DIFFRACTION OF ELECTROMAGNETIC WAVES ON AN ANISOTROPIC CYLINDRICAL INHOMOGENEITY INSIDE A PLANE WAVEGUIDE

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Abstract

Excitation of a plane waveguide with perfectly conducting walls by an elementary vertical electric dipole in the presence of an anisotropic cylindrical inhomogeneity has been studied. Diffraction of normal waves on this inhomogeneity has also been considered. The interaction and transformation analysis of TE and TM modes at the boundary of cylindrical inhomogeneity is given. The exact expression for the longitudinal component of electric field intensity presented in the double series form is obtained. Numerical calculations of the scattering and total electric field are performed and compared with the similar dependencies for the case of an isotropic cylindrical inhomogeneity placed inside the plane waveguide. Two limit cases are analyzed: the Relay approximation and the ray approximation for the case of long and short wavelength, respectively.

1. INTRODUCTION.

In the recent time significant attention has been paid to the investigating of electromagnetic waves propagation features inside plane waveguides containing uniform inhomogeneities. This considerable interest has been associated with wide use of the plane waveguides model in hydroacoustics, seismology and, in particular, in the theory of LF radiowaves propagation nearby the land [1]. As an inhomogeneity a borehole, mine, volcanic channel and other structures both of natural and artificial origin can be considered. These inhomogeneities are known to have a substantial influence on the behavior of electromagnetic fields distribution inside waveguides [2-4]. In [5] the affect of step-form inhomogeneity on propagation of electromagnetic waves inside a plane waveguide was analyzed. The problem of electromagnetic waves diffraction on a perfectly conducting cylinder located inside a plane waveguide with perfectly conducting walls was studied in [6]. Excitation of a plane perfect waveguide in the presence of a magnetodielectric cylinder by some elementary dipoles was also investigated [7]. In this paper the excitation problem of electromagnetic waves by a vertical electric vibrator located inside a plane waveguide with perfectly conducting walls in the presence of an anisotropic

inhomogeneity in the circular cylinder form has been considered as well as the diffraction problem of normal waves on this inhomogeneity.

2. BASIC EQUATIONS AND BOUNDARY CONDITIONS.

Consider the plane waveguide with perfectly conducting walls inside which the anisotropic cylindrical inhomogeneity is located. In Figure 1 the model geometry is demonstrated: it is the side view on the plane waveguide of the width L with the anisotropic cylinder of the radius b, as well as the top view in the section of plane $z = z_0$. The cylindrical coordinate system (r, φ, z) is selected with the axis z oriented along the cylinder axis. As the source we have used an elementary vertical electric dipole placed outside the

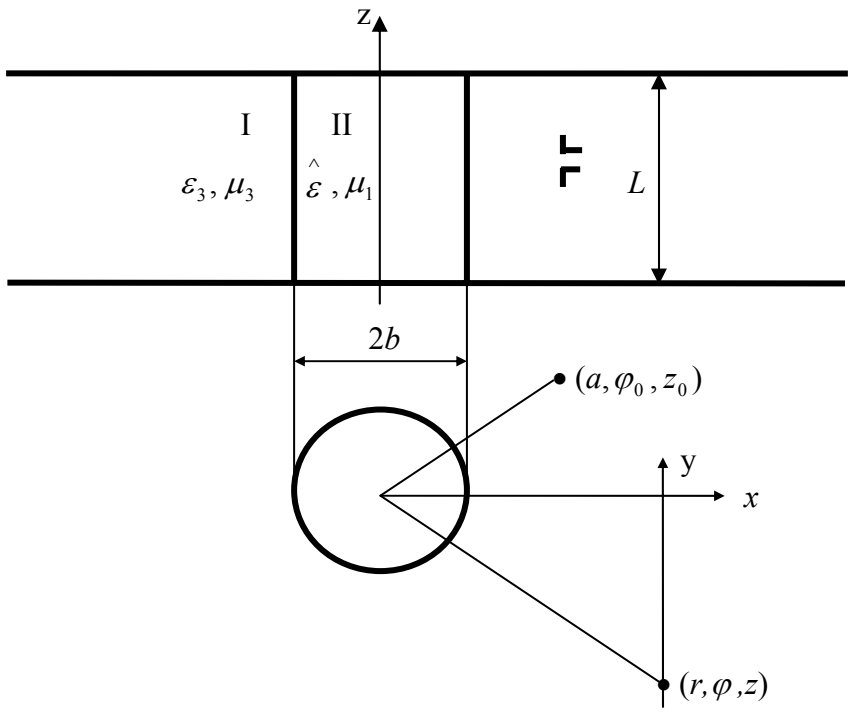

Figure 1. Schematic representation of the problem model.

cylinder at the point with the coordinates (a, φ_0, z_0). Properties of an anisotropic medium are characterized by the relative dielectric permittivity tensor

$$\hat{\varepsilon} = \begin{pmatrix} \varepsilon_1 & 0 & 0 \\ 0 & \varepsilon_1 & 0 \\ 0 & 0 & \varepsilon_2 \end{pmatrix} \tag{1}$$

and the relative magnetic permeability μ_1, and medium properties of the waveguide are the constants ε_3 and μ_3.

For the vertical electric dipole the vector electric and magnetic potentials $\vec{A}$ and $\vec{F}$, respectively, have the longitudinal components only. The equation assumed for the vertical components of this potentials can be presented in the following form:

- *inside the cylinder (region I)*

$$\Delta_{\perp} A_1 + \frac{1}{\Lambda}\frac{\partial^2 A_1}{\partial z^2} + k_2^2 A_1 = 0 \tag{2}$$

$$\Delta F_1 + k_2^2 F_1 = 0 \tag{3}$$

- *outside the cylinder (region II)*

$$\Delta A_3 + k_3^2 A_3 = -j_z \tag{4}$$

$$\Delta F_3 + k_3^2 F_3 = 0\,. \tag{5}$$

Here k_2 and k_3 are the wavenumbers inside and outside the cylinder, respectively: $k_2 = k_0\sqrt{\varepsilon_2\,\mu_1}$, $k_3 = k_0\sqrt{\varepsilon_3\,\mu_3}$; $k_0 = \omega / c$ is the wavenumber in vacuum, ω is the circular frequency, c is the speed of light in vacuum; $\Lambda = \dfrac{\varepsilon_1}{\varepsilon_2}$ is the anisotropy coefficient; z-component of the extraneous current is as follows

$$j_z = \frac{I_0 l}{\sqrt{r\ a}}\delta(r-a)\delta(\varphi-\varphi_0)\delta(z-z_0), \tag{6}$$

where I_0 is the current amplitude at the frequency ω, l is the radiator size, $\delta(x)$ is the Dirac delta-function symbol.

The following expressions for the components of electric and magnetic fields intensity using the Lorenz calibration of the potentials will be used in such way [7]:

$$E_r = \frac{i}{\omega\varepsilon\varepsilon_0}\frac{\partial^2 A}{\partial r\partial z} - \frac{1}{r}\frac{\partial F}{\partial\varphi} \tag{7}$$

$$E_\varphi = \frac{i}{\omega\varepsilon\varepsilon_0}\frac{1}{r}\frac{\partial^2 A}{\partial\varphi\partial z} + \frac{\partial F}{\partial r} \tag{8}$$

$$E_z = \frac{i}{\omega\varepsilon\varepsilon_0}\left(\frac{\partial^2 A}{\partial z^2} + k^2 A\right) \tag{9}$$

$$H_r = \frac{i}{\omega\mu\mu_0}\frac{\partial^2 F}{\partial r \partial z} + \frac{1}{r}\frac{\partial A}{\partial \varphi} \tag{10}$$

$$H_\varphi = \frac{i}{\omega\mu\mu_0}\frac{1}{r}\frac{\partial^2 F}{\partial \varphi \partial z} - \frac{\partial A}{\partial r} \tag{11}$$

$$H_z = \frac{i}{\omega\mu\mu_0}\left(\frac{\partial^2 F}{\partial z^2} + k^2 F\right) \tag{12}$$

Formulate boundary conditions on the waveguide walls and the cylindrical surface. Firstly, the tangential components of electric field intensity on a perfect surface equal zero ($E_r = 0, E_\varphi = 0$) and in accordance with (7) and (8) for $z = 0$ and $z = L$ the boundary conditions can be written as

$$\frac{\partial A}{\partial z} = 0, \quad F = 0. \tag{13}$$

Secondly, on the circular anisotropic cylinder at $r = b$ the following boundary conditions have to be performed

$$\begin{aligned} &E_{\varphi 1} = E_{\varphi 3},\ H_{\varphi 1} = H_{\varphi 3}, \\ &E_{z1} = E_{z3},\ H_{z1} = H_{z3}. \end{aligned} \tag{14}$$

3. BOUNDARY PROBLEM SOLUTION.

To solve the problem stated ((2-6), (13) and (14)) we shall apply the expansion of sources and desired field in terms of the normal waves of the plane waveguide with the perfectly conducting walls, as well as the variable separation method [10]. The problem eigenfunctions satisfied the boundary conditions (13) are well known and can be presented as [6]

$$f(z) = \cos\left(n\pi\frac{z}{L}\right)$$

$$g(z) = \sin\left(n\pi\frac{z}{L}\right)$$

For the δ-functions included in the (6) we shall use the following representation in the series form [8]

$$\delta(\varphi-\varphi_0)=\frac{1}{2\pi}\sum_{s=-\infty}^{\infty}\cos(s\varphi_0)\ \cos(s\varphi)$$
$$\delta(z-z_0)=\frac{1}{L}\sum_{n=-\infty}^{\infty}\cos\left(n\pi\frac{z_0}{L}\right)\ \cos\left(n\pi\frac{z}{L}\right). \tag{15}$$

Using the expressions (15) the solution of the problem stated will be given in the double series form:

$$A_1=\sum_{n=-\infty}^{\infty}\sum_{s=-\infty}^{\infty}A_{1ns}(r)\cos\left(n\pi\frac{z_0}{L}\right)\cos\left(n\pi\frac{z}{L}\right)\cos\left[s(\varphi-\varphi_0)\right] \tag{16}$$

$$F_1=\sum_{n=-\infty}^{\infty}\sum_{s=-\infty}^{\infty}F_{1ns}(r)\cos\left(n\pi\frac{z_0}{L}\right)\sin\left(n\pi\frac{z}{L}\right)\sin\left[s(\varphi-\varphi_0)\right] \tag{17}$$

$$A_3=\sum_{n=-\infty}^{\infty}\sum_{s=-\infty}^{\infty}A_{3ns}(r)\cos\left(n\pi\frac{z_0}{L}\right)\cos\left(n\pi\frac{z}{L}\right)\cos\left[s(\varphi-\varphi_0)\right] \tag{18}$$

$$F_3=\sum_{n=-\infty}^{\infty}\sum_{s=-\infty}^{\infty}F_{3ns}(r)\cos\left(n\pi\frac{z_0}{L}\right)\sin\left(n\pi\frac{z}{L}\right)\sin\left[s(\varphi-\varphi_0)\right] \tag{19}$$

Substituting (6), (15)-(19) into the equations (2)-(5) we can obtain the following system of the ordinary differential equations for the $A_{1ns}(r)$, $A_{3ns}(r)$, $F_{1ns}(r)$ and $F_{3ns}(r)$

$$\frac{d^2A_{1ns}}{dr^2}+\frac{1}{r}\frac{dA_{1ns}}{dr}+\left[k_2^2-\frac{1}{\Lambda}\left(\frac{n\pi}{L}\right)^2-\frac{s^2}{r^2}\right]A_{1ns}=0 \tag{20}$$

$$\frac{d^2F_{1ns}}{dr^2}+\frac{1}{r}\frac{dF_{1ns}}{dr}+\left[k_2^2-\left(\frac{n\pi}{L}\right)^2-\frac{s^2}{r^2}\right]F_{1ns}=0 \tag{21}$$

$$\frac{d^2A_{3ns}}{dr^2}+\frac{1}{r}\frac{dA_{3ns}}{dr}+\left[k_3^2-\left(\frac{n\pi}{L}\right)^2-\frac{s^2}{r^2}\right]A_{3ns}=-\frac{I_0l}{2\pi L}\frac{\delta(r-a)}{\sqrt{r-a}} \tag{22}$$

$$\frac{d^2F_{3ns}}{dr^2}+\frac{1}{r}\frac{dF_{3ns}}{dr}+\left[k_3^2-\left(\frac{n\pi}{L}\right)^2-\frac{s^2}{r^2}\right]F_{3ns}=0 \tag{23}$$

Represent the general solution of the equation (22) in the form

$$A_{3ns}=\frac{iI_0l}{4L}\left\{\begin{bmatrix}J_s(\kappa_{3n}r)\ \ H_s^{(1)}(\kappa_{3n}a), r\le a\\ J_s(\kappa_{3n}a)\ \ H_s^{(1)}(\kappa_{3n}r), r\ge a\end{bmatrix}+R_{ns}^{ee}H_s^{(1)}(\kappa_{3n}r)\right\}. \tag{24}$$

The expression between the square brackets is the particular solution of the equation (22) and corresponds to the forward wave which is propagated from the source to the observation point. The second summand in the (24) conforms to the wave reflected from the

inhomogeneity, and R_{ns}^{ee} is the reflection coefficient of the electric wave with indexes n and s from the cylinder. Here $J_s(x)$ and $H_s^{(1)}(x)$ are the first kind Bessel and Hankel functions of the s order, respectively. In (24) the following designation for the radial wavenumber in the waveguide is used

$$\kappa_{3n} = \sqrt{k_3^2 - \left(\frac{n\pi}{L}\right)^2}, \quad n = 0,1,2...$$

In analogy the particular solution of the uniform equations (20), (21) and (23) can be written as

$$A_{1ns} = \frac{iI_0 l}{4L} \sum_{n=-\infty}^{\infty} \sum_{s=-\infty}^{\infty} T_{ns}^{ee} \; J_s(\kappa_{2n} r) \cos\left(n\pi \frac{z_0}{L}\right) \cos\left(n\pi \frac{z}{L}\right) \cos\left[s(\varphi - \varphi_0)\right] \tag{25}$$

$$F_{1ns} = \frac{iI_0 l}{4L} \zeta_0 \sum_{n=-\infty}^{\infty} \sum_{s=-\infty}^{\infty} T_{ns}^{em} \; J_s(\kappa_{2n} r) \cos\left(n\pi \frac{z_0}{L}\right) \sin\left(n\pi \frac{z}{L}\right) \sin\left[s(\varphi - \varphi_0)\right] \tag{26}$$

$$F_{3ns} = \frac{iI_0 l}{4L} \zeta_0 \sum_{n=-\infty}^{\infty} \sum_{s=-\infty}^{\infty} R_{ns}^{em} \; H_s^{(1)}(\kappa_{3n} r) \cos\left(n\pi \frac{z_0}{L}\right) \sin\left(n\pi \frac{z}{L}\right) \sin\left[s(\varphi - \varphi_0)\right]. \tag{27}$$

Here we have introduced: ζ_0 is the vacuum characteristic impedance equal 120π, Om and the radial wavenumbers

$$\kappa_{2n} = \sqrt{k_2^2 - \frac{1}{\Lambda}\left(\frac{n\pi}{L}\right)^2}, \quad \kappa_n = \sqrt{k_2^2 - \left(\frac{n\pi}{L}\right)^2}.$$

The coefficients T_{ns}^{ee} and T_{ns}^{em} in the formulas (25)-(26) describe the amplitude of electric and magnetic waves, respectively, transmitted into the cylinder. They result from transformation of electric waves incident on the cylinder surface. R_{ns}^{em} is the transformation coefficient of electric waves into magnetic as a result of reflection from the cylinder. These coefficients are defined from the boundary conditions on the cylinder surface (14). The solution of the applicable equations system yields

$$R_{ns}^{ee} = -\frac{\left\{D_n^{\mu} M_n^{\varepsilon} - s^2 C_n^{\mu} C_n^{\varepsilon}\right\}}{\left\{D_n^{\mu} D_n^{\varepsilon} - s^2 C_n^{\mu} C_n^{\varepsilon}\right\}} \frac{J_s(\beta_{3n})}{H_s^{(1)}(\beta_{3n})} H_s^{(1)}(\alpha_{3n}) \tag{28}$$

$$R_{ns}^{em} = -\frac{2\mu_3}{\pi\beta_{3n}^2} \frac{s\, C_n^{\varepsilon}}{\left\{D_n^{\mu} D_n^{\varepsilon} - s^2 C_n^{\mu} C_n^{\varepsilon}\right\}} \frac{H_s^{(1)}(\alpha_{3n})}{\left[H_s^{(1)}(\beta_{3n})\right]^2} \tag{29}$$

$$T_{ns}^{ee} = -i\frac{2\varepsilon_1}{\pi\beta_{1n}^2}\frac{D_n^{\mu}}{\left\{D_n^{\mu}D_n^{\varepsilon} - s^2 C_n^{\mu}C_n^{\varepsilon}\right\}}\frac{H_s^{(1)}\left(\alpha_{3n}\right)}{J_s\left(\beta_{2n}\right)H_s^{(1)}\left(\beta_{3n}\right)} \tag{30}$$

$$T_{ns}^{em} = -i\frac{2\mu_1}{\pi\beta_n^2}\frac{sC_n^{\varepsilon}}{\left\{D_n^{\mu}D_n^{\varepsilon} - s^2 C_n^{\mu}C_n^{\varepsilon}\right\}}\frac{H_s^{(1)}\left(\alpha_{3n}\right)}{J_s\left(\beta_n\right)H_s^{(1)}\left(\beta_{3n}\right)}. \tag{31}$$

In the formulas mentioned above for the reflection and transmission coefficients (28)-(31) we use the denotations:

$$D_n^{\mu} = \frac{\mu_1}{\beta_n}\frac{\dot{J}_s\left(\beta_n\right)}{J_s\left(\beta_n\right)} - \frac{\mu_3}{\beta_{3n}}\frac{\dot{H}_s^{(1)}\left(\beta_{3n}\right)}{H_s^{(1)}\left(\beta_{3n}\right)},\ D_n^{\varepsilon} = \frac{\varepsilon_1}{\beta_{1n}}\frac{\beta_{2n}}{\beta_{1n}}\frac{\dot{J}_s\left(\beta_{2n}\right)}{J_s\left(\beta_{2n}\right)} - \frac{\varepsilon_3}{\beta_{3n}}\frac{\dot{H}_s^{(1)}\left(\beta_{3n}\right)}{H_s^{(1)}\left(\beta_{3n}\right)}$$

$$M_n^{\varepsilon} = \frac{\varepsilon_1}{\beta_{1n}}\frac{\beta_{2n}}{\beta_{1n}}\frac{\dot{J}_s\left(\beta_{2n}\right)}{J_s\left(\beta_{2n}\right)} - \frac{\varepsilon_3}{\beta_{3n}}\frac{\dot{J}_s\left(\beta_{3n}\right)}{J_s\left(\beta_{3n}\right)},\ C_n^{\varepsilon} = \frac{n\pi}{k_0 L}\frac{\beta_{3n}^2 - \beta_{1n}^2}{\beta_{1n}^2\beta_{3n}^2},\ C_n^{\mu} = \frac{n\pi}{k_0 L}\frac{\beta_{3n}^2 - \beta_n^2}{\beta_n^2\beta_{3n}^2}.$$

We also assume that

$$\beta_n = \kappa_n b;\ \beta_{jn} = \kappa_{jn} b, \text{ where } j = 1,2,3;\ \alpha_{3n} = \kappa_{3n} a;$$

$$\kappa_{1n} = \sqrt{k_1^2 - \Lambda\left(\frac{n\pi}{L}\right)^2}.$$

Mention should be made that the diffracted electromagnetic fields are completely described by potentials A_3 and F_3, i.e. the coefficients R_{ns}^{ee} and R_{ns}^{em}. To define electromagnetic fields inside the cylinder we must use the potentials A_1 and F_1 and the expressions for the transmission coefficients (30), (31).

The vertical component of the electric field intensity E_z is of great importance for measurements. Using the relations (9), (24) and applying the addition theorem for the cylindrical functions [9] to the expression (24) described the field without the dielectric inhomogeneity for the outside cylinder region we reduce

$$\begin{aligned} E_{3z} = -\frac{I_0 l}{4L}\frac{\zeta_3}{k_3}\sum_{n,s=-\infty}^{\infty}\kappa_{3n}^2\left[H_0^{(1)}\left(\kappa_{3n}R\right) + R_{ns}^{ee}H_s^{(1)}\left(\kappa_{3n}r\right)\right]\times \\ \times\cos\left(n\pi\frac{z_0}{L}\right)\cos\left(n\pi\frac{z}{L}\right)\cos\left[s\left(\varphi-\varphi_0\right)\right], \end{aligned} \tag{32}$$

where R_{ns}^{ee} is determined (28), $\zeta_3 = \sqrt{\mu_3\mu_0 / \varepsilon_3\varepsilon_0}$ is the medium characteristic impedance filled the waveguide, and $R = \sqrt{r^2 + a^2 - 2ra\cos\left(\varphi - \varphi_0\right)}$.

It should be noted that in the plane waveguide the electric waves contain «telegraph mode» as the fundamental wave ($n = 0$), i.e. TEM-wave. In this case from the relation (32) the longitudinal component of the electric field can be easily defined

$$E_{3z} = E_{3z}^{(1)} + E_{3z}^{(2)}, \text{ where} \tag{33}$$

$$E_{3z}^{(1)} = -\frac{I_0 l}{4L} k_3 \zeta_3 H_0^{(1)}\left(k_3 R\right), \tag{34}$$

$$E_{3z}^{(2)} = -\frac{I_0 l}{4L} k_3 \zeta_3 \sum_{s=-\infty}^{\infty} R_{0s}^{ee} H_s^{(1)}\left(k_3 r\right) \cos\left[s\left(\varphi - \varphi_0\right)\right], \tag{35}$$

$$R_{0s}^{ee} = -\frac{M^{\varepsilon}}{D^{\varepsilon}} \frac{J_s\left(\beta_3\right)}{H_s^{(1)}\left(\beta_3\right)} H_s^{(1)}\left(\alpha_3\right); \quad \beta_j = k_j b, j = 1,2,3; \quad \alpha_3 = k_3 a .$$

The expression (34) describes the forward wave which is propagated from the source to the observation point, and the expression (35) can be interpreted as the diffracted field.

Using the expression (32) the numerical calculations of the total and reflected electric fields intensities inside the waveguide are performed. Figure 2 and 3 show the plots of the electric fields normalized to the quantity $E_0 = \frac{I_0 l}{2L} k_3 \zeta_3$ as functions of the distance $y = x_1 \, tg\varphi$, where $x_1 = 3$ and the azimuth angle φ ranges from 0° to 80°. These figures conform to the cases of isotropic and anisotropic inhomogeneity, respectively. Here we have applied the following lines designations: the solid line is $\varepsilon = 3$ (Figure 2); the dot-dashed line is $\varepsilon = 9$ (Figure 2) and $\varepsilon_1 = 3$, $\varepsilon_2 = 9$ (Figure 3); the dashed line is $\varepsilon = 30$ (Figure 2) and $\varepsilon_1 = 3$, $\varepsilon_2 = 30$ (Figure 3). Note that these lines designations will be here-after used.

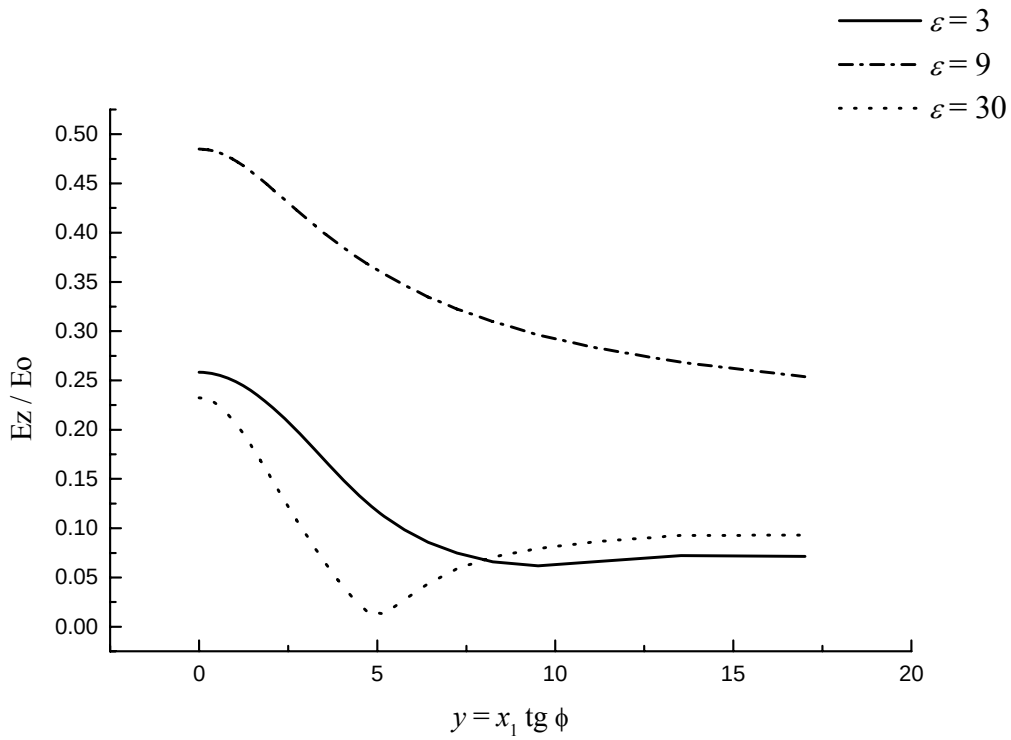

Figure 2. Total E_z - component as a function of a distance for an isotropic inhomogeneity.

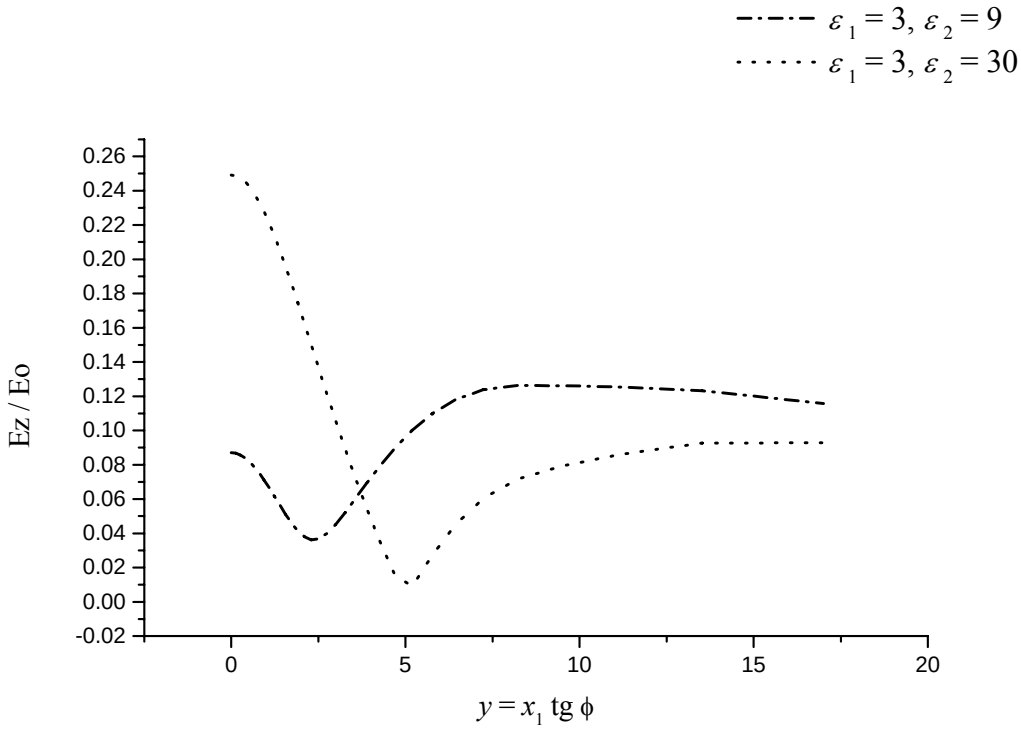

Figure 3. Total E_z - component as a function of a distance for an anisotropic inhomogeneity.

In this calculations we use the following values of the nondimensional parameters: $k_0 b = 1$, $k_0 a = 3$, $k_0 L = 6$, $k_0 r = x/\cos\varphi$, $\varphi_0 = \pi$, $\varepsilon_3 = 1$. The calculation results have shown insignificant effect of the cylinder anisotropic properties on the scattering field structure at high values of the permittivity. However this affect comes into particular prominence at relatively small values of the permittivity.

Figure 4-7 illustrate varying of a scattering indicatrix for the isotropic (Figure 4, 6) and anisotropic (Figure 5, 7) inhomogeneity depending on the distance to the observation point $k_0 r = 4$ (Figure 4, 5) and $k_0 r = 10$ (Figure 6, 7). Here we also assume $\varphi_0 = 0$. At high values of the permittivity the scattering indicatrixes for the isotropic and anisotropic cylinders (Figure 4, 5 or Figure 6, 7) agree in type while they differ noticeably at small values of the permittivity.

In Figure 8-11 (by analogy with Figure 4-7) the azimuth dependencies of total electric fields intensities are demonstrated. It should be mentioned that the cylinder anisotropy is not a factor.

4. ASYMPTOTIC APPROCHES

The expressions obtained for the z-components of electric field intensity (32-35) are rather complex that makes their analysis difficult. Therefor consider two limit cases: long and short wave approximations.

a) The Relay approximation.

The wavelength in the waveguide is assumed to be much less in comparison with the cylinder radius, i.e.

$$\left|k_3 b\right| << 1. \tag{36}$$

Under this condition the series in s in the expression (35) for the field reflected are fast converged. Therefore the two first terms corresponded to $s = 0$ и $s = 1$ can be limited.

We will also use the approximated expressions of the cylindrical functions for the small values of the arguments $x << 1$ [9]

$$J_s(x) \approx \frac{1}{s!}\left(\frac{x}{2}\right)^s, \quad H_0^{(1)} \approx \frac{2i}{\pi} ln\left(\gamma \frac{x}{2}\right). \tag{37}$$

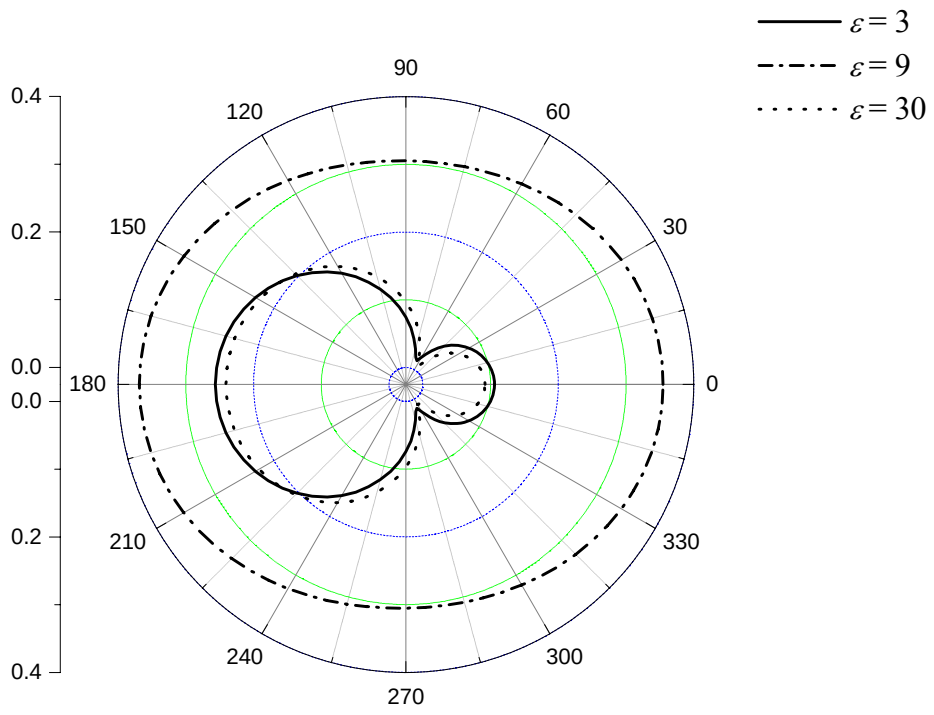

Figure 4. Scattering indicatrixes for an isotropic inhomogeneity at $k_0 r = 4$

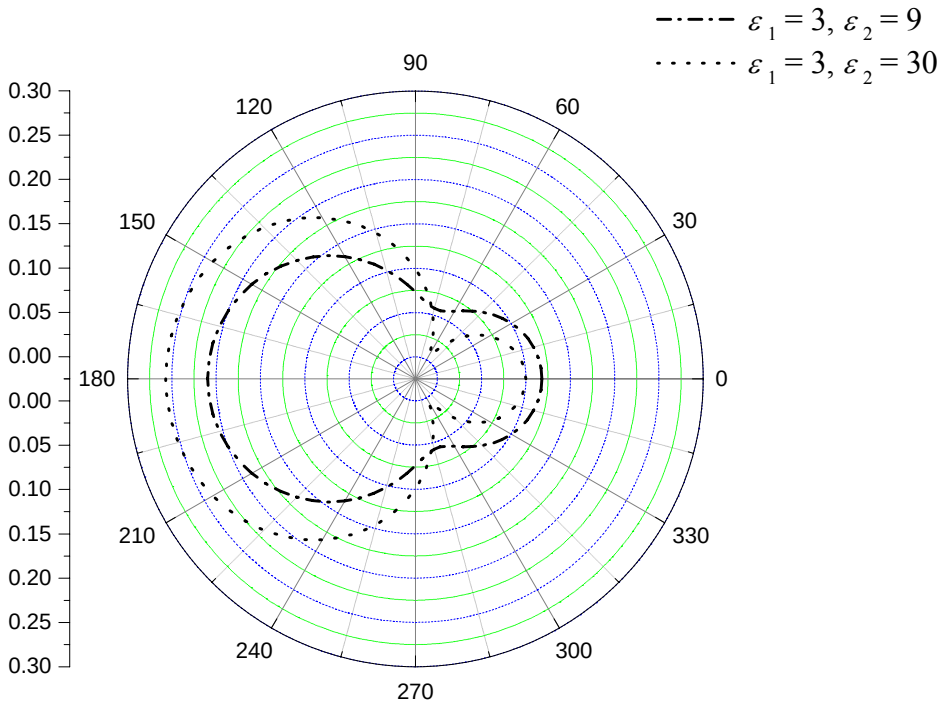

Figure 5. Scattering indicatrixes for an anisotropic inhomogeneity at $k_0 r = 4$

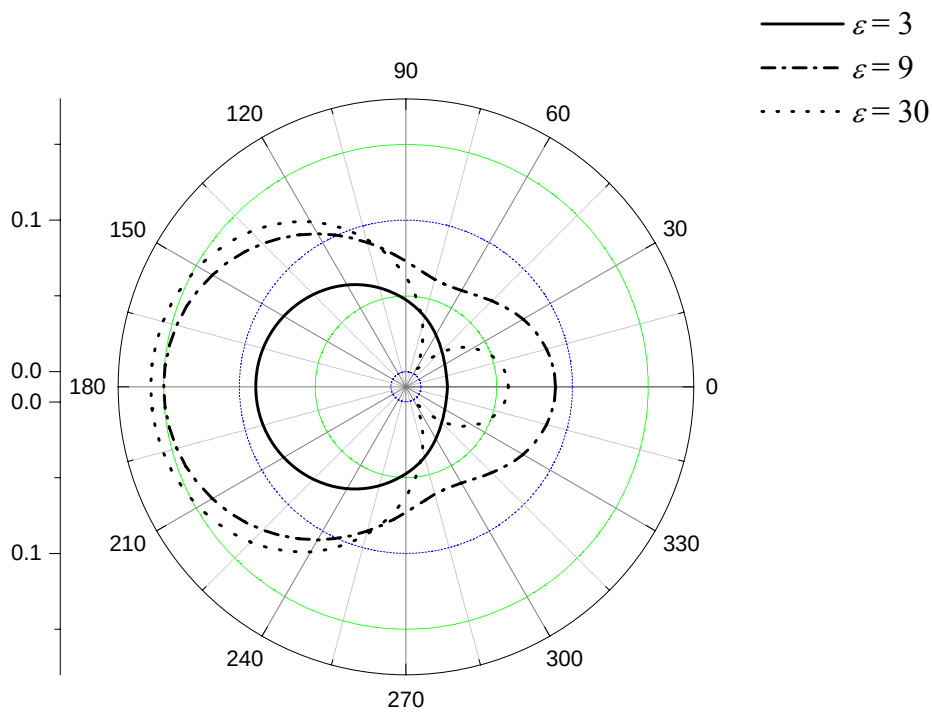

Figure 6. Scattering indicatrixes for an isotropic inhomogeneity at $k_0 r = 10$

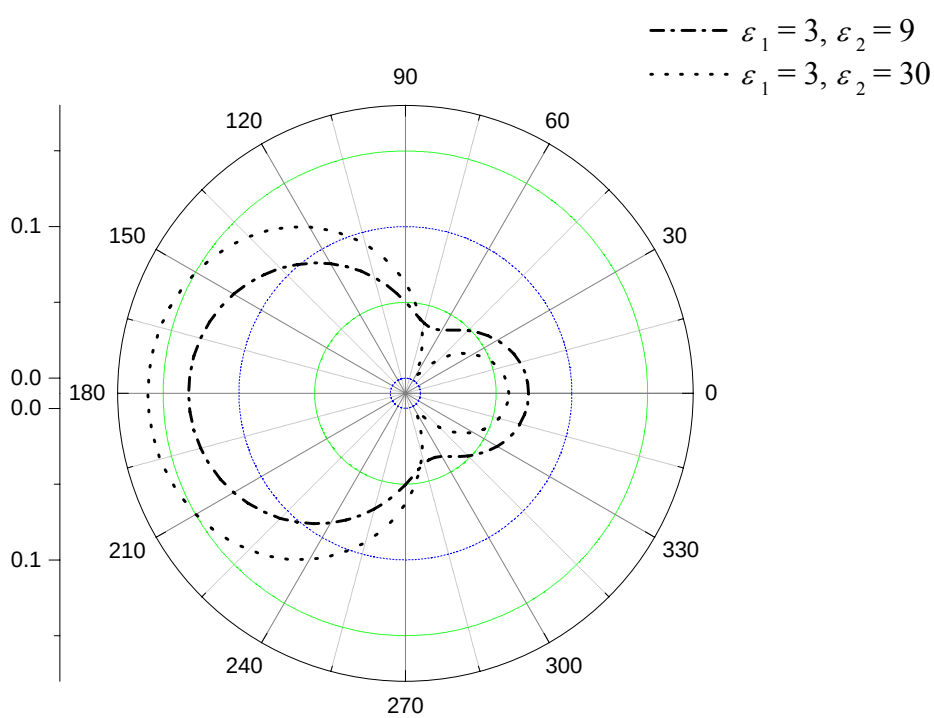

Figure 7. Scattering indicatrixes for an anisotropic inhomogeneity at $k_0 r = 10$

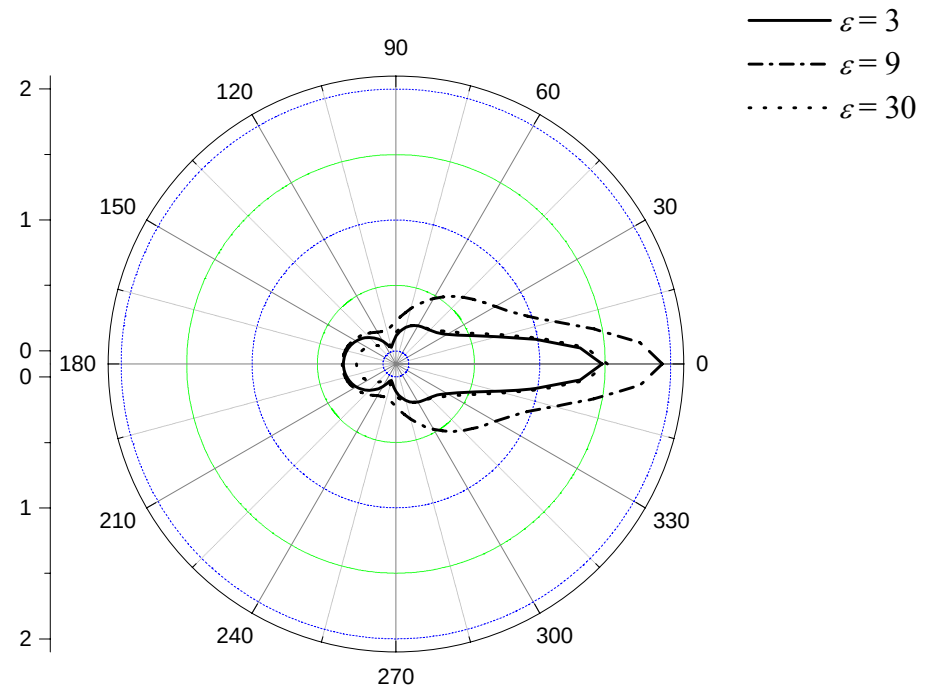

Figure 8. Azimuth diagrams of total E_z - component for an isotropic inhomogeneity at $k_0 r = 4$

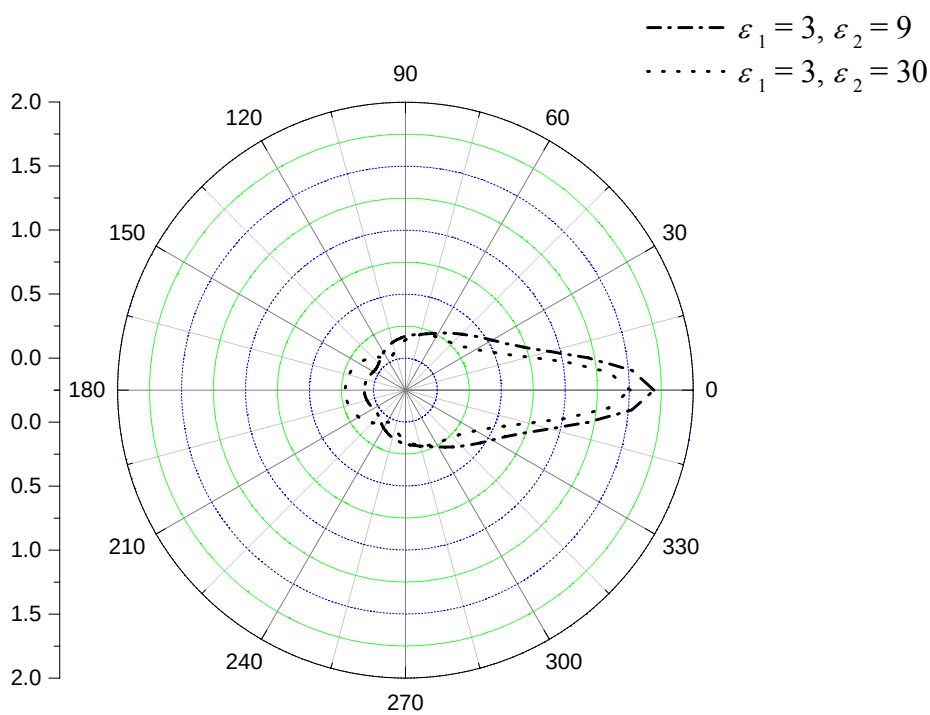

Figure 9. Azimuth diagrams of total E_z - component for an anisotropic inhomogeneity at $k_0 r = 4$

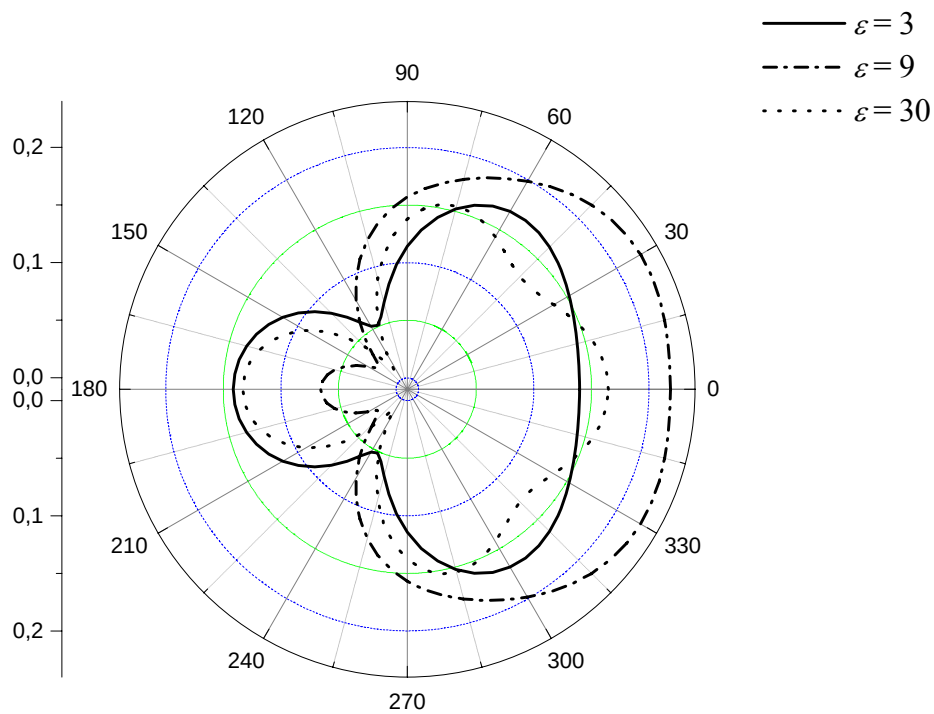

Figure 10. Azimuth diagrams of total E_z - component for an isotropic inhomogeneity at $k_0 r = 10$

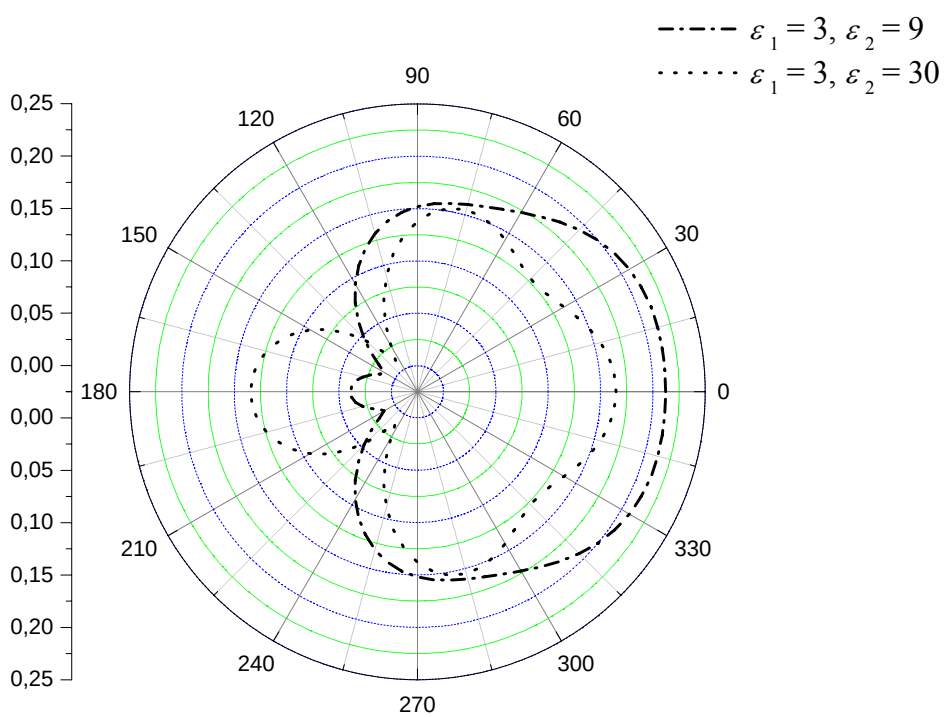

Figure 11. Azimuth diagrams of total E_z - component for an anisotropic inhomogeneity at $k_0 r = 10$

As a result substitution of (37) into (34) and (35) yields the following formula for the z-components of electric field intensity inside the waveguide

$$E_{3z} = -\frac{I_0 l}{4L} k_3 \zeta_3 \left\{ H_0^{(1)}(k_3 R) + i\pi \left[A\left(\frac{\beta_3}{2}\right) H_0^{(1)}(\alpha_3) H_1^{(1)}(k_3 r) - \right.\right.$$
$$\left.\left. - B\left(\frac{\beta_3}{2}\right)^2 H_1^{(1)}(\alpha_3) H_1^{(1)}(k_3 r) cos(\varphi - \varphi_0) \right] \right\} \tag{38}$$

where the coefficients A and B are as follows

$$A = \frac{1 - \Lambda \frac{\varepsilon_3}{2\varepsilon_1} \beta_1 \frac{J_0(\beta_2)}{J_1(\beta_2)}}{\beta_3 \ln\left(\gamma \frac{\beta_3}{2}\right) + \Lambda \frac{\varepsilon_3}{\varepsilon_1} \frac{\beta_1}{\beta_3} \frac{J_0(\beta_2)}{J_1(\beta_2)}}; \quad B = \frac{1 - \Lambda \frac{\varepsilon_3}{\varepsilon_1} \frac{\beta_1}{\beta_3^2} \left(1 - \left(\frac{\beta_3}{2}\right)^2\right) \frac{J_1(\beta_2)}{\dot{J}_1(\beta_2)}}{1 + \Lambda \frac{\varepsilon_3}{\varepsilon_1} \frac{\beta_1}{\beta_3^2} \frac{J_1(\beta_2)}{\dot{J}_1(\beta_2)}}.$$

When the wavelength inside the cylindrical inhomogeneity is also much less in comparison with the cylinder radius, i.e.

$$|k_2 b| << 1. \tag{39}$$

the simplified expression for the electric field can be written in the form

$$E_{3z} = -\frac{I_0 l}{4L} k_3 \zeta_3 \left\{ H_0^{(1)}(k_3 R) + i\pi \left(\frac{\beta_3}{2}\right) \left(\frac{\varepsilon_1 - \Lambda \varepsilon_3}{\Lambda \varepsilon_3}\right) \times \right.$$
$$\left. \times \left[H_0^{(1)}(\alpha_3) H_0^{(1)}(k_3 r) + \frac{1}{2}\left(\frac{\beta_3}{2}\right) H_1^{(1)}(\alpha_3) H_1^{(1)}(k_3 r) \cos\left[s(\varphi - \varphi_0)\right] \right] \right\} \tag{40}$$

It is worth noting that in the Relay approximation for a perfectly conducting cylindrical inhomogeneity the similar expression for the vector potential was obtained in the paper [6].

b) The ray approximation.

There is another limit case which allows to obtain the simplified problem solution. It is the ray approximation. The cylinder radius is suggested to be much less in comparison with the wavelength inside and outside the waveguide, i.e.

$$|k_2 b| >> 1 \tag{41}$$

$$|k_3 b| >> 1.$$

In this case it is convenient to use the Watson transformation [10] which allows to transform the formula (35) into the series fast converged. For this purpose transform the series of the form

$$S = \sum_{m=-\infty}^{\infty} A_m \exp\left[i\, m\left(\varphi - \varphi_0\right)\right]$$

into the contour integral

$$S = \frac{i}{2} \oint \frac{\exp\left[i\, \nu\left(\varphi - \varphi_0 - \pi\right)\right]}{\sin\left(\nu\pi\right)} A_\nu d\nu .$$

Moreover we shall use the Debay asymptotic expansion for the Hankel functions [9]

$$H_\nu^{(1)}(z) \approx \sqrt{\frac{2}{\pi^2 z \sin\gamma}}\, e^{-i\pi/4} e^{i\, z\left(\sin\gamma - \gamma\cos\gamma\right)}$$

$$H_\nu^{(2)}(z) \approx \sqrt{\frac{2}{\pi^2 z \sin\gamma}}\, e^{-i\pi/4} e^{-i\, z\left(\sin\gamma - \gamma\cos\gamma\right)} .$$

As the result substitution these relations into the formula (35) reduces the following expression for the vertical component of the electric field intensity

$$E_{3z}^{(2)} = \frac{I_0 l}{4L} k_3 \zeta_3 \sum_{m=0}^{\infty} \oint f_\perp(\nu) F(\nu) e^{i\Phi(\nu)} d\nu , \qquad (42)$$

where

$$f_\perp(\nu) = \frac{\zeta_1 \sin\gamma_{3b} - \zeta_3 \sin\gamma_{1b}}{\zeta_1 \sin\gamma_{3b} + \zeta_3 \sin\gamma_{1b}}; \quad F(\nu) = \left(2\pi k_3 \sqrt{a \sin\gamma_{3a}\, r \sin\gamma_{3r}}\right)^{-1};$$

$$\Phi(\nu) = k_3 a\left(\sin\gamma_{3a} - \gamma_{3a}\cos\gamma_{3a}\right) + k_3 r\left(\sin\gamma_{3r} - \gamma_{3r}\cos\gamma_{3r}\right) -$$
$$- 2k_3 b\left(\sin\gamma_{3b} - \gamma_{3b}\cos\gamma_{3b}\right) + \nu\left[\left(\varphi - \varphi_0\right) + 2m \cdot 2\pi\right];$$

$$\nu = k_3 a \cos\gamma_{3a} = k_3 r \cos\gamma_{3r} = k_3 b \cos\gamma_{3b} .$$

The rays geometry is shown in Figure 12. To calculate the integrals included in the formula (42) we apply the stationary phase method [10]. The stationary phase point $\nu = \nu^*$ is defined from the equality to zero of a derivative of the function *Φ(ν)* which can be written as follows

$$\begin{gathered} \gamma_{3a} + \gamma_{3r} - 2\gamma_{3b} = \left(\varphi - \varphi_0\right) \\ \sin\gamma_{3b} = \cos\theta_3 \end{gathered} \qquad (43)$$

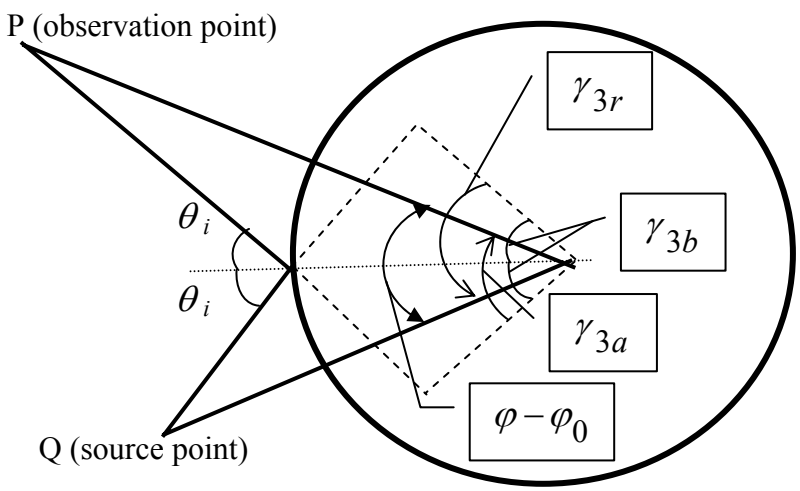

Figure 12. Physical interpretation of the ray approximation.

Figure 12 shows that the stationary phase point is the reflection point of the spherical wave incident on the cylinder from its surface. Hence, the angle θ_3 is the angle of a wave incidence on the cylindrical inhomogeneity surface.

As a result we have obtained the following expression for the vertical component of the electric field of the wave reflected from the inhomogeneity

$$E_{3z}^{(2)} = E_0(R) f_\perp^* e^{i k_3 R'} G_3 . \tag{44}$$

Here $E_0(R)$ describes the field of a wave incident on the cylindrical inhomogeneity

$$E_0(R) = \frac{I_0 l}{2L} k_3 \zeta_3 \frac{\exp i\left(k_3 R - \pi/4\right)}{\sqrt{2\pi k_3 R}} .$$

The quantity $f_\perp^*$ is the Frenel coefficient of the plane wave from the anisotropic dielectric

$$f_\perp^* = \frac{\Lambda\zeta_1 \cos\theta_3 - \zeta_3 \cos\theta_1}{\Lambda\zeta_1 \cos\theta_3 + \zeta_3 \cos\theta_1} .$$

It should be marked that having passed to the case of isotropic inhomogeneity we deduce the expression for Frenel coefficient presented in [10].

The multiplier G_3 is the ray tube divergence

$$G_3 = \sqrt{\frac{b\cos\theta_3}{2R' + \left(1 + R'/R\right) b\cos\theta_3}} ,$$

and the distances R and R' are defined by the formulas

$$R = a\sin\gamma_{3a} - b\sin\gamma_{3b}$$
$$R' = r\sin\gamma_{3r} - b\sin\gamma_{3b}.$$

5. CONCLUSION

In this paper the excitation problem of electromagnetic waves by a vertical electric vibrator inside a plane waveguide with perfectly conducting walls in the presence of an anisotropic cylindrical inhomogeneity and the diffraction problem of normal waves on this inhomogeneity has been considered. Using the expansion of sources and fields in terms of the normal waves of the plane waveguide together with the variable separation method yields the exact solution of the diffraction and excitation problem by the extraneous source of the inhomogeneity in the anisotropic circular cylinder form. This solution is presented in the double sums form over the eigenfunctions of the perfect plane waveguide and the azimuth eigenfunctions of the cylinder what is very convenient for numerical calculations. The anisotropy affect on the field diffracted in the waveguide has been analysed. It is shown that due to the mutual transformation of TE and TM modes the vertical electric dipole excites both electric and magnetic waves.

The numerical results of the scattering indicatrix and the total field structure inside the waveguide have also been performed. The two important particular cases have been considered in detail: diffraction on the thin cylinder (the Relay approximation) and diffraction on the cylinder of a large electric radius (the ray approximation).

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