while the $\overline{\overline{V}}_{ij}$, $\overline{\overline{P}}_{ij}$ and $\overline{\overline{H}}_{ij}$ (i=1,2,…, j=1,2,…) elements are related to the coupling between input/output microstrip lines and circuit cell elements.

The right hand side elements are associated with the incident wave excitation.

In the context of the above calculation improper integrals such as

$$\int_{-\infty}^{0} dy e^{-j\beta_o y} e^{jk_y y} \tag{12}$$

are encountered.

Since the incident wave propagation constant β_o is being taken without loss term, the tackling of the above integrals needs particular care in the context of the generalized function theory. Furthermore, the following equation

$$\int_{-\infty}^{+\infty} dy' \int_{-w/2}^{w/2} \overline{\overline{G}}(\bar{r},\bar{r}') \cdot \overline{w}(x') e^{-j\beta_o y'} = 0 \tag{13}$$

when $\bar{r}$ is on the infinite microstrip line is employed to elaborate the computation of necessary matrix elements. Notice that eq. (13) is the propagation condition on the double side infinite microstrip line.

3. NUMERICAL COMPUTATIONS

The computational technique presented in the previous section has been employed to compute various structures. In order to evaluate the accuracy of the developed numerical code, exhaustive convergence tests on various levels of the code has been carried out such as the accuracy of the numerical integrations in computing the matrix elements in eq. (11) and the right hand side elements. Furthermore, the validity of energy conservation is also checked in every case as well as the convergence of the computed R and T coefficients. In computing the matrix elements in eq. (11) and to accelerate the numerical computation parallel processing programming techniques are implemented [11], [12].

The case of a longitudinal discontinuity has been studied. In Fig. 2 the geometry of the structure under study is shown. The computed R and T coefficients are shown in Table I for various discontinuity cases.

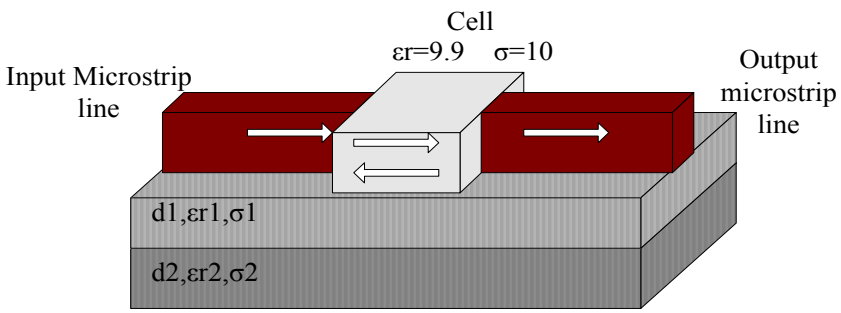

Figure 2. Longitudinal Discontinuity Geometry

The geometry shown in Fig. 2 represents a general structure with the following initial characteristics:

Frequency of operation: 50 GHz

Substrate characteristics

1st layer's height: $d_1 = 22.7\mu m$ 2nd layer's height: $d_2 = 17.1\mu m$

1st layer's dielectric constant: $\varepsilon_{r1} = 10$ 2nd layer's dielectric constant: $\varepsilon_{r2} = 12.85$

1st layer's conductivity: $\sigma_1 = 0.00278$ 2nd layer's conductivity: $\sigma_2 = 0.00278$

Complex propagation constant: $\beta = 2755.78 - j18.193$

Dimensions of microstrip lines and cells:

Input microstrip line dimensions: $l_x = 30\mu m$, $l_y = 0.5m$, $l_z = 5\mu m$

Output microstrip line dimensions: $l_x = 30\mu m$, $l_y = 0.5m$, $l_z = 5\mu m$

Dielectric cell dimensions: $l_x = 30\mu m$, $l_y = 953\mu m$, $l_z = 5\mu m$

Electrical characteristics of the cell:

Dielectric constant: $\varepsilon_r = 9.9$

Conductivity: $\sigma = 10$

The arrows in Fig. 2 represent the number and directions of plane waves assumed in the geometry under consideration. Varying some of the above characteristics of the

structure, several results are derived depending on different discontinuity cases and are shown in the following Table I.

Longitudinal Discontinuity type (cases in respect to Fig. 2)	R	T
Case A: Fig. 2 with identical Transverse dimensions of cell and I/O microstrip lines	$0.639 - j0.446$	$7.87 \cdot 10^{-2} - j0.253$
Case B: case A assuming one plane wave at the cell	$0.695 - j0.42$	$4.43 \cdot 10^{-2} - j0.506$
Case C: case B with $\sigma_1 = \sigma_2 = 0.139$ layers' Conductivity	$0.768 - j0.553$	$-5.028 \cdot 10^{-2} - j3.61 \cdot 10^{-2}$
Case D: case C with cell conductivity $\sigma = 500$	$0.865 - j0.597$	$-0.146 - j4.67 \cdot 10^{-2}$
Case E: case A with $\sigma_1 = \sigma_2 = 0.139$ layers' Conductivity, assuming free space instead of cell	$0.86 - j0.58$	$-3.47 \cdot 10^{-2} - j4.23 \cdot 10^{-2}$
Case F: Fig. 2	$0.95 - j0.59$	$-2.99 \cdot 10^{-2} + j8.33 \cdot 10^{-2}$

Table 1 Numerical Results for Longitudinal Discontinuities

4. CONCLUSION

A Generalized Three Dimensional full wave dynamic solution to analyze MMIC structures has been presented. The method is based on the use of Galerkin technique and allows the analysis of arbitrary geometries. Numerical results are presented for specific longitudinal discontinuities.

ACKNOWLEDGMENTS

Authors wish to express their thanks to Dr. A. Marsh for his support in parallel programming parts of the numerical code.

REFERENCES

[1] Pisti B. Katehi, "A Generalized Method for the Evaluation of Mutual Coupling in Microstrip Arrays", IEEE Trans. on Antennas and Propagation, vol. AP-35, p.p. 125-133, 1987

[2] R. Goyal (Editor), "Monolithic Microwave Integrated Circuits: Technology & Design", Artech House, 1989, Boston

[3] "Special Issue on Modeling and Design of Coplanar Monolithic Microwave and Millimeter-Wave Integrated Circuits", IEEE Trans. on Microwave Theory and Techniques vol. 41, September 1993

[4] A. Sommerfeld, "Partial Differential Equations of Physics", Academic Press, 1949, New York

[5] Tai Chen-To, "Dyadic Green's Functions in Electromagnetic Theory", London, International Textbook Co.. 1971

[6] N.K. Uzunoglu, N.G. Alexopoulos and J.G. Fikioris, "Radiation Propertie of Microstrip Dipoles", IEEE Trans. on Antennas and Propagation, vol. AP-27, p.p. 853-858, 1979

[7] D.S. Jones, "Theory of Electromagnetism", Pergamon Press, London, 1964

[8] A.R. Holt, N.K. Uzunoglu and B.G. Evans, "An Integral Equation Solution to the Scattering of Electromagnetic Radiation by Dielectric Spheroids and Elipsoids", IEEE Trans. on Antennas and Propagation, vol. AP-26, p.p. 706-712, 1978

[9] Roger F. Harrington, "Matrix Methods for Field Problems", Proceedings of the IEEE, vol. 55, No. 2, p.p. 136-149, February 1967

[10] M.N. Neu, "Method of Moments as applied to electromagnetic problems", IEEE Trans on Microwave Theory and Techniques, vol MTT-33, p.p. 972-980, Oct. 1985

[11] L. Grandinetti, J. Kowalik, M. Vajtersic (Editors), "Advances in High Performance Computing", NATO ASI Series, Kluwer Academic Publishers, 1997

[12] A. Marsh and D.I. Kaklamani, "Applying High Performance Computing in Electromagnetics", p.p. 303-320, Advances in High Performance Computing, Kluwer Academic Publishers, 1997