NOISE STABILITY OF SPREAD-SPECTRUM COMMUNICATION SYSTEMS

G.Chalikyan, M.Markosyan, G.Simonyan

Yerevan Telecommunication Research Institute
Republic of Armenia, Yerevan
email: mark@arminco.com

Abstract

The method of information transmission by means of a circular shift of a pseudo-random sequence is reviewed, relevant formulas proved and curves of probability of error dependence on numbers of shifts and time of correlation in the receiver are given.

1. INTRODUCTION

The communication systems using the property of spread-spectrum signals (SSS) have received broad usage thanks to their high noise stability, security and code addressing of a large number of subscribers in a common band [1]. However communication systems with SSS occupy a broad band, providing low rate of information transmission [2]. In [3] the method of information transmission by means of circular shift of a pseudo-random sequence (PRS) is reviewed, which allows increase of transmission rate. However in [3] the noise stability of such systems is not calculated. The aim of this work is to calculate the probability of error dependence on number of shifts of a PRS and time of correlation in the receiver.

2. ANALYSIS OF CODING SYSTEM

According to the analysis of [3], at the transmitting end the initial information sequence is divided into groups containing m-binary bits, then each possible combination is brought in correspondence to PRS with definite circular shift. The maximum number of shifts is determined by $Q=2^m$. In the case when 3 information bits are simultaneously transmitted, table 1 shows appropriate PRS for each possible combination

Information sequence	Transmitted PRS
000	$a_1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots a_N$
001	$a_N a_1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots a_{N-1}$
010	$a_{N1} a_N a_1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots a_{N-2}$
001	$a_{N2} a_{N1} a_N a_1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots a_{N-3}$
100	$a_{N3} a_{N2} a_{N1} a_N a_1 \ldots\ldots\ldots\ldots\ldots\ldots a_{N-4}$
101	$a_{N4} a_{N3} a_{N2} a_{N1} a_N a_1 \ldots\ldots\ldots\ldots\ldots a_{N-5}$
110	$a_{N-5} a_{N4} a_{N3} a_{N2} a_{N1} a_N a_1 \ldots\ldots\ldots\ldots a_{N-6}$
111	$a_{N-6} a_{N-5} a_{N4} a_{N3} a_{N2} a_{N1} a_N a_1 \ldots\ldots\ldots a_{N-7}$

Table 1

As it is seen in table 1, PRS is transmitted without shift only in case of 000 combination transmission. In other cases, PRS is transmitted with shift on 1,2 .. 7 chips. The block diagram of the transmitter is shown in figure 1 and the block diagram of the receiver is shown in figure 2. For simplicity the basic blocks are brought only.

In the transmitter (figure 1), from an information source (IS) the binary sequences are fed on the first input of a source encoder (ES). On the second input PRS are fed from the PRS generator (PRSG). The ES analyzes 3 (generally - m) bits of information sequence and according to this PRS shift is realized. In figure 3a the information sequence and in figure 3b the appropriate PRS are represented.

In the receiver (figure 2) the transmitted PRS by means of the matched filter (MF) correlate with a reference PRS. In figure 3c the correlation resulting signals (main lobes of correlation functions CF) are shown, where it is seen that depending on the number of circular shifts with which PRS was transmitted, these signals origin at $0\tau_0$, $2\tau_0$, $7\tau_0$ (where τ_0 - PRS chip duration) later than in the case of transmission of a PRS without shift (figured by a dotted line).

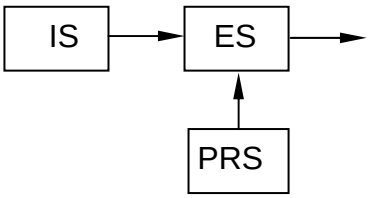

Figure 1

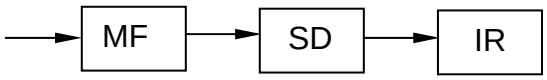

Figure 2

The decoder of a source (SD) determining time differences establishes the transmitted information sequence and feeds it to the receiver of the information (IR).

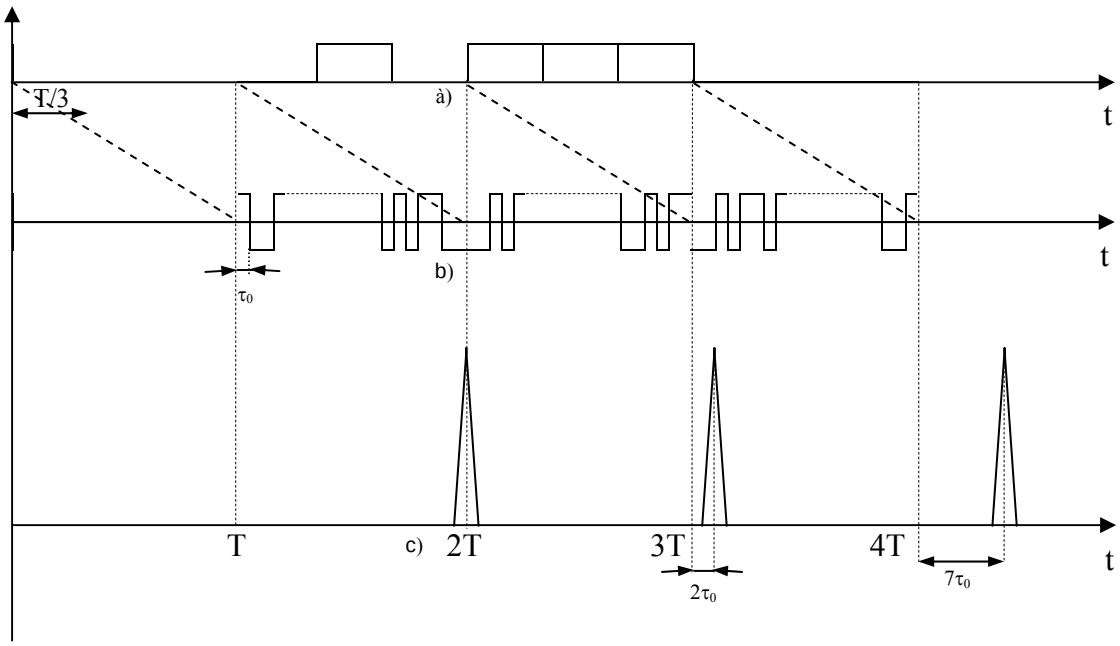

Figure 3

In figure 3c the principle of the method CF is shown, where the side lobes of CF are not shown, and also the influence of the main lobes CF of the subsequent PRS is not taken into account. Let's now calculate the probabilities of an error. As it is known from [4] the response of the MF to a signal with which it is matched, has CF of the last $K_s(t)$, displaced at time T in the side of delay, i.e.

$$S_{\text{out}}(t) = K_S(t-T) = \int_{-\infty}^{\infty} S(\theta) S(\theta - (t-T)) d\theta$$

Therefore, the maximum amplitude of a signal after MF is given by:

$$U_{mout}=S_{out}(T)=K_s(0)=E \quad (1)$$

The CF of the white noise in the output of MF:

$$K_{out}(\tau)=0.5N_0K_s(\tau) \quad (2)$$

and

$$K_{out}(0)=N_0E/2 \quad (3)$$

However (1) is correct in case of transmission of a PRS with zero shift. In all other cases the signal amplitude on the output of MF depends on the subsequent PRS. It is possible to calculate the amplitude of a signal in the case of discrete variation τ by taking advantage the formula of account CF of phase-shift keyed signals [1]

$$K(\tau) = \sum_{n=n_n}^{n_2} a_n b_{n-\tau} \quad (4)$$

where

$n_1 = \tau + 1, \quad n_2 = N$ when $\tau \geq 0$

$n_1 = 1, \quad n_2 = N - |\tau|$ when $\tau < 0$

$\tau[-N; N]$

Having taken advantage (4) and having omitted intermediate expressions, we shall write the expression permitting to calculate the values of the main and side CF lobes.

$$K(\tau) = \sum_{n=\tau+1}^{N} a_n b_{n-\tau} + \sum_{n=1}^{\tau} a_n c_{n-\tau+N} \quad (5)$$

where: a_n - code sequence with which MF is matched and which corresponds to PRS without shift, b_n - the code sequence which corresponds to the transmitted PRS, and c_n – corresponds to transmitted PRS immediately after b_n. Here τ [0, Q-1].

In each possible couple b_n and c_n calculating the value of the main and side lobes, it is possible to calculate the probability of a false alarm and skip of a signal for each τ.

The probability of a false alarm is obtained to be:

$$P_{\tau\, fa}=\exp(-h^2/2) \quad (6)$$

where $h = z_{th} / \sqrt{N_0 E / 2}$

The probability of skip of a signal is also determined to be:

$$P_{\tau\,\mathrm{ss}} = \int_0^{z_{\mathrm{th}}} \frac{z}{N_0 E/2} \exp\left(\frac{z^2 + E^2}{2\frac{N_0 E}{2}} \right) I_0\left(\frac{zE}{N_0 E/2} \right) dz \tag{7}$$

where z_{th}=0.5E has a constant value.

The common probability of an error in case of equal prior probabilities of transmission PRS can be calculated under the formula (8), where $P_{\tau\,\mathrm{fa}}$ and $P_{\tau\,\mathrm{ss}}$ are the average probabilities.

$$P_{\mathrm{er}} = 1 - \frac{1}{Q}\left[\sum_{i=1}^{Q} \left(1 - P_{\mathrm{ss}_I}\right) \prod_{j=1}^{i-1} \left(1 - P_{\mathrm{fa}_j}\right) + P_{\mathrm{ss}_1} \prod_{i=2}^{Q} \left(1 - P_{\mathrm{fa}_i}\right) \right] \tag{8}$$

In expression (8) has been taken into account that if all temporal moments t are below the threshold, a PRS with zero shift is considered. If the threshold is exceeded by many temporal moments, then the first exceeding of the threshold is considered.

The curves of probability of error dependence on the signal-to-noise ratio in the input of the threshold device, if PRS is M-sequence with N=127, are shown in figure 4

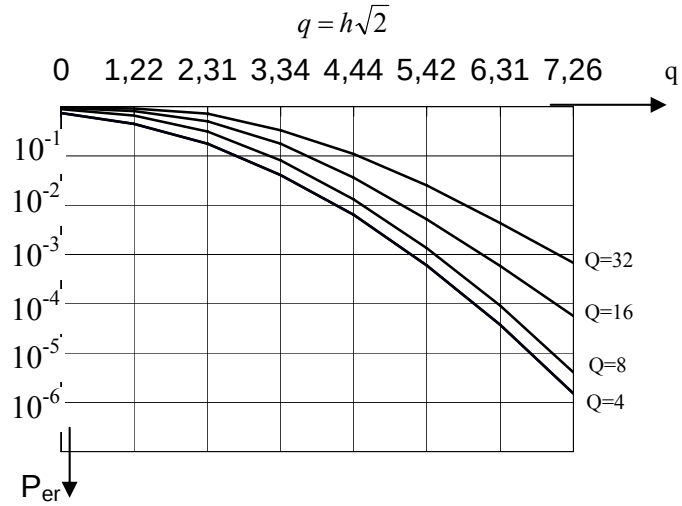

Figure 4

In order to reduce the probability of an error it is possible to increase the time of integration (figure 5). As it is seen in figure 5 (Q=4), P_{er} diminishes with increase in time of integration T_i until T_i=T+3τ_0, and then it increases. Selecting T_i =T+3τ_0 it is possible to reduce the probability of an error (r= (T_i-T) /τ_0).

Figure 5

REFERENCES

[1] L.E Varakin., “Sistemy svazi s shumopodobnymi signalami” .-M.: Radio i svaz, 1985. –384s. (in russian)

[2] Z87200 Spread - Spectrum Transcelver, Product Specification

[3] P20010048 24.10.2001y Patent of Republic of Armenia

[4] V.I. Tixonov, “Optimalnyj priem signalov.” –M.: Radio i svaz, 1983.-320c. (in russian)