APPLICATION OF THE MODIFIED METHOD OF AUXILIARY SOURCES FOR THE ANALYSIS OF MICROSTRIP ANTENNAS

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Abstract

The modified method of auxiliary sources (mMAS) is applied for the analysis of planar microstrip antennas. The principle idea of mMAS is based on the concept of assuming auxiliary unknown current and charge density distribution on an auxiliary surface, which is usually taken to be conformal to the real surface. Applying the current contribution law the charge density is determined out of the current components flowing on the auxiliary surface. The respective boundary conditions on the conductors and the substrate material are utilized to solve for the auxiliary current on the auxiliary surface. The substrate material could be lossless, loss or inhomogeneous dielectric as well as complex material. Numerical results are presented for planar microstrip antenna cases, concerning near field and far field calculations. The validation of the results is confirmed by the satisfaction of all appropriate boundary conditions.

1. INTRODUCTION

Microstrip antennas have several unique features, which make them attractive candidates for many antenna applications, ranging from mobile communication to phased array radar systems [1-23]. Recent advances in packaging technology of microwave and millimeter wave antennas have engender a considerable amount of interest in the development of computer aided design tool for these packages. Creation of efficient method for numerical solution of these problems is of great practical interest.

Over the past several years, many techniques have been developed [1]-[23] for the analysis of microstrip antennas. Even though the analysis in the past has been done utilized various circuit theory and other approximations, recently work has been performed utilizing the Green's functions/Sommerlfeld approach [24]-[28]. In this analysis, it is generally assumed that the ground planes as well as the dielectric regions are of infinite extent. Finite, single layer dielectric structures have been treated in [29]-[31]. An integral equation approach to the analysis of finite microstrip antennas is done [32].

In Section II, the general formulation of mMAS for the analysis EM problem is given. Section III describes the way mMAS is applied for the analysis of planar microstrip antennas while in

Section IV numerical results are presented concerning the near field and far field radiation patterns of a microstrip antenna.

2. THE MMAS FORMULATION

Let assume a linearly polarized, monochromatic electromagnetic wave with time dependence $\exp(-j\omega t)$, incident on a penetrable body with surface *S*. It is known that the given problem is reduced to the solution of the Helmholtz equation, in the inner and the outer region of the scatterer, as

$$\nabla^2 \underline{F}^{sc}(\underline{r}) + k_\ell^2 \underline{F}^{sc}(\underline{r}) = 0, \qquad \ell = 1,2 \tag{1}$$

and the boundary conditions on the surface *S* of the scatterer, as

$$\pounds\left\{\underline{F}^{sc}(\underline{r}) + \underline{F}^{inc}(\underline{r})\right\} = 0, \quad \underline{r} \in S \tag{2}$$

where $k_1 = k_{in}$ or $k_2 = k_{out}$ represent the wavenumber inside or outside the scatterer, respectively, $\underline{F}^{sc}(\underline{r})$ and $\underline{F}^{inc}(\underline{r})$ are the functions describing the scattered and incident fields, respectively, and $\pounds$ denotes the boundary conditions operator.

The principle idea of MAS is based on the concept of distributing a set of auxiliary current sources with unknown coefficients on virtual surfaces usually conforming to the real surfaces of the examined scattering structures [33]-[45]. Thus, auxiliary sources are distributed on auxiliary surfaces σ_ℓ, $\ell = 1,\ldots,L$ lying inside and outside the scatterer, to describe the external and internal excited fields, respectively. For conducting objects, *L*=1, since only one (internal) surface is required to describe the external field, while for dielectric media *L*=2, since two surfaces are required to describe both the external and internal fields. Then, using as auxiliary sources fundamental solutions $\underline{\Pi}(|\underline{r}_n - \underline{r}|)$ of the Helmholtz equation (1), where $\underline{r}_n$ are the positions of the auxiliary sources on the auxiliary surface σ_ℓ, a new family of functions is constructed as,

$$\underline{\Psi}(\underline{r}_n, \underline{r}) = \square\left\{\underline{\Pi}(\underline{r}_n, \underline{r})\right\}, \text{ n=1,2,…} \tag{3}$$

where $\square$ is a linear operator and

$$\underline{\Pi}(\underline{r}_n, \underline{r}) = \frac{\underline{p}_n}{|\underline{r}_n - \underline{r}|} e^{jk_\ell |\underline{r}_n - \underline{r}|} \tag{4}$$

In [8], it is shown that, the family of functions $\{\underline{\Pi}(\underline{r}_n,\underline{r})\}$ and $\{\underline{\Pi}(\underline{r}_n,\underline{r})\}$ satisfy the following properties:

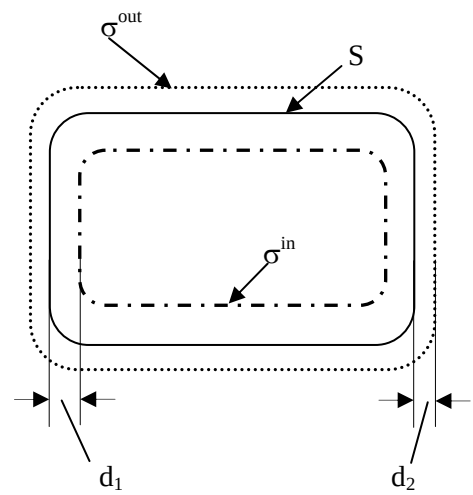

Figure 1. The MAS geometry

a. For each function of the family $\{\underline{\Pi}(\underline{r}_n,\underline{r})\}$ which satisfies equation (1), a new function of the family $\{\underline{\Pi}(\underline{r}_n,\underline{r})\}$ on the surface *S* always exists.

b. The family of functions $\{\underline{\Pi}(\underline{r}_n,\underline{r})\}$ is complete and linearly independent on the surface *S* for the functional space L_2.

c. The weights of the auxiliary sources, which are the vector coefficients $\underline{p}_n$ of equation (4), can be calculated by forcing the conditions of the best (in the meaning of L_2) decomposition of the family $\{\underline{\Pi}(\underline{r}_n,\underline{r})\}$, using the first *N* functions of the family (3). Thus, the scattered field on the surface *S* can be represented by a linear combination of $\underline{\Psi}(\underline{r}_n,\underline{r})$ functions

$$\underline{F}^{sc}(\underline{r}_s) \approx \sum_{n=1}^{N} \underline{\Pi}_n(\underline{r}_n,\underline{r}_s) = \sum_{n=1}^{N} \frac{\underline{p}_n}{|\underline{r}_n - \underline{r}_s|} e^{jk_\ell |\underline{r}_n - \underline{r}_s|}, \quad \underline{r}_s \in S \tag{5}$$

which tends to the exact solution as $N \to \infty$.

The properties (a)-(c) of the fundamental solutions guarantee the existence of the vector coefficients $\{\underline{p}_n\}_{n=1}^{N}$, which ensure the validity of equation (2). These coefficients can be defined using the collocation or point matching technique, by imposing the boundary conditions at a finite number of points located on the interfaces between different media. For example, in Fig. 1 the collocation surface (*S*) and the auxiliary surfaces ($\sigma^{in}, \sigma^{out}$) are illustrated for the problem of an infinite rectangular dielectric cylinder. Note that no external auxiliary surface is needed for the case of perfectly conductors, since the internal field is equal to zero everywhere. In the problem considered in the next section of the paper, all the necessary auxiliary surfaces are similarly defined.

Thus, in analyzing three-dimensional (3-D) electromagnetic structures using the MAS [7]-[11], the electric field at an arbitrary point $\underline{r}$ can be represented as a superposition of fields,

$$\underline{E}^{sc}(\underline{r}_n,\underline{r}) = \square\, \underline{\Pi}_n(\underline{r}_n,\underline{r}) \tag{6}$$

with $\square = \nabla(\nabla\cdot) + k_\ell^2$. This electric field corresponds to the Green's function of an electric dipole placed at the point $\underline{r}_n$, with a dipole moment $\underline{p}_n$ and $\underline{\Pi}_n(\underline{r}_n,\underline{r}) = \frac{\underline{p}_n}{R_n} e^{jk_\ell R_n}$, $R_n = \sqrt{(x_n - x)^2 + (y_n - y)^2 + (z_n - z)^2}$, where k_ℓ denotes the wavenumber of the region where the observation point is located.

At each point lying on the auxiliary surfaces (σ^{in}, σ^{out}), a set of orthogonal axes (u,v) can be defined, tangential to the surface, with unit vectors along the axes u and v given by the equations

$$\underline{n}^u = N_x^u \hat{x} + N_y^u \hat{y} + N_z^u \hat{z}$$
$$\underline{n}^v = N_x^v \hat{x} + N_y^v \hat{y} + N_z^v \hat{z}$$

where $\hat{x}, \hat{y}, \hat{z}$ represent the unit vectors of the reference Cartesian coordinates system.

For each auxiliary source, placed on the considered auxiliary surfaces, the unknown vector $\underline{p}_n$ is written in the local coordinates system as $\underline{p}_n = p_n^u \underline{n}^u + p_n^v \underline{n}^v$ or in the global Cartesian coordinates system as, $\underline{p}_n = p_n^x \hat{x} + p_n^y \hat{y} + p_n^z \hat{z}$, with

$$\begin{aligned} p_n^u &= p_n^x N_x^u + p_n^y N_y^u + p_n^z N_z^u \\ p_n^v &= p_n^v N_x^v + p_n^y N_y^v + p_n^z N_z^v \end{aligned}, \quad n = 1,2,\ldots,N$$

Thus, for each auxiliary source, a set of two unknown vector current components are encountered.

Then, according to the collocation method, the real (dielectric or conducting) surfaces S are divided into M subsurfaces, with an orthogonal set $\left\{\underline{k}_m^u, \underline{k}_m^v\right\}_{m=1}^{M}$ of tangential vectors, attached to the center of each subsurface, given by the equations,

$$\underline{k}^u = K_x^u \hat{x} + K_y^u \hat{y} + K_z^u \hat{z}$$
$$\underline{k}^v = K_x^v \hat{x} + K_y^v \hat{y} + K_z^v \hat{z}$$

By imposing the boundary conditions for the tangential electric and magnetic field components at a finite number of M points lying on the real surfaces S, the unknown weighting coefficients of the auxiliary sources can be determined and then the fields at any point of interest can be easily obtained.

When MAS is used to analyze thin and open structures, a large number of auxiliary sources must be introduced for the accurate description of the unknown fields. This is due to the fact that the fields generated by the elementary infinitesimal dipole "auxiliary sources" exhibit a distance dependence like r^{-3}, r^{-2} and r^{-1}. Thus, when treating thin and open structures, since the distance between the auxiliary and the actual surface(s) is small compared to the wavelength, the dominant term of the field (calculated on the physical surface(s)) is r^{-3} and r^{-2}. This leads to numerical instability of the obtained solution and a large number of auxiliary sources are required to adequately represent the unknown quantities.

In order to overcome the aforementioned problem, a modification of the standard MAS is applied by using both current and charge densities on the auxiliary surface(s) as unknowns, instead of the dipole moments, while the produced fields are represented by vector and scalar potentials. However, in the proposed mMAS, the only independent unknowns are the current distribution components J^u and J^v tangential to the auxiliary surface(s) (see Fig. 2), while the charge density ρ is calculated from the current distribution by applying the continuity equation. The basic idea of the described technique stems from the Finite Difference Time Domain (FDTD) method, since similar grid scheme and derivative equations are applied to solve for the charge density, as shown in Fig. 2.

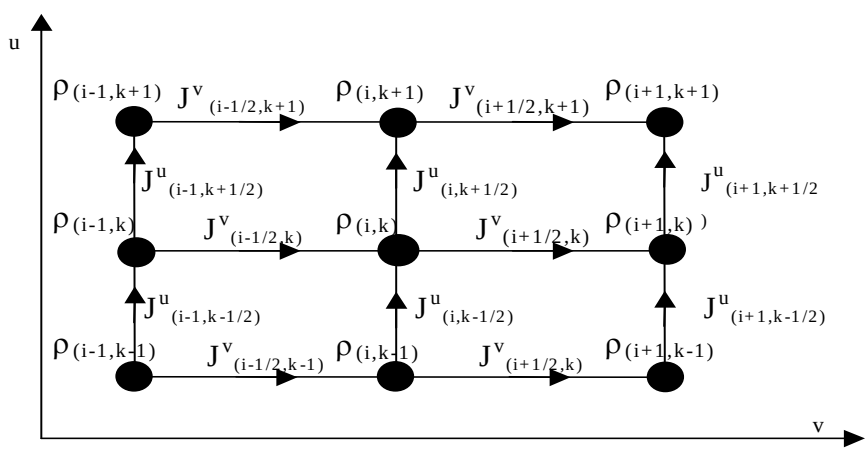

Figure 2. Grid and unknown current components and charge distributions on the auxiliary surface, used in the

Applying the proposed mMAS, the electric field produced by the auxiliary current and charge distributions is written in terms of the vector potential $\underline{A}$ and the scalar potential φ, as

$$\underline{E}(\underline{r}) = j\omega\underline{A}(\underline{r}) - \nabla\varphi(\underline{r}) \tag{7}$$

where

$$\underline{A}(\underline{r}) = \frac{\mu_0}{4\pi}\int_{\sigma^{aux}} \frac{\underline{J}(\underline{r}')e^{jk_0|\underline{r}-\underline{r}'|}}{|\underline{r}-\underline{r}'|}ds', \qquad \varphi(\underline{r}) = \frac{1}{4\pi\varepsilon_0}\int_{\sigma^{aux}} \frac{\rho(\underline{r}')e^{jk_0|\underline{r}-\underline{r}'|}}{|\underline{r}-\underline{r}'|}ds' \tag{8}$$

with σ^{aux} indicating the auxiliary surface.

The surface integrals appearing in equations (8) are replaced by finite summations over the auxiliary sources. Furthermore, by using a finite difference approximation of the continuity equation, the charge density ρ is expressed in terms of the current components J^u and J^v (see Fig. 2), as

$$j\omega\rho(i,k) = \frac{J^u(i,k+1/2) - J^u(i,k-1/2)}{\Delta u} + \frac{J^v(i+1/2,k) - J^v(i-1/2,k)}{\Delta v} \tag{9}$$

3. FORMULATION OF THE PROBLEM

Microstrip antennas as shown in Figure 3 consist of a very thin metallic strip placed a small fraction of a wavelength above the ground surface. A dielectric sheet separates the strip and ground plane. In solving such problem using mMAS two auxiliary surface inside and outside the microstrip antenna volume are introduced (Figure 3).

The current and charge densities distributed on the auxiliary surfaces inside (σ^{inner}) and outside (σ^{outer}) the antenna volume are used to describe the fields propagating outside and inside the antenna volume respectively.

The unknown current distributions on the introduced surfaces are calculated using the appropriate boundary conditions, ie, the :

$$\vec{E}^{sc}_{1,2}(\vec{r})\Big|_{tan} = \vec{0}, \qquad \text{for} \quad \vec{r} \in S_{cond}$$

$$\begin{cases} \vec{E}^{sc}_1(\vec{r}) - \vec{E}^{sc}_2(\vec{r})\Big|_{tan} = \vec{0}, \\ \vec{H}^{sc}_1(\vec{r}) - \vec{H}^{sc}_2(\vec{r})\Big|_{tan} = \vec{0}, \end{cases} \qquad \text{for} \quad \vec{r} \in S_{diel} \tag{10}$$

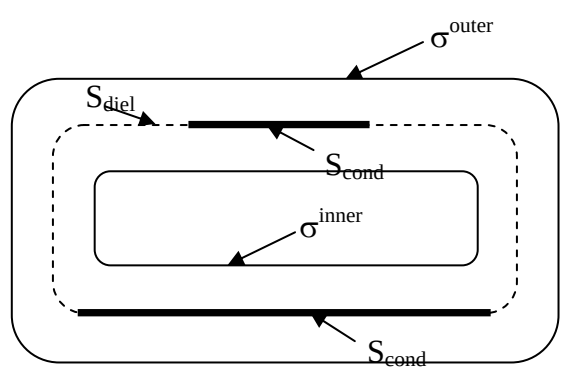

Figure 3. Geometry of problem

Where $\vec{E}_1^{sc}(\vec{r})$, $\vec{E}_2^{sc}(\vec{r})$ and $\vec{H}_1^{sc}(\vec{r})$, $\vec{H}_2^{sc}(\vec{r})$ are the propagating electric and magnetic fields for observation point $\vec{r}$ placed inside and outside the antenna volume respectively. Hence

$$\vec{E}^{sc}(\vec{r}) = \begin{cases} \vec{E}_1^{sc}(\vec{r}) = L_{out}[\vec{J}_{out}] \\ \vec{E}_2^{sc}(\vec{r}) = L_{in}[\vec{J}_{in}] \end{cases}, \quad \vec{H}^{sc}(\vec{r}) = \begin{cases} \vec{H}_1^{sc}(\vec{r}) = L_{out}^{h}[\vec{J}_{out}] \\ \vec{H}_2^{sc}(\vec{r}) = L_{in}^{h}[\vec{J}_{in}] \end{cases} \tag{11}$$

where

$$L_{in,out}[\vec{J}_{in,out}] = j\omega A_{in,out} - \vec{\nabla}\phi_{in,out} = \frac{j\omega\mu_0}{4\pi} \iint_{\text{Inner,Outeer Aux. Surface}} \vec{J}_{in,out}(\vec{r}\,')G(\vec{r},\sqrt{\varepsilon}k_0,\vec{r}\,')ds' - \frac{\vec{\nabla}}{4\pi\varepsilon\varepsilon_0} \iint_{\text{Inner,Oteter Aux. Surface}} \rho_{in,out}(\vec{r}\,')G(\vec{r},\sqrt{\varepsilon}k_0,\vec{r}\,')ds' \tag{12}$$

where $G(\vec{r};k_0,\vec{r}\,') = \exp(jk_0R)/R$ is free space Green's function, ε is the dielectric permittivity of the region where the field is calculated.

The charge density $\rho_{in,out}$ on the auxiliary surfaces is related to the electric currents on the corresponding auxiliary surface by the equation continuity law

$$\vec{\nabla}'\vec{J}_{in,out} = j\omega\rho_{in,out} \tag{13}$$

By combining (12) and (13) we have the fields produced by auxiliary sources:

$$L_{in,out}[\vec{J}_{in,out}] = \frac{j}{4\pi\omega\varepsilon\varepsilon_0}\left[\varepsilon k_0^2 \iint_{\text{Inner,Outer Aux. Surface}} \vec{J}_{in,out}(\vec{r}\,')G(\vec{r},\sqrt{\varepsilon}k_0,\vec{r}\,')ds' - \vec{\nabla} \iint_{\text{Inner,Outer Aux. Surface}} \vec{\nabla}'\vec{J}_{in,out}(\vec{r}\,')G(\vec{r},\sqrt{\varepsilon}k_0,\vec{r}\,')ds'\right] \tag{14}$$

and

$$L_{in,out}^{h}[\vec{J}_{in}] = \left[\vec{\nabla}\times\vec{A}_{in,out}\right] = \frac{\mu_0}{4\pi}\left[\vec{\nabla}\times \iint_{\text{Inner,Outer Aux. Surface}} \vec{J}_{in,out}(\vec{r}\,')G(\vec{r},\sqrt{\varepsilon}k_0,\vec{r}\,')ds'\right] \tag{15}$$

The equations (10) form a system of linear equations with unknowns the current distributions $\vec{J}_{in}$ and $\vec{J}_{out}$ on the auxiliary surfaces σ^{inner} and σ^{outer}. A collocation technique is used in order to solve this system. According this technique the boundary conditions are forced by requiring the continuity of electric and magnetic field on the dielectric surfaces as well as the short-circuit condition on the conducting surfaces at a finite number of points, equal to the total number of unknowns. By solving this system the auxiliary currents $\vec{J}_{in}$ and $\vec{J}_{out}$ are determined and the electric and magnetic field can be calculating using equation (11).

4. NUMERICAL RESULTS

As example, consider a square microstrip antenna fed at the middle of one side be a transmission lane as shown in Figure 4. The square antenna is axa (a=5mm) and fed by a 2.5mm long transmission line of width c=2mm. The ground plane is dxd (d=10mm) and dielectric substrate of 10x10x2mm (h=2mm) of dielectric permittivity $\varepsilon = 2.32$. The wavelength considered is 3cm. The antenna is excited by a constant voltage source of $E_z = 1V/m$ placed at the beginning of the microstrip feeding line as shown in

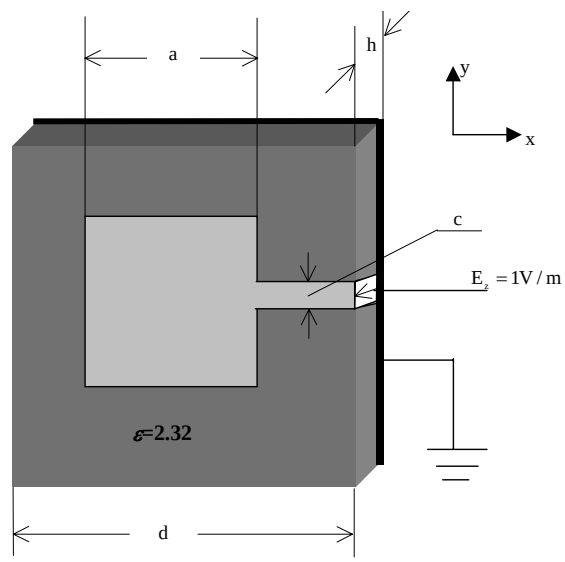

Figure 4. Geometry of microstrip antenna

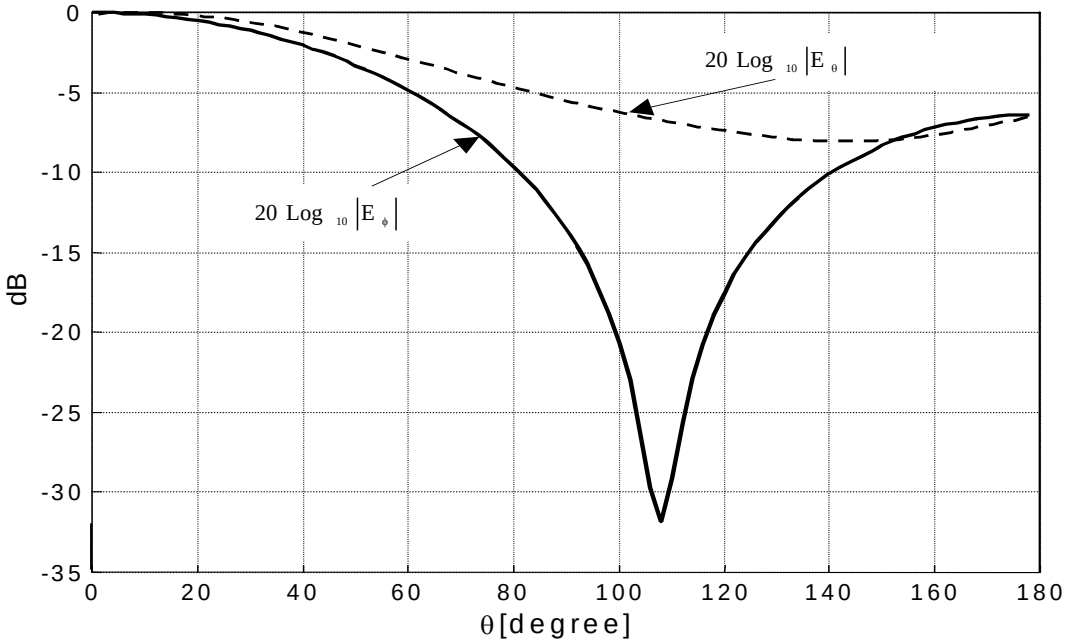

Figure 5. The E_x component of the electric field on plane x=0

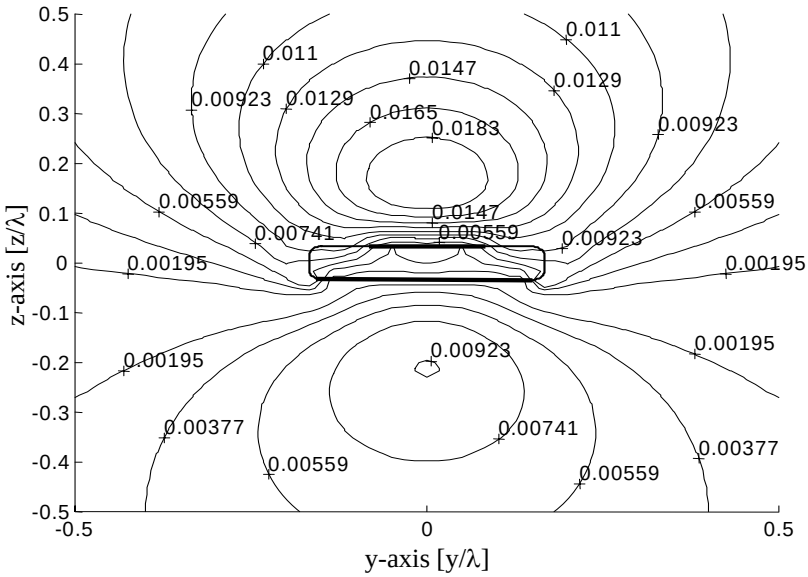

Figure 6. E_θ and E_ϕ components of the radiation pattern

Fig. 5. The origin of the coordinate system is taken to be in the center of the parallelepiped

that is defined by the dielectric substrate, with the ground plane lying on the z=-h/2 , -d/2<x,y<d/2 plane, and the patch antenna lying on the z=h/2 , -a/2<x,y<a/2 plane. The total number of unknowns used in both auxiliary surfaces (Figure 3) are 3880 end the CPU time for

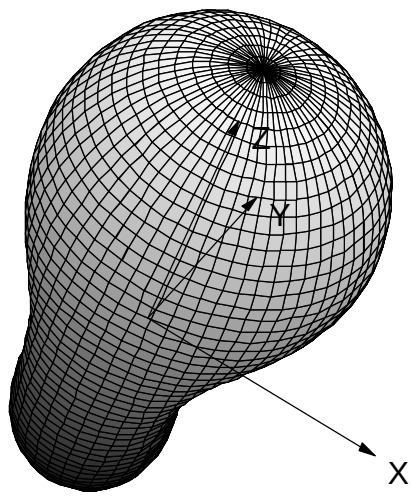

Figure 7. Volume representation of the patch antenna radiation pattern.

calculation of the matrix elements where 3 minute while the CPU time for the solution of the matrix was 30 minute.

In Figure 5 the E_x component of the electric field on the plane x=0 is shown. The validity of the boundary conditions can be checked through this figure since both short-circuit and continuity laws in the conducting and dielectric borders are satisfied. Finally, in Figure 6 both E_θ (ϕ=0) and E_ϕ ($\phi=\pi/2$) components of the radiation pattern as well as their volume representation in Figure 7 are shown.

5. CONCLUSION

The modified method of auxiliary sources is developed for the analysis of EM structures. In this paper additionally mMAS is applied for the analysis of a planar microstrip antenna. Numerical results are given and the advantages of the proposed method in both ease of

application as well as computational cost are exhibited. The authors are planning to expand the method by introducing also non-uniform grids in order to enable the solution of more complicated electromagnetic structures.

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