RIGHT-ANGLE DIELECTRIC WEDGE AS AN ANTENNA SIMULATION

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Abstract

The method of solving the right-angle dielectric wedge (RADW) boundary-value problem (BVP) consists of integral representation of the inside and outside fields and matching these fields on the boundaries. As a result, a set of integral equations (IE) of the 1-st kind was obtained. A new iterative method is proposed for such equations solving. This method produces a robust and adequate solution to the ill-conditioned matrix equation (ME) resulted from IE while discretization. With the aid of this method, the fields radiated into the empty quarter-space are investigated

1. INTRODUCTION

The electromagnetic diffraction by a dielectric wedge is of a great interest in the theory of dielectric waveguide matching, the theory of resonators and antennas. The problem of diffraction of an E-polarized plane wave by a RADW has been treated by a number of authors [1-6]. The aim of these authors was to obtain an exact closed form solution to the 2-D Fredholm IE of electromagnetics [7]. However, the rigorous solution to this problem is not available to date.

Some approximate solutions are also known [8-10]. In all this papers, far-fields were obtained approximately, but there were no backscattered field estimations in this works. However, it is of great importance to know the field behavior in the vicinity of the tip of the wedge and the backscattered far-fields.

The solution obtained in this paper, permits to compute a scattered field in any observation point. Asymptotically approximation of the obtained solution gives both the backscattered and forward scattered far-fields.

In the presented paper, the BVP for the TM plane wave diffraction by a RADW for a frontal incidence is solved numerically, but the obtained solution is valid at the whole space due to having an analytical form. The canonical method of fields matching on the boundaries is used. For the fields inside the wedge and outside of it the plain wave spectral representation [11] is used in the right-angle coordinates. Then, the boundary conditions satisfaction leads to a set of integral equations of the 1-st kind for six spectral functions. The Laplace transform of the obtained integral

equations results in an ill-conditioned matrix equation (ME) with the aid of the quadratures. For such equation solving, a new iterative scheme is proposed, because of the known iterative methods do not work. This iterative scheme gives a stable solution in the sense of Hadamard that does not depend practically on sampling.

2. BOUNDARY-VALUE PROBLEM

It is assumed that the dielectric body with the relative permittivity ε occupies the 2nd , 3rd and 4th quadrants. The permittivity of the body will be a complex constant value:

$$\varepsilon = \varepsilon' - j\varepsilon'' \tag{1}$$

where ε'' is assumed to be positive. Inside this area a plane wave which has only a z-component

$$E_0(x,y) = e^{-jk_{01}x - jk_{02}y} \tag{2}$$

is radiated and diffracted by the empty quarter-space (see Fig.1) with dielectric constant $\varepsilon_0 = 1 - j0$. Here are denoted:

$$k_{01} = \sqrt{\varepsilon_0}\sin i_0 \ , \quad k_{02} = \sqrt{\varepsilon_0}\cos i_0 \tag{3}$$

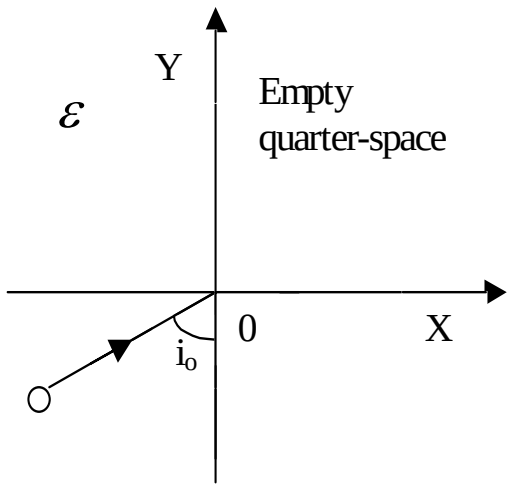

Figure 1.

where i_0 is the incident angle between $\vec{k}_0$ and OY. The frontal incidence is assumed with an incident angle: $0 \le i_0 \le \pi/2$. We put in this paper $\omega/c = 1$. To solve a BVP for 2-D waveequation, the total electric field in the 1st quadrant was represented as theWitteker's integrals [11]:

$$E_1(x,y) = \int_{-\infty}^{\infty} A_1(\xi) e^{-j\xi x - jh_0 y} d\xi + \int_{-\infty}^{\infty} A_2(\xi) e^{-jh_0 x - j\xi y} d\xi . \tag{4}$$

Inside the dielectric body we used the analogous representation of the scattered field in the 2-nd quadrant:

$$E_2(x,y) = \int_{-\infty}^{\infty} A_3(\xi) e^{jh x - j\xi y} d\xi + \int_{-\infty}^{\infty} A_4(\xi) e^{j\xi x - jh y} d\xi \tag{5}$$

and in the lower half-space ($y \le 0$) for the scattered field we have:

$$E_3(x,y) = \int_{-\infty}^{\infty} A_5(\xi) e^{-j\xi x + jh y} d\xi + \int_{-\infty}^{\infty} A_6(\xi) e^{j\xi x + jh y} d\xi \tag{6}$$

where $h = \sqrt{\varepsilon - \xi^2} = -j\sqrt{\xi^2 - \varepsilon}$, $h_0 = \sqrt{\varepsilon_0 - \xi^2} = -j\sqrt{\xi^2 - \varepsilon_0}$. The functions (4-6) satisfy the corresponding wave equations and radiation conditions inside and outside the body for arbitrary integrable function $A_i(\xi)$. The representations (4-6) have a physical sense of the plain wave decomposition and they must account all waves of a physical problem. Inside the empty wedge, waves $\sim \exp(-j\xi x - jh_0 y)$ in Eq. (4) move in the positive y-direction (y^+) and in all x-directions, but waves $\sim \exp(-jh_0 x - j\xi y)$ move in the positive x-direction (x^+) and in all y-directions. Then, such waves in Eq. (4) take into account any physical situations inside the 1st quadrant for mentioned incident angle. In the 2-nd quadrant there are waves that move in x^--direction like $\sim \exp(-jhx - j\xi y)$ and waves that move in y^+-direction like $\sim \exp(j\xi x - jhy)$. Then, the representation (5) accounts all possible waves in the 2-nd quadrant. In the lower half-space (y<0) all scattered waves move in y^--direction like $\sim \exp(\pm j\xi x + jhy)$ and then Eq. (6) accounts all such waves.

The boundary conditions for the electric and magnetic fields lead to the set of 1-D IE of the 1-st kind for unknown spectral functions:

$$E_1(x,0) - E_3(x,0) = E_0(x,0) \tag{7}$$

$$\partial_y E_1(x,0) - \partial_y E_3(x,0) = \partial_y E_0(x,0), \quad x \ge 0 \tag{8}$$

$$E_1(0,y) - E_2(0,y) = E_0(0,y) \tag{9}$$

$$\partial_x E_1(0,y) - \partial_x E_2(0,y) = \partial_x E_0(0,y), \quad y \ge 0 \tag{10}$$

$$E_2(x,0) - E_3(x,0) = 0 \tag{11}$$

$$\partial_y E_2(x,0) - \partial_y E_3(x,0) = 0 \ , \qquad x \le 0 \tag{12}$$

for corresponding boundaries: $y = 0,\ x \ge 0$; $x = 0,\ y \ge 0$ and $y = 0,\ x \le 0$.

The Laplace transform that is applied to the Eqs. (7-12), leads us to the set of IE:

$$\int_{-\infty}^{\infty} A_1 \frac{d\xi}{\nu-\xi} + \int_{-\infty}^{\infty} A_2 \frac{d\xi}{\nu-h_0} - \int_{-\infty}^{\infty} A_5 \frac{d\xi}{\nu-\xi} - \int_{-\infty}^{\infty} A_6 \frac{d\xi}{\nu+\xi} = \frac{1}{\nu-k_{01}} \tag{13}$$

$$\int_{-\infty}^{\infty} A_1 \frac{d\xi}{\nu-h_0} + \int_{-\infty}^{\infty} A_2 \frac{d\xi}{\nu-\xi} - \int_{-\infty}^{\infty} A_3 \frac{d\xi}{\nu-\xi} - \int_{-\infty}^{\infty} A_4 \frac{d\xi}{\nu-h} = \frac{1}{\nu-k_{02}} \tag{14}$$

$$\int_{-\infty}^{\infty} A_1 \frac{\xi\, d\xi}{\nu-h_0} + \int_{-\infty}^{\infty} A_2 \frac{hd\xi}{\nu-\xi} + \int_{-\infty}^{\infty} A_3 \frac{h_0 d\xi}{\nu-\xi} + \int_{-\infty}^{\infty} A_4 \frac{\xi\, d\xi}{\nu-h} = \frac{k_{01}}{\nu-k_{02}} \tag{15}$$

$$\int_{-\infty}^{\infty} A_3 \frac{d\xi}{\nu-h} + \int_{-\infty}^{\infty} A_4 \frac{d\xi}{\nu-\xi} - \int_{-\infty}^{\infty} A_5 \frac{d\xi}{\nu+\xi} - \int_{-\infty}^{\infty} A_6 \frac{d\xi}{\nu-\xi} = 0 \tag{16}$$

$$\int_{-\infty}^{\infty} A_1 \frac{h_0\, d\xi}{\nu-\xi} + \int_{-\infty}^{\infty} A_2 \frac{\xi\, d\xi}{\nu-h_0} + \int_{-\infty}^{\infty} A_5 \frac{hd\xi}{\nu-\xi} + \int_{-\infty}^{\infty} A_6 \frac{h\, d\xi}{\nu+\xi} = \frac{k_{02}}{\nu-k_{01}} \tag{17}$$

$$\int_{-\infty}^{\infty} A_3 \frac{\xi\, d\xi}{\nu-h} + \int_{-\infty}^{\infty} A_4 \frac{hd\xi}{\nu-\xi} + \int_{-\infty}^{\infty} A_5 \frac{hd\xi}{\nu+\xi} + \int_{-\infty}^{\infty} A_6 \frac{hd\xi}{\nu-\xi} = 0\,. \tag{18}$$

The existence of inverse transform (21) ensures the equivalence of sets (7-12) and (13-18) therefore we solve the transformed set with the aid of quadratures. The matrix of the algebraic linear system is very ill-conditioned. The known iterative methods do not work for such matrices hence a new iterative scheme is developed for such systems solving.

3. ITERATIVE TECHNIQUE

A new iterative scheme is proposed to solve an ill-conditioned ME of the form:

$$KX = B \tag{19}$$

where K is a square matrix, X and B are vectors or matrices. The method is based on the LU-decomposition:

$$K = L + U \tag{20}$$

where

$$L = \begin{pmatrix} \alpha & 0 & 0 & \cdots \\ k_{21} & \alpha & 0 & \cdots \\ k_{31} & k_{32} & \alpha & \cdots \\ \cdots & \cdots & \cdots & \ddots \end{pmatrix}, \qquad U = \begin{pmatrix} k_{11}-\alpha & k_{12} & k_{13} & \cdots \\ 0 & k_{22}-\alpha & k_{23} & \cdots \\ 0 & 0 & k_{33}-\alpha & \cdots \\ \cdots & \cdots & \cdots & \ddots \end{pmatrix}.$$

Then, the Eq. (11) is represented in the iterative form:

$$X = \tilde{F} + \tilde{K}X$$

where

$$\tilde{F} = L^{-1}F, \quad \tilde{K} = -L^{-1}U .$$

This equation gives the iterative process

$$X^{(n)} = \tilde{F} + \tilde{K}X^{(n-1)} \tag{21}$$

that produces a set of approximate solutions

$$X^{(n)} = \sum_{i=0}^{n} \tilde{K}^{i}\tilde{F} \tag{22}$$

with $\tilde{F}$ as an initial guess. The corresponding discrepancy

$$d_n = \left\| KX^{(n)} - F \right\| \tag{23}$$

tends to zero for consequent $X^{(n)}$ if the iterative process (21) converges. The convergence depends on the right-hand side (F) and the decomposition parameter (α). This is typical for ill-conditioned ME which has a solution if and only if the right-hand side is in the range of the matrix. Numerical investigations show that for some F and α the following convergence condition fulfills:

$$\left\| K^{n}\tilde{F} \right\| = \beta_n \left\| \tilde{F} \right\|, \quad \sum_{n=0}^{\infty} \beta_n = S < \infty .$$

In that case the solutions (22) make up a compact set in a Banach space. Then, the tendency of discrepancy (23) to zero for consequent $X^{(n)}$ ensures the tendency of approximate solution $X^{(n)}$ to an exact solution X_e of Eq. (11):

$$\left\| X^{(n)} - X_e \right\| \to 0 ,$$

according to Tikhonov's investigation [12]. Here $\|\cdot\|$ denotes a vector norm. For complex matrices, like used for integral equation set (13-18) solving, it is necessary to multiply the Eq. (19) by the complex conjugate matrix K^* and then to solve the equation

$$K^{*}KX = K^{*}B \tag{24}$$

with the aid of the mentioned iterative scheme. To find the optimal value of decomposition parameter α it is enough to test $\alpha = 10^{n}$ for n=1,2,3 and 4. The convergence rate of this iterative process is very high as a rule and discrepancies of the Eqs. (19) and (24) are approximately equal.

This iterative technique gives a stable solution in the sense of Hadamard. We can not affirm that proposed method always converges but it is so for a wide class of

ill-conditioned ME. Numerical investigation shows that the more symmetric the matrix K is, the better a convergence is of the iterative scheme (21). The existence of diagonally predominance of the matrix K makes worse the convergence.

With the aid of this iterative scheme the ME corresponding to the set of IE of the 1-st kind (13-18) was solved. The obtained solution is immune to the sampling. In the same time, the known iterative methods do not give some convergent series for this ill-conditioned ME.

4. NUMERICAL RESULTS

The discretization of the IE set (13-18) by the quadratures is used in this paper with the aid of trapezoidal formula. The integration range was reduced to a finite one due to the physical considerations that permit to affirm the spectral functions $A_i(\xi)$ to be concentrated not far from geometric-optical points and the numerical calculations prove that it is really so. We used the same meshing for ξ and ν. The range [-8, 8] was used while integration in (13-18) if the dielectric constant is less than 25. The interval $\xi \in [-1,1]$ was sampled at 0.1 and the rest of interval was sampled at 0.25. The enlarging of the integration range up to [-20, 20] was tested and the solution was not change. For larger permittivity the integration range must be enlarged. A parameter σ (19) was varied in the range: 0.5 – 0.9, a minimal discrepancy to find. The mentioned discretization produces the matrix K of the size 354 x 354 and the CPU time to the ME solving with the aid of iterative scheme (21) with $\alpha = 800$ and tolerance level $tol = \|A\tilde{X} - B\|$ equals to 0.05 is of about 15 min on a PC Pentium Pro 200 MHz (with the code written in Matlab). For the mentioned parameters the boundary condition matching error is of about 0.002-0.005. The tolerance level decreasing to 0.01 tends to accuracy increasing but the solution does not change appreciably. Therefore we take tol = 0.05, the CPU time to save.

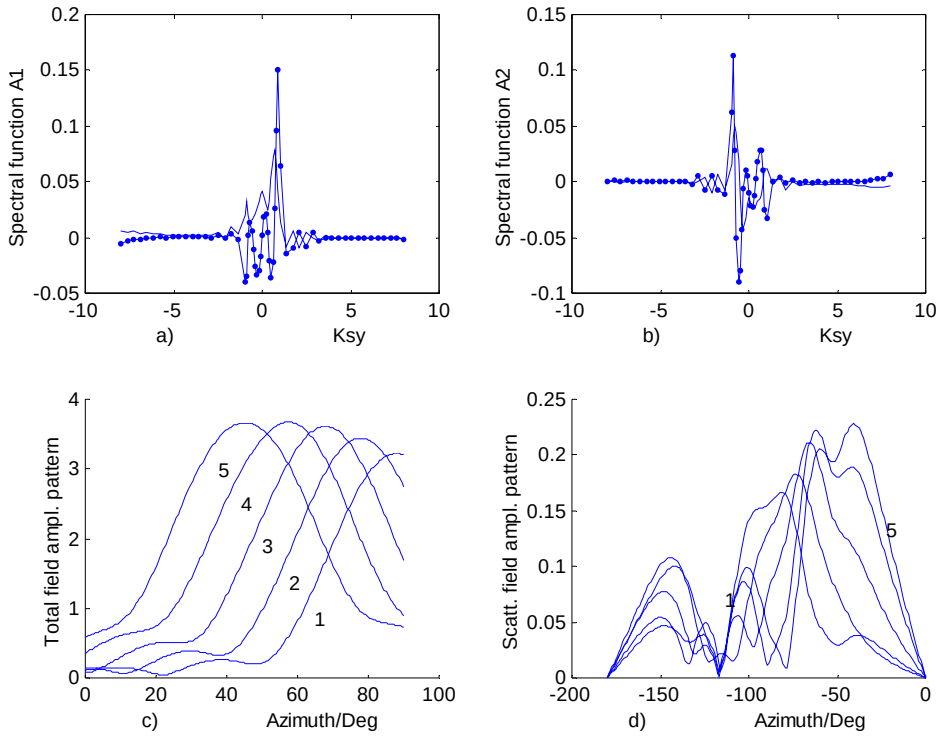

Figure 2. (a, b) the real (solid) and imaginary (dotted) parts of spectral functions for total field in the empty wedge; (c) the focusing effect in the empty wedge via incident angle: (1) $i_0 = 5^\circ$, (2) $i_0 = 15^\circ$, (3) $i_0 = 25^\circ$, (4) $i_0 = 35^\circ$, (5) $i_0 = 45^\circ$; (d) the scattered field in the low half-space for the same i_0. The dielectric constant of the body is $\varepsilon = 1.2$.

The typical solution of the ME corresponding to the IE set (13-18) for ε=1.2, $i_0 = 45^\circ$ is shown in Fig. 2 (a,b) for total field inside the wedge ($A_1(\xi)$, $A_2(\xi)$). The functions $A_5(\xi)$, $A_6(\xi)$ derive the scattered field in the lower half-space (y<0). Because of immunity of the solution from sampling, the far-field is evaluated asymptotically from (4-6) as described in [13]. It is represented as

$$E_1(\rho, \vartheta) = F(\vartheta) e^{-j\rho} / \sqrt{\rho} \tag{25}$$

with

$$F(\vartheta) = \sqrt{2\pi \varepsilon_0}\, e^{j\pi/4} \left(A_1(\sqrt{\varepsilon}_0 \cos\vartheta) \sin\vartheta + A_2(\sqrt{\varepsilon}_0 \sin\vartheta) \cos\vartheta \right) \tag{26}$$

for the total field inside the empty wedge ($0^\circ \le \vartheta \le 90^\circ$) and with

$$F(\vartheta) = -\sqrt{2\pi \varepsilon}\, e^{j\pi/4} \left(A_5(\sqrt{\varepsilon} \cos\vartheta) + A_6(-\sqrt{\varepsilon} \cos\vartheta) \right) \sin\vartheta \tag{27}$$

for the scattered field in the lower half-space ($-180^\circ \le \vartheta \le 0^\circ$). The amplitude patterns $|F(\vartheta)|$ (26) of the total far-field inside the wedge for various incident angles are presented in Fig. 2c. We can see that the focusing effect of Rawlins [8] occurs for an empty wedge as well if the dielectric constant of a body is small. This effect vanishes for $\varepsilon > 1.5$. The main lob of the radiated field amplitude pattern varies significantly his direction with the incident angle changing that is important for antennas managing. The significant interest is to find the fields scattered in the back direction (y<0). In Fig. 2d we can see the scattered far-field amplitude patterns $|F(\vartheta)|$ (27) in the lower half space for the same incident angles. We see here the field scattered by the edge (the first maxima near $\vartheta = -135^\circ$), which maximum's directions are almost independent from the incident angle. It contains also the backscattered field ($\vartheta = -\pi/2 - i_0$), that is absent in geometric-optical approximation. The backscattered field was not obtained also in mentioned papers [8-10]. The rest of the amplitude pattern is connected with the field reflected from the side. Both field amplitudes are of the same order in value (≤ 0.2). Then, the reflected field is significantly less than the field radiated in the empty wedge.

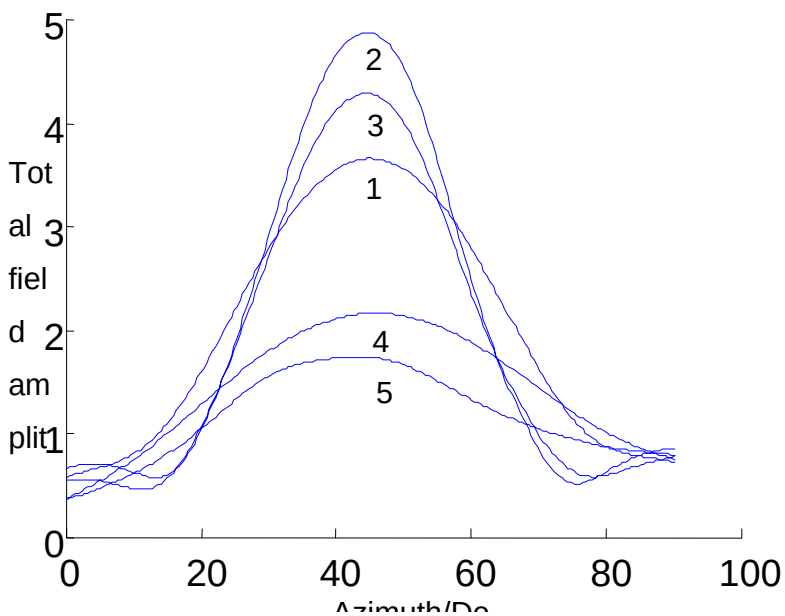

Figure 3. The focusing effect in the empty wedge via absorption level in the dielectric body for $\varepsilon' = 1.2$, $i_0 = 45^\circ$. (1) ε''=0.001; (2) ε''=0.01; (3) ε''=0.1; (4) ε''=0.5; (5) ε''=1.

The focusing effect of the total field in the empty wedge reveals the extremal property via an absorption level in dielectric body as we can see in Fig. 3 where the best focusing is observed for $\varepsilon'' = 0.01$ but not for $\varepsilon'' = 0.001$, and then it decreases with an absorption growth.

5. CONCLUSION

The BVP solving method used in presented paper has no simplifications and is rigorous in this meaning. The unknown spectral functions $A_i(\xi)$ are derived approximately as a solution of a set of IE of the 1-st kind. A new iterative scheme is proposed to derive a solution of an ill-conditioned ME that obtained while IE discretization, which is stable in the sense of Hadamard and is immune to the sampling variation. The obtained solution ensures a boundary condition matching error less than 0.005 at any point on the boundary. The accuracy increases with the meshing subinterval value decreasing. The obtained spectral functions $A_i(\xi)$ determine the analytical form (4-6) of the BVP solution that permits to compute fields and their derivatives at any point without additional calculations. The immunity of the solution to the sampling variation permits to obtain the far-fields with the aid of an asymptotic evaluation of integrals (4-6).

The total far-field amplitude patterns are obtained in the empty wedge. The focusing effect of Rawlins is detected for an empty wedge for small permittivity of the body. The main lob's direction depends significantly on the plane wave's incident angle. The focusing effect reveals an extremal property via absorption level in dielectric body. The best focusing is observed for a non-zero absorption.

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