SOLUTION OF THREE-DIMENSIONAL ELECTROMAGNETIC SCATTERING PROBLEMS VIA A HYBRID METHOD OF MOMENTS / METHOD OF AUXILIARY SOURCES (MOM/MAS) TECHNIQUE

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Abstract

An efficient computational method is developed to solve three-dimensional (3D) electromagnetic (EM) scattering problems. The advantages of a standard Method of Moments (MoM) formulation are combined with those of a standard Method of Auxiliary Sources (MAS) approach, which, especially for the case of -even arbitrarily-curved conducting surfaces, has been proved to give accurate results with significantly low computational cost. Namely, a novel hybrid MoM/MAS technique is developed, in order to determine the response of 3D EM structures to a time harmonic incident field. The electric fields inside the dielectric parts of the examined 3D scatterers are represented by pulse piecewise functions, while equivalent current sources are employed to represent the secondary fields generated by the conducting parts of the scatterers. The validity and accuracy of the method is checked and numerical results are presented, proving the suitability of the proposed technique to treat diverse EM scattering problems.

1. INTRODUCTION

The motivation of the present work has been the development of an efficient computational method in solving complex and arbitrary structures in terms of both their geometry and constituents. Scientific and technology areas needing such a computational tool include design of microwave antennas and microwave-millimetre wavelength integrated circuits, assessment of electromagnetic (EM) interference and EM compatibility phenomena and investigation of the EM interaction with the human body.

During the last decades numerous efforts have been carried out to develop generalized computational EM techniques. Initially the emphasis was given to method of moments (MoM) techniques [1] and diverse MoM versions, such as Galerkin technique, point matching technique etc., using integral equation formulations. In all these cases, the use of the Green's functions provides a significant reduction of the unknown field region and, moreover, guarantees the satisfaction of the far-field radiation conditions, known as Sommerfeld-Muller conditions. It has been established

that MoM techniques are characterized by a very high accuracy level. However, a drawback of MoM is the computational complexity in evaluating the "system matrix elements" (which usually require the numerical computation of multiple integrals, thus, increasing drastically the relevant computational cost), as well as the numerical singularities related to the MoM impedance matrix self terms. Furthermore, another difficulty in applying MoM techniques has been the high complication arising in solving realistic geometrical models.

In fact, the effort to analyze "real world" problems, has motivated the development of other numerical techniques, such as the Finite Elements (FE) [2] and the Finite Difference Time Domain (FDTD) [3]. However, both FE and FDTD suffer by lower accuracy and the difficulty in satisfying the Sommerfeld-Muller radiation conditions, which is usually accomplished by using "Artificial Boundary Conditions".

According to the above considerations, a new MoM technique is introduced, based on the following principles:

a) Complex objects consisted of both dielectrics and perfect electric conductors (PECs) should be modeled easily. The examined structure should be arbitrary in terms of its geometry and consistence.

b) Physical considerations should be embedded in selecting the set of basis functions describing the electric field inside the dielectric objects, while the same principle should be applied for the case of the PECs.

As already mentioned, the principal motivation on developing the present technique has been to introduce a MoM incorporating easy modeling of structures and low computational cost. Nevertheless, in attempting to develop a generalized numerical code to cope with arbitrary dielectric and conducting bodies, the essential difficulty in matching the mathematical description of the 3D geometries and the formulation to be used to obtain the numerical results is faced. To this end, on one hand, based on past experience [4,5], the description of field penetrable objects can be easily described by using cubical cells of sufficiently small dimensions. On the other hand, the arbitrary shape of the 3D conducting surfaces discourages the use of a similar approach. Therefore, the Method of Auxiliary Sources (MAS) [6], which has already been proved very efficient in solving conducting geometries, is selected as the method to treat the conducting points of the examined structures. Moreover, MAS inherently eliminates the singularities problems of a typical MoM impedance matrix,

while does not require the integral evaluations at any stage of the MAS solution (i.e. matrix fill, field calculation etc).

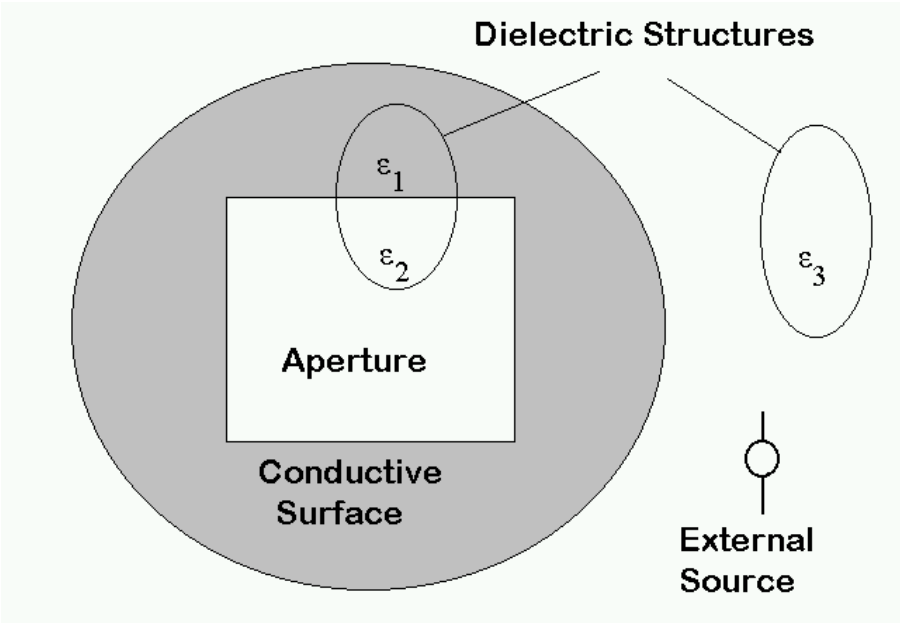


Figure 1. Generalized geometry.

The geometry of an arbitrary EM structure is given in Fig. 1, where one can see conducting surfaces, which may have apertures and (lossy and/or inhomogeneous) dielectric bodies inside and/or outside the conducting surfaces. This generalized structure is illuminated by an external known source, such as a dipole, a horn antenna or a plane wave. In analyzing of the given EM problem, it is necessary to calculate the induced electric currents on the conducting surfaces and the polarization currents inside the dielectric bodies. The format of the paper is as follows: A system of coupled integral equations is derived in section 2, which is solved via the proposed hybrid MoM/MAS technique. The MoM part employed in treating the dielectric bodies and the MAS part employed in treating the conductors are presented in sections 3 and 4 respectively, while the hybridization of the two methods is given in section 5. The validation of the proposed technique is checked in section 6 and numerical results are presented for several cases, while concluding remarks and suggestions for future improvements are given in section 7. In the following, a time dependence $e^{j\omega t}$ is assumed for all the field quantities and is suppressed throughout the analysis.

2. INTEGRAL EQUATION FORMULATION

Applying the Green's theorem, the unknown electric field $\vec{E}(\vec{r})$ at an arbitrary point $\vec{r} = (x, y, z)$ satisfies the equation:

$$\vec{E}(\vec{r}) = \vec{E}_0(\vec{r}) + \vec{E}_{diel}(\vec{r}) + \vec{E}_{cond}(\vec{r}) \text{ or} \tag{1}$$

$$\vec{E}(\vec{r}) = \vec{E}_0(\vec{r}) + \iiint_{V_{diel}} \overline{\overline{G}}(\vec{r}, \vec{r}') \cdot k_0^2(\varepsilon_r(\vec{r}') - 1)\vec{E}(\vec{r}')d\vec{r}' - j\omega\mu_0 \oiint_{S_{cond}} \overline{\overline{G}}(\vec{r}, \vec{r}_s') \cdot \vec{J}_s(\vec{r}_s')d\vec{r}_s',$$

where $\vec{E}_0(\vec{r})$ is the primary field due to the known external source, $\vec{E}_{diel}(\vec{r})$ and $\vec{E}_{cond}(\vec{r})$ denote the contribution of the dielectric and conducting parts of the structure respectively, $\varepsilon_r(\vec{r}')$ denotes the relative dielectric permittivity (complex for lossy dielectrics and function of $\vec{r}'$ for inhomogeneous dielectrics), $\vec{J}_s(\vec{r}_s')$ denotes the unknown conductivity currents distribution induced on the conductors surfaces S_{cond}, V_{diel} stands for the dielectric parts volume, $k_0 = \omega\sqrt{\varepsilon_0\mu_0}$ is the free-space propagation constant, with ω being the angular frequency and ε_0, μ_0 being the free-space dielectric permittivity and magnetic permeability respectively and

$$\overline{\overline{G}}(\vec{r}, \vec{r}') = (\overline{\overline{I}} + k_0^{-2}\nabla\nabla)G(\vec{r}, \vec{r}') = (\overline{\overline{I}} + k_0^{-2}\nabla\nabla)\frac{\exp(-jk_0 \mid \vec{r} - \vec{r}' \mid)}{4\pi \mid \vec{r} - \vec{r}' \mid} \tag{2}$$

is the free-space electric field Green's dyadic function, with $\overline{\underline{I}} = \hat{x}\hat{x} + \hat{y}\hat{y} + \hat{z}\hat{z}$ being the unit dyadic.

In order to solve equation (1), the observation point $\vec{r} = (x, y, z)$ is first restricted on the conducting surfaces S_{cond} and then inside the dielectric parts' volume V_{diel}. Then, two dimensional (2D) integral equations arise, by requiring the fulfillment of the boundary conditions on the conducting surfaces S_{cond}, where the total tangential electric field should be zeroed. On the other hand, 3D integral equations arise, by requiring the satisfaction of equation (1) inside the dielectric parts volume V_{diel}. In order to solve the derived system of coupled integral equations in terms of the conducting currents and the electric field inside the dielectrics, a hybrid MoM/MAS technique is developed, as described in the following sections.

3. METHOD OF MOMENTS TECHNIQUE

The contribution of the dielectric parts of the analyzed structure, denoted in equation (1) by the term $\vec{E}_{diel}(\vec{r})$, is taken into consideration using "traditional" MoM [1]. To this end, all the dielectric parts are subdivided into cubic elementary cells, each of which can be characterized by a different dielectric permittivity, in a way that even inhomogeneous (lossy) media can be considered. MoM approach is implemented, by assuming that the electric field inside each elementary cell is constant. Therefore, the electric field inside the dielectric structure is described by cubic basis functions of the type

$$\vec{E}(\vec{r}) = \sum_{i=1}^{N} \vec{A}_i W(\vec{r} - \vec{r}_i), \tag{3}$$

where $W(\vec{r}) = \begin{cases} 1 & \text{inside the cubic cell} \\ 0 & \text{elsewhere} \end{cases}$, $\vec{A}_i$ are the unknown expansion coefficients of the electric field and $\vec{r}_i$ denotes the ith cell centre of gravity.

Using equation (3) and assuming that the structure under consideration is consisted only of dielectric parts, equation (1) becomes

$$\vec{E}(\vec{r}) = \vec{E}_0(\vec{r}) + k_0^2(\bar{\bar{I}} + k_0^{-2}\nabla\nabla)\sum_{j=1}^{N}\left(\varepsilon(r_j) - 1\right)\iiint_{V_j} G(\vec{r}, \vec{r}_j') \cdot \vec{E}(\vec{r}_j') d\vec{r}_j' \text{ or}$$

$$\vec{E}(\vec{r}_i) = \vec{E}_0(\vec{r}_i) + \sum_{j=1}^{N} \bar{\bar{\Gamma}}_{ij} \cdot \vec{A}_j, \tag{4}$$

$$\text{where } \bar{\bar{\Gamma}}_{ij} = k_0^2(\bar{\bar{I}} + k_0^{-2}\nabla\nabla)\left(\varepsilon_j - 1\right)\iiint_{V_j} G(\vec{r}_i, \vec{r}_j') d\vec{r}_j'. \tag{5}$$

In calculating the $\bar{\bar{\Gamma}}_{ij}$ terms, two cases are encountered. First, when i=j (self term) and second, when i≠j (mutual term). In the first case, the calculation of $\bar{\bar{\Gamma}}_{ii}$ requires special attention, since $G(\vec{r}_i, \vec{r}_i')$ has a singularity. To this end, we calculate analytically the volume integral (see eq. (5)) over the inscribed sphere (where the singularity exists) and we calculate numerically the integral over the remaining volume of the cell. In the second case, the integral can be calculated easily, since there is no singularity of the integrand in the integration volume and several numerical methods of integration (Gauss, Simpson) or even average value approximation can be

used. More specifically, we calculate numerically the integral over the six adjacent cells, while for all the rest cells we employ an average value of the Green's function.

4. METHOD OF AUXILIARY SOURCES

The electric field scattered by the conducting parts of the analyzed structure, denoted in equation (1) by the term $\vec{E}_{cond}(\vec{r})$, is taken into consideration using conventional MAS [6]. More specifically, in order to briefly denote the principles of MAS, one could assume the diffraction problem of a linearly polarized, monochromatic EM wave incident on a body of external surface S, placed in the free space. For the needs of the present work, the body is assumed to be a PEC, although MAS can also treat scatterers penetrable to EM waves. The given problem is reduced to the solution of the corresponding wave equation in the outer region of the conducting scatterer and the validity of the boundary condition on the surface S, as

$$\vec{E}_{cond}(x,y,z) + \vec{E}_0(x,y,z) = 0, \qquad (x,y,z) \in S,$$

where $\vec{E}_{cond}(x,y,z)$ and $\vec{E}_0(x,y,z)$ are the functions describing the scattered and incident fields respectively. According to a standard MAS solution, N number of auxiliary sources, radiating in the free space without the presence of the scatterer, are homogeneously distributed on the auxiliary surface σ, which is selected to be always inside the physical area of the scatterer and is conformal to S, as shown in Fig. 2.

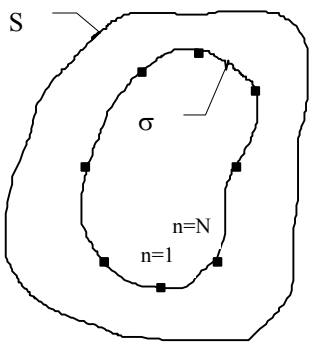


Figure 2. Conducting scatterer geometry for the MAS formulation.

Then, the scattered from the conducting surface field is described as a summation of the fields generated by the auxiliary sources:

$$\vec{E}_{cond}(x,y,z) = \sum_{n=1}^{N} \overline{\overline{G}}(\vec{r}_n, \vec{r}_s) \cdot \vec{B}_n \tag{6}$$

where $\overline{\overline{G}}(\vec{r}_n, \vec{r}_s)$, is given by (2) and $\vec{B}_n$ are the unknown weights of the auxiliary sources.

The fulfillment of the boundary conditions on the conducting surfaces, assuming the tangential vector $\hat{p}(\vec{r}_m) = \vec{p}_x(\vec{r}_m) + \vec{p}_y(\vec{r}_m) + \vec{p}_z(\vec{r}_m)$ on the conducting surface point $\vec{r}_m$, is given by the equations

$$\vec{p}_x(\vec{r}_m) \cdot \sum_{n=1}^{N} \overline{\overline{G}}(\vec{r}_n, \vec{r}_m) \cdot \vec{B}_n = -\vec{p}_x(\vec{r}_m) \cdot \vec{E}_0(\vec{r}_m)$$

$$\vec{p}_y(\vec{r}_m) \cdot \sum_{n=1}^{N} \overline{\overline{G}}(\vec{r}_n, \vec{r}_m) \cdot \vec{B}_n = -\vec{p}_y(\vec{r}_m) \cdot \vec{E}_0(\vec{r}_m)$$

$$\vec{p}_z(\vec{r}_m) \cdot \sum_{n=1}^{N} \overline{\overline{G}}(\vec{r}_n, \vec{r}_m) \cdot \vec{B}_n = -\vec{p}_z(\vec{r}_m) \cdot \vec{E}_0(\vec{r}_m)$$

where $\overline{\overline{G}}(\vec{r}_n, \vec{r}_m)$ is defined in equation (2), $\vec{E}_0(\vec{r}_m)$ denotes the primary field and $\vec{B}_n = B_{nx}\hat{x} + B_{ny}\hat{y} + B_{nz}\hat{z}$, (n = 1,…,N) are the expansion coefficients of the auxiliary sources to be determined. The last three equations can be rewritten as

$$\hat{p}(\vec{r}_m) \cdot \sum_{n=1}^{N} \overline{\overline{G}}(\vec{r}_n, \vec{r}_m) \cdot \vec{B}_n = -\hat{p}(\vec{r}_m) \cdot \vec{E}_0(\vec{r}_m) \text{ or } \sum_{n=1}^{N} \overline{\overline{H}}_{nm} \cdot \vec{B}_n = -\hat{p}(\vec{r}_m) \cdot \vec{E}_0(\vec{r}_m)$$

where $\overline{\overline{H}}_{nm} = \hat{p}(\vec{r}_m) \cdot \overline{\overline{G}}(\vec{r}_n, \vec{r}_m)$. Therefore, the scattered by the conducting surfaces field at an arbitrary point $\vec{r}$ can be expressed as

$$\vec{E}_{cond}(\vec{r}) = \sum_{n=1}^{N} \overline{\overline{G}}(\vec{r}_n, \vec{r}) \cdot \vec{B}_n \tag{7}$$

5. COMBINATION OF THE TWO METHODS

The contribution of the dielectric parts on the conducting surfaces equations is given by calculating the scattered by the dielectric bodies electric field, according to the equation

$$\vec{E}^s(\vec{r}_i) = \sum_{j=1}^{N} \overline{\overline{\Gamma}}_{ij} \cdot \vec{A}_j \tag{8}$$

Since the boundary condition on the conducting surfaces is related to the corresponding tangential electric field, equation (8) becomes

$$\hat{p}(\vec{r}_i)\cdot\vec{E}^s(\vec{r}_i)=\hat{p}(\vec{r}_i)\cdot\sum_{j=1}^{N}\overline{\overline{\Gamma}}_{ij}\cdot\vec{A}_j=\sum_{j=1}^{N}\overline{\overline{\Delta}}_{ij}\cdot\vec{A}_j$$

where $\hat{p}(\vec{r}_i)$ denotes the tangential unit vector.

The contribution of the conducting currents to the dielectric parts is taken into consideration using equation (7), with the observation point restricted inside the electric body (i.e. $\vec{r}=\vec{r}_i$ for the ith cell). Then, equation (7) becomes

$$\vec{E}^s(\vec{r}_i)=\sum_{j=1}^{N}\overline{\overline{G}}(\vec{r}_j,\vec{r}_i)\cdot\vec{B}_j=\sum_{j=1}^{N}\overline{\overline{Z}}_{ij}\cdot\vec{B}_j$$

The derived overall matrix is of the form

$$\begin{bmatrix}\overline{\overline{\Gamma}}_{ij} & \overline{\overline{Z}}_{ij}\\ \overline{\overline{\Delta}}_{ij} & \overline{\overline{H}}_{ij}\end{bmatrix}\cdot\begin{bmatrix}\vec{A}_j\\ \vec{B}_j\end{bmatrix}=\begin{bmatrix}\vec{E}_0\\ -\hat{p}_i\cdot\vec{E}_0\end{bmatrix} \tag{9}$$

where $\vec{A}_j$ and $\vec{B}_j$ denote the unknown expanding coefficients of the electric field inside the dielectric bodies and the weights of the auxiliary sources inside the conducting surfaces respectively. The system of linear equations (9) is solved using a bi-conjugate gradient method. After the calculation of the unknown coefficients, the electric field at every point of the space can be calculated using equation (1).

6. NUMERICAL RESULTS

Based on the previously presented analysis, a numerical code has been developed and its accuracy has been checked. Firstly, the MAS part of the code has been separately checked by considering “pure” conducting structures, then, the MoM part of the code has been also separately checked by considering “pure” dielectric objects and, finally, the validity of the overall code (hybrid MoM/MAS technique) has been checked by analyzing structures consisted of both dielectric and conducting parts. Indicative 3D scattering problems have been solved and the derived results for “simple” geometries have been compared with known previously published data. In all cases, detailed convergence checks have been carried out, both in terms of the number and positions of the auxiliary sources placed inside the conductors and in

terms of the number of elementary dielectric cells, while the satisfaction of the proper boundary conditions has been verified.

First, the problem of a conducting sphere placed in the free space and illuminated by an x-polarized and z-incident plane wave is considered. According to a standard MAS formulation auxiliary sources are homogeneously distributed on a spherical surface inside the sphere and the amplitudes and phases of the auxiliary sources are determined, using a point matching technique, requiring the fulfillment of the boundary condition for zero tangential electric field on the boundary of the sphere. In Fig. 3, the normalized monostatic radar cross section $\sigma_{3D} = \lim_{r\to\infty}[4\pi r^2 |\vec{E}^s|^2 / |\vec{E}^i|^2]$

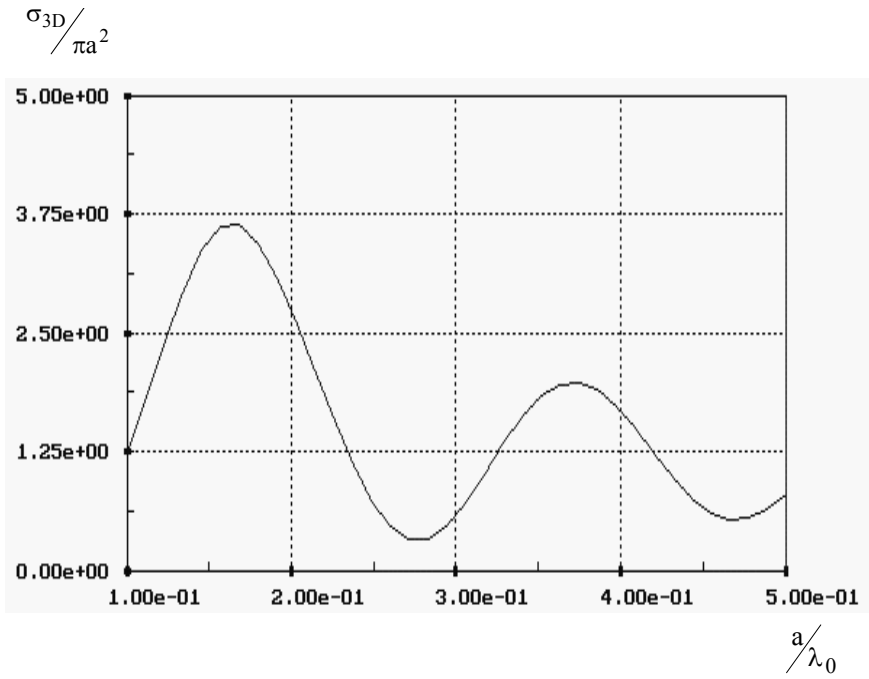


Figure 3. Normalized monostatic radar cross section (RCS) of a conducting sphere of radius a as a function of the wavelength λ_0 in the free space.

(where $\vec{E}^s$ and $\vec{E}^i$ denote the scattered and incident fields respectively)is plotted as a function of the free space wavelength and the results are found to be in excellent agreement with previously published data [7].

Then, the problem of a dielectric sphere of radius a = 0.08λ (f = 90 MHz, a = 26.5 cm) and dielectric permittivity $\varepsilon_r = 16$ placed in the free space and illuminated by an x-polarized and z-incident plane wave is investigated. According to the methodology described in section 3, the sphere is enclosed by a cube of side 0.24λ (80 cm), which is subdivided to elementary cubic cells. In Fig. 4, the amplitude of the x-component of

the total (incident + scattered) field $|E_x|$ along the x-axis is illustrated. The convergence is checked by increasing the number of elementary cells, while the results are found to be in agreement with both the Mei theory [8] and results derived using an independently developed FE code [9].

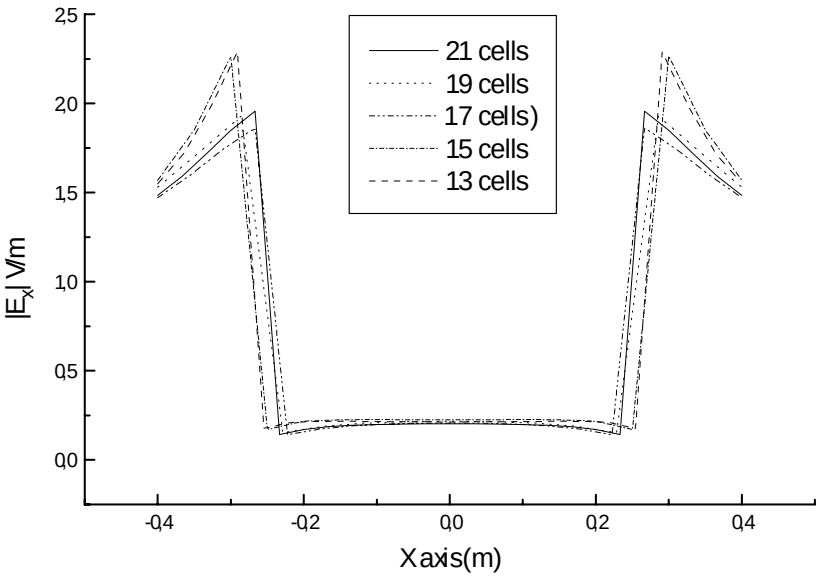
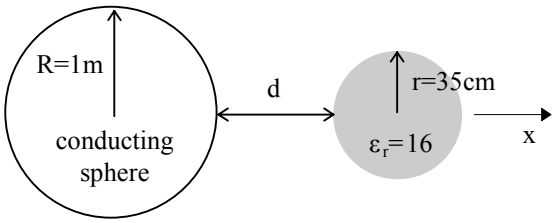


Figure 4. $|E_x|$ field along the x-axis of a dielectric sphere of radius 0.265 m illuminated by an x-polarized and z-incident plane wave at f = 90 MHz: Convergence in terms of finer cell division.

Finally, the overall code is checked by considering the presence of both a dielectric and a conducting sphere in the free space. In Fig. 5, the amplitude $|E_x|$ along the x-axis inside a dielectric sphere of dielectric permitivity $\varepsilon_r = 16$ and radius 0.03 m illuminated by an x-polarized and z-incident plane wave at f = 900 MHz is plotted, while a conducting sphere of radius 1 m is also present at a distance d. It can be easily seen that, as expected, for large d values ($d \geq \lambda$), the electric field inside the dielectric sphere is not affected by the presence of the conducting sphere and the Mei solution is again obtained.



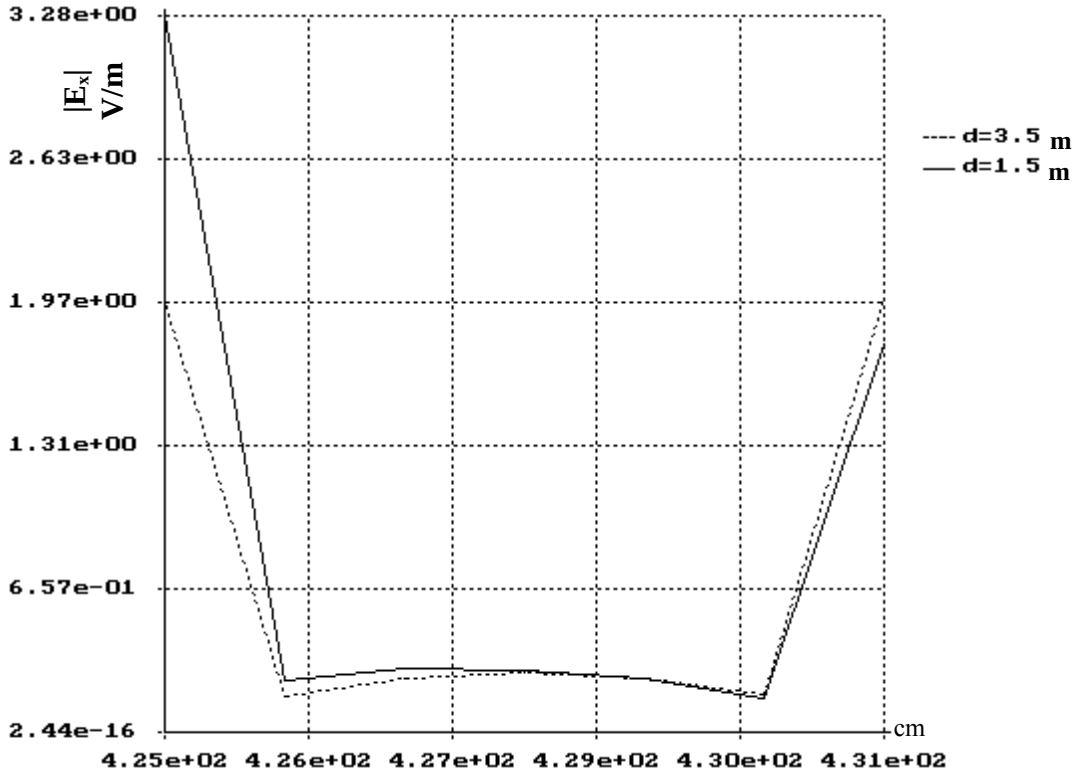


Figure 5. Dielectric sphere of radius 0.35 m illuminated by an x-polarized and z-incident plane wave at f = 90 MHz in the presence of a conducting sphere of radius 1 m: $|E_x|$ field along the x-axis of the dielectric sphere for different distances d between the two spheres.

7. CONCLUSIONS - FUTURE WORK

An efficient numerical method has been developed in analyzing 3D EM scatterers consisted of conducting and dielectric parts. The well known advantages of MoM have been combined with those of MAS, which, especially for the case of curved conducting surfaces, has been proved to give accurate results with significantly low computational cost. Therefore, the electric fields inside the dielectric parts of the analyzed structure were represented by pulse piecewise functions using conventional MoM, while equivalent sources were employed to represent the currents on the

conducting parts. The validity and accuracy of the method has been checked and numerical results have been presented, proving the suitability of the proposed technique to treat diverse scattering problems.

The main challenge for future work would be to proceed with treating radiation problems, as well as larger-scale EM problems, meaning more complex EM structures in terms of their shape, constituents, electrical dimensions and/or external excitation. Therefore, the associated computational cost, in terms of consumed CPU time and memory requirements should be decreased to a reasonable level. In this context, both parallelization techniques, as well as optimization of the developed sequential MoM/MAS code should be considered.

As far as the MAS part of the code is concerned, it would be interesting to incorporate into the overall MoM/MAS code an optimized version of MAS, rather than the conventional one introduced in the present paper. According to this "optimized" MAS [10,11], solving first the direct and then the inverse problem, the optimum -and therefore fewest- positions of the auxiliary sources are determined. As shown in [10], these positions coincide to the scattered field singularities. Nevertheless, it has been proved that in many practical cases, the most "expensive" part of the MoM/MAS code in terms of both CPU time and memory requirements is the MoM part, while conventional (unoptimized) MAS provides quite "fast" results for the conducting parts of the structure (fewer auxiliary sources compared to the number of dielectric elementary cells and extremely faster computation for the contribution at each collocation point compared to the CPU time needed to calculate the contribution of each dielectric elementary cell). An interesting version/optimization of the MoM part of the code would be to employ -instead of the pulse cubic basis functions described by eq. (3)- overlapping rooftops of the type

$$f_n(t) = \begin{cases} 1 - \dfrac{(t - n\alpha)^2}{\alpha^2} & t \in \aleph_n \\ 0 & t \notin \aleph_n \end{cases} \tag{10}$$

with $\aleph_n = (n\alpha - \alpha/2, n\alpha + \varepsilon/2)$ for $t \equiv x$ and $\aleph_n = (n\alpha - \alpha, n\alpha + \alpha)$ for $t \equiv y,z$ (α being the cell side). The unknown electric field would then be

$$E_x = \sum_{i,j,k} A_{ijk} f_j(y) f_k(z), \quad E_y = \sum_{i,j,k} A_{ijk} f_i(x) f_k(z), \quad E_z = \sum_{i,j,k} A_{ijk} f_i(x) f_j(y) \tag{11}$$

and the basis function given by eq. (10) would, in addition, satisfy the boundary condition at the cells interfaces, providing the same accuracy with fewer number of cells.

Finally, parallelization of the most time-consuming part of the overall code should be implemented, together with an optimum “rearrangement” between sequentially and parallel executed part of the MoM/MAS code.

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