CALCULATION OF THE CHARACTERISTIC IMPEDANCE OF TWO-CONDUCTORS MICROWAVE STRIP TRANSMISSION LINE

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Abstract

The characteristic impedance Z (TEM - approximation) of a microwave transmission line composed by two rectangular conductors disposed above an infinitive dielectric slab (two - conductors strip transmission line) is calculated. The extended mirror images method together with the universal method for calculation the characteristic impedance of strip transmission lines are used. Calculations are made for two cases. In first case both strip conductors are identical. The graphics for several widths (w\d = 0.1 ; 0.2 ; 0.5 ; 1.0 ; 2.0) and wide range of thicknesses (t/d = 0.01 - 1.0) are shown.. The current density distribution over down and up walls of the conductors is clarified also. In the second case calculations are made when the conductors have different sizes. The values of Z when the sizes of the first conductor remain the same (w_1/d = 1. and t_1/d = 0.1) and the sizes of the second conductor change (w_2/d = 0.1 - 0.5 and t_2\d = 0.01 - 0.5) are shown. In all calculations the thickness of the dielectric slab is 2d, its dielectric constant is ε_r = 2.7 and the distance between conductors is s\d = 0.2 . The expected errors are enough small to meet the design requirements.

1. INTRODUCTION

The propagating of microwaves by the open strip line structures with finite thickness of the strip conductors is recently discussed [1], [2], [3]. Lines with significant thickness of the strip conductors are used when it is necessary to design microwave devices transmitting signals with intermediate power. Also it is necessary to take care of the thickness of the strip conductors in monolithic microwave integrated circuits if they are operating with frequencies above 10 GHz. In this paper an attempt is made to calculate the quasi - TEM characteristic impedance Z for the transmission line built from two conductors with rectangular profile disposed on an infinitely in x and z directions dielectric slab - two-conductors strip transmission line (Fig. 1). If $w_1 = w_2 = w$ and $t_1 = t_2 = t \neq 0$, the two-conductors line is considered as a coplanar parallel strips line with thick strip conductors. Some results concerning the characteristic impedance Z of the coplanar

parallel strips transmission line in a case of equally wide ($w_1 = w_2$) and infinitely thin conductors ($t_1 = t_2 = 0$) are reported in [4], [5], [6].

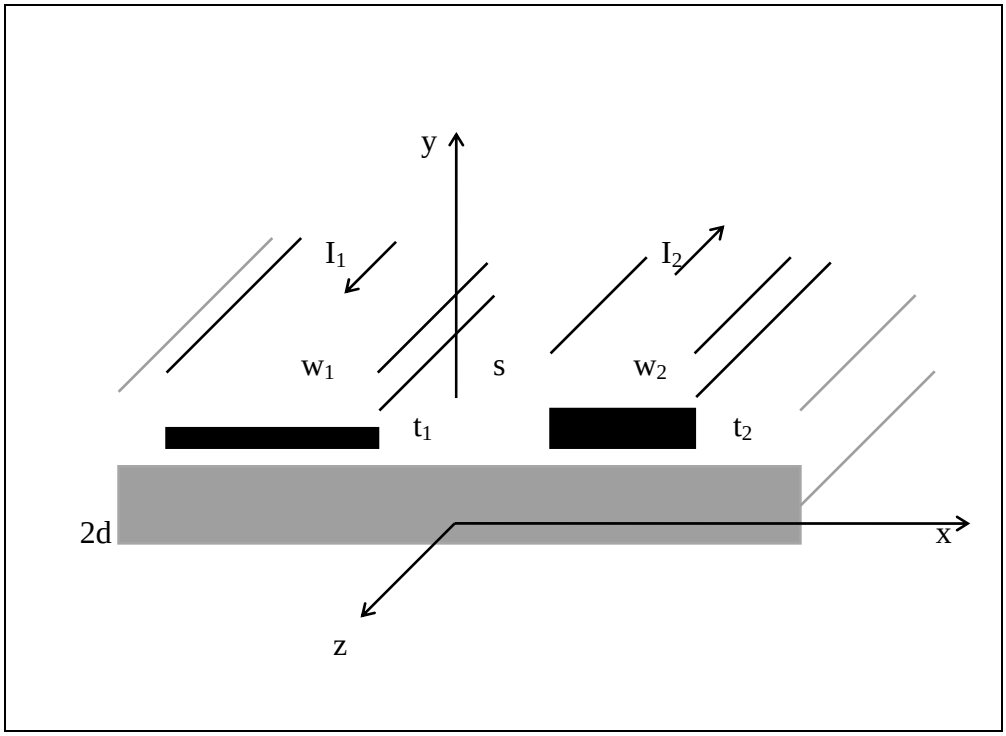

Figure 1. Two-conductors strip transmission line

2. THEORY

The characteristic impedance Z of strip transmission lines is defined in terms of power and current using next formulas [7]:

$$(1) \qquad Z = Z_0\sqrt{\frac{Z_1}{Z_0}}; \qquad Z_0 = \frac{1}{\sqrt{\varepsilon_0\mu_0}}\frac{1}{I^2}N; \qquad Z_1 = \frac{1}{\sqrt{\varepsilon_0\mu_0}}\frac{1}{\sqrt{\varepsilon_r\mu_r}}\frac{1}{I^2}N$$

where I is the current flowing along one of the line's conductors (the currents on the two conductors are equal and have opposite directions ($I = I_1 = -I_2$) ;

(2) $$N = \int_S J(x,y).A(x,y)dxdy$$

is the full electromagnetic power per unit length (in z - direction), S is the cross section of the line, J(x,y) is the current density and A(x,y) is the vector potential of the electromagnetic field.

If the conductors are considered as ideal, the function J(x,y) ≠ 0 only on the perimeters of the conductors and the integration procedure over the aria S in (2) is replaced by integration procedure along the line as follow:

(3) $$N = \sum_{i=1}^{2} \int_{P_i} J(x,y).A(x,y)dxdy \ ,$$

where P_i is the perimeter of the i-th conductor (i = 1, 2).

The vector potential A(x,y) created by a current fiber located in a free space parallel of axis z crossing the plane z=0 in point (x_0 ,y_0) has only a z - component (A_0) and it is up to the requirements of two dimensional Poison equation. The solution is well known and is expressed with the next formula:

(4) $$A_0(x,y) = -\frac{\mu_0}{4\pi} I_0 \ln\left[(x-x_0)^2 + (y-y_0)^2\right] = -\frac{\mu_0}{4\pi} I_0 \ln R^2 ,$$

where

$$R^2 = (x - x_0)^2 + (y - y_0)^2 \ .$$

and I_0 is the value of the current and it flows in z direction.

A vector potential A(x,y) created by a fibre, located above a dielectric slab was defined by three different formulas for each of the space regions ($y \geq d$; $-d \leq y \leq d$ and $y \leq -d$) using the extended mirror images method [10]. For the purpose of calculations in this paper it is necessary to know A(x,y) for the region $y \geq d$ (see Fig. 1). If the dielectric slab is thick 2d and its relative dielectric constant is ε_r , the vector potential of electromagnetic field is expressed with next formula [10]:

$$
\begin{aligned}
A(x, y) = -\frac{\mu_0 I_0}{4\pi} \{ & \left[(x - x_0)^2 + (y - y_0)^2\right] + \\
& + k \ln\left[(x - x_0)^2 + (y - (d - a))^2\right] - \\
& - \sum_{n=1}^{\infty} c_n \ln\left[(x - x_0)^2 + (y + (4n - 1)d + a)^2\right] \},
\end{aligned} \tag{5}
$$

where

$$k = \frac{1 - \varepsilon_r}{1 + \varepsilon_r}$$

and

$$c_n = (1 - k^2) k^{2n-1}.$$

This result shows that the real case - current fiber disposed above an infinite dielectric slab can be replaced by an equivalent model - infinite number of current fibbers in free space with suitable currents and coordinates [10]. The absolute value of c_n is bellow zero and c_n changes alternatively its sign ($k < 0$) so the infinite sum in (5) is fast converging sum and can be replaced by finite one.

The equation (5) is used to find the vector potential of the discussed line. It is proposed that the perimeter of each conductor is made of infinite number of parallel current fibers. The current of the fibers in different points over the perimeters agree with current density distribution $J(\xi,\eta)$ in the same points. According to this the characteristic impedance is expressed by following formula [8]:

$$Z = \frac{-1}{4\pi I^2}\sqrt{\frac{\mu_0}{\varepsilon_0}}\sum_{i=1}^{2}\int_{P_i} dxdyJ(x,\ y)\sum_{n=1}^{\infty}\int_{P_n} c_n J(\xi,\ \eta)\ \ln\left[(x-\xi)^2+(y-\eta)^2\right]d\xi d\eta.$$

The universal method for determining the characteristic impedance of the strip transmission lines [8] gives the solution of last integral with two restrictions:

a) The perimeter P_i of each conductor is divided to straight line sections L_i. This is possible because the cross sections of the conductors are rectangular.

b) The real current densities $J_i(x,y)$ on the line sections L_i are replaced by their average values J_i. This means that the real current density distributions on every wall of the conductors is approximated by step function model. This replacement influences negligible the characteristic impedance Z if the perimeters are divided in sufficient number properly chosen straight-line sections. The values of J_j are calculated by using the extended self-consistent field method [9].

3. NUMERICAL RESULTS

All calculations for the characteristic impedance Z of the two-conductors strip transmission line are made for the dielectric slab with relative dielectric constant $\varepsilon_r = 2{,}7$. The geometric sizes are normalized regarding d (Fig. 1). The normalized thickness of the slab is D = 2d/d = 2, the normalized widths of the conductors are $W_1 = w_1/d$, $W_2 = w_2/d$, the normalized thicknesses $T_1 = t_1/d$ and $T_2 = t_2/d$ and the normalized separation between them S = s/d = 0,2 . The perimeters of the conductors are divided into 38 straight-line sections each. The length of the sections decreases toward the edges of the conductors (Fig. 2). In this way the step function model with 38 steps on each conductor is close to the real current density distribution.

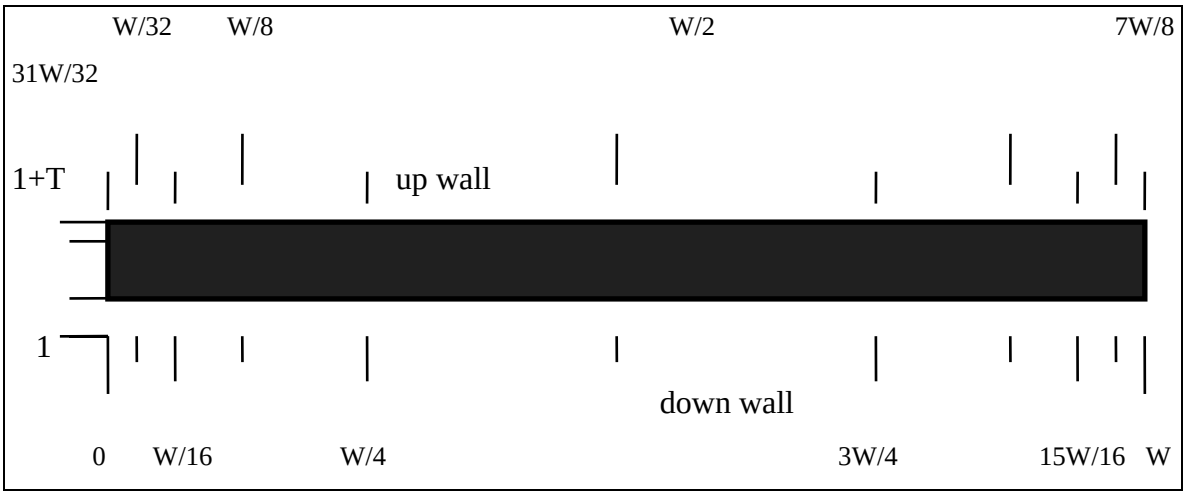

Figure 2. The splitting up the conductor perimeter into unequal in length straight-line sections

The results of calculations for the case corresponding to coplanar strip transmission line with thick conductors (see Fig. 1) are shown in Fig. 3. The task was to clarify the influence of the thickness of the strip conductors on the characteristic impedance Z, in the figure there are curves showing how Z depend on the size T for several widths of the strip conductors. The average values of the current density on the sections are received like a mutual result . The supposed real current density distributions (absolute value) for the two-conductors strip transmission line with normalized sizes W1 = W2 = 1.0 ; T1 = T2 = 0.2 and S=0.2 is shown in Fig. 4.

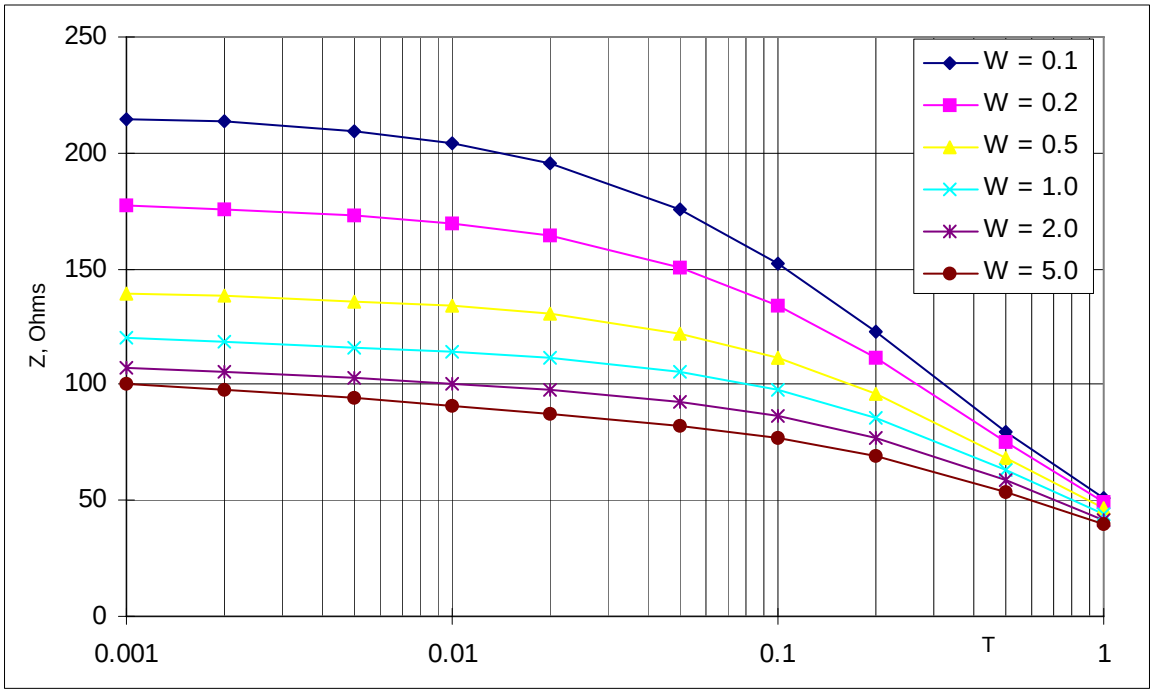

Figure 3. Characteristic impedance Z of the coplanar strip transmission line versus thickness T of the conductors.

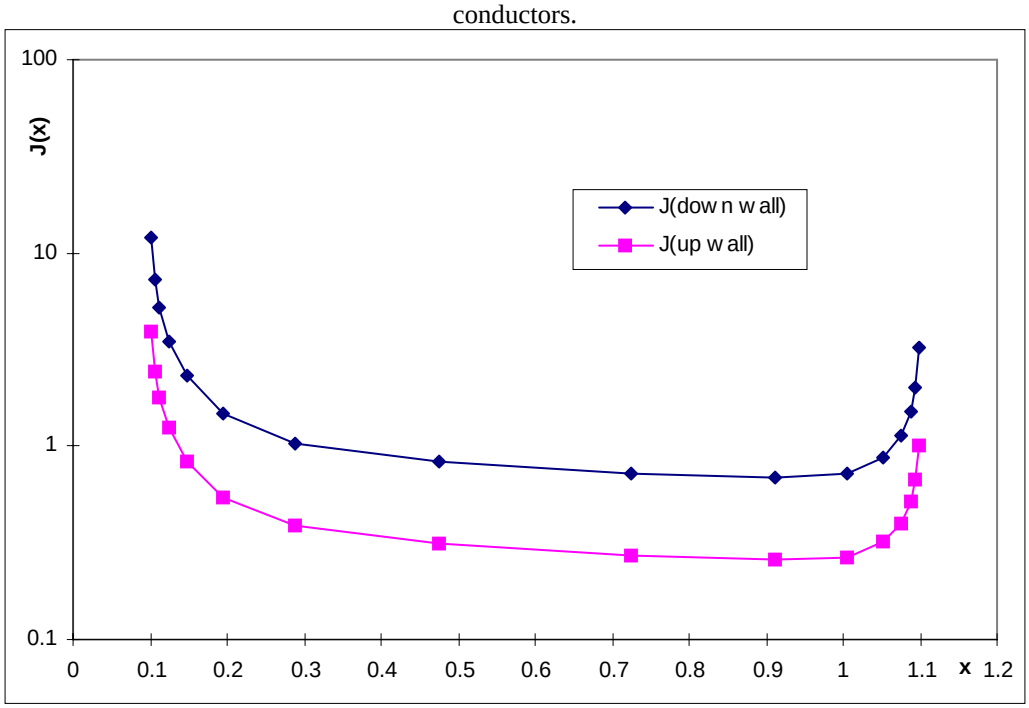

Figure 4. Current density distribution on the down and up walls of the right conductor in case of coplanar strip transmission line (W = 1.0 ; T = 0.1; S = 0.2; ε_r = 2.7)

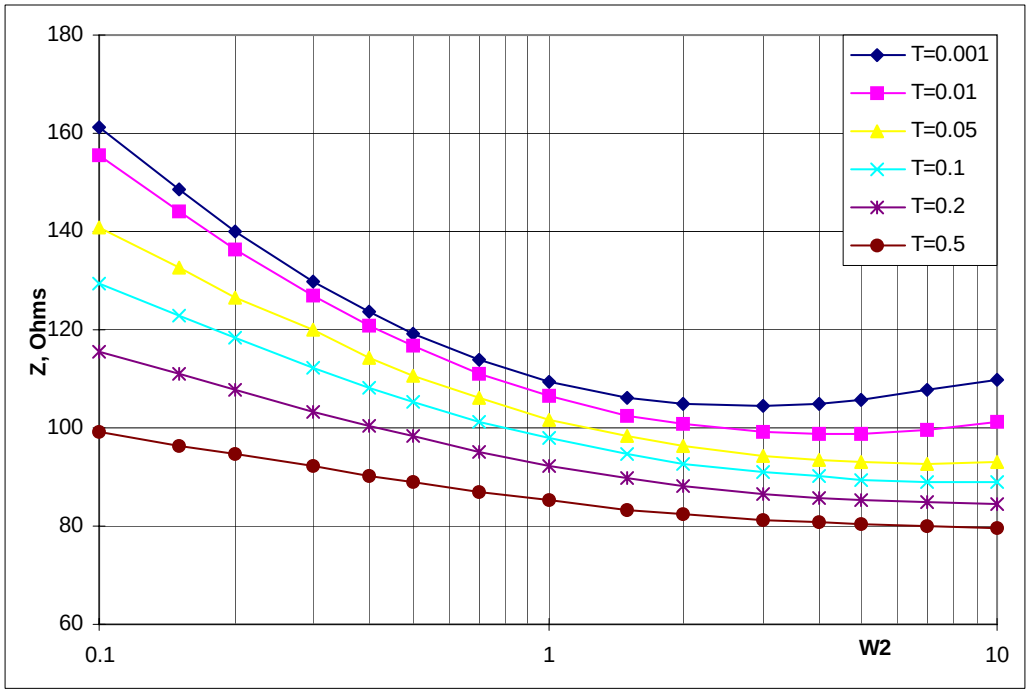

Figure 5. The characteristic impedance of the two-conductor strip transmission line versus the normalized width (W_2) and thickness (T_2) of the second conductor if the sizes of the first conductor and separation remain constant (W_1= 1. , T_1= 0.1 , S = 0.2)

Calculations for the characteristic impedance are made also for the less symmetrical case of two-conductors strip transmission line built from strip conductors with different in size rectangular cross sections (Fig. 5). The results for Z when the left conductor saves its sizes (W_1 = 1.0 , T_1 = 0.1) and the right conductor changes its width - W_2 and thickness - T_2 are presented in Fig. 6. The separation between the conductors remains the same S = 0.2 .

4. CONCLUSION

The extended mirror images method together with enlarged self- consisted field method and universal method for calculating the characteristic impedance (Z) of strip transmission lines applied in previous works to calculate Z for microstrip, coupled and three conductors microstrip lines is successfully used in this article to calculate the

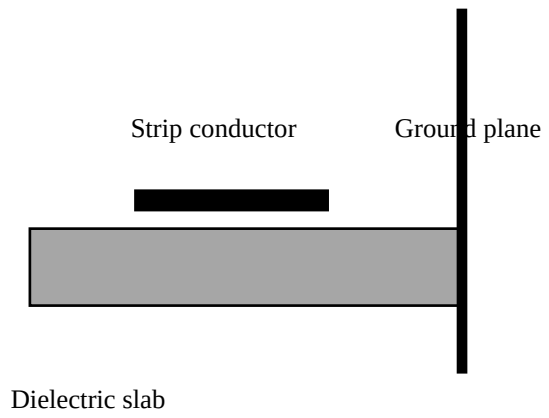

Figure 6. Strip transmission line composed by strip conductor, dielectric slab and infinite ground

characteristic impedance of two conductor strip transmission line without any restrictions of the conductor sizes W and T. The comparison the results for the symmetrical case (coplanar line) with very thin strip conductors (T=0.001) with the data in [4] shows a good agreement. This fact gives the confidence to assume that all other results are calculated with sufficient for application.

The symmetry of the task permits to consider the results received for the corresponding to a coplanar case of transmission line (Fig. 3), as a doubled characteristic impedance of the line shown in fig. 6.

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