FINITE ELEMENT METHOD FOR THE ANALYSIS OF GUIDED MODE REFLECTION FROM THE FACET OF A DIELECTRIC WAVEGUIDE

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Abstract

The reflectivity of a guided mode from the facet of a dielectric waveguide is examined by the finite element method in scalar and vector formulations. The three dimensional problem is reduced to the two dimensional one using a variational formula for the reflection coefficient. Numerical tests are provided.

1. INTRODUCTION

The information about facet reflectivity of the dielectric waguides is important in the design of the antennas on dielectric waveguides, integral optic devices, elements for the output of the light from the fibers and others. The complexity of the problem is conditioned by two factors. Firstly, if the dielectric waveguide has noncircular cross section, then the task becomes essentially three dimensional. Second factor is that the dielectric waveguide is an open structure, and it is necessary to take into account the radiating modes of continuous spectrum during the modelling. Direct three dimensional modelling of the problem is very expensive in the sense of CPU time and memory. Here we propose another approach, that allows to reduce the problem to the two dimensional one. Our approach is based on the combination of a variational formula for the reflection coefficient of a guided mode from the facet of a dielectric waveguide and two dimensional finite element method.

In Section 2 we present the variational formulas for the reflection coefficient in vector case and in scalar case. In Section 3 some questions of the practical implementation of the algorithm in both vector and scalar cases will be discussed. In Section 4 the numerical results of the computation using our approach will be presented and discussed.

2. FORMULAS FOR THE REFLECTION COEFFICIENT

The geometry of the problem under consideration is shown in Fig. 1. There is a dielectric waveguide with arbitrary shape of its cross section. The refractive indeces of the core and the envelope are n_1 and n_2 respectively. In the plane $z = 0$ the waveguide abruptly ends and the refractive index of the right semi-space is denoted as n_0.

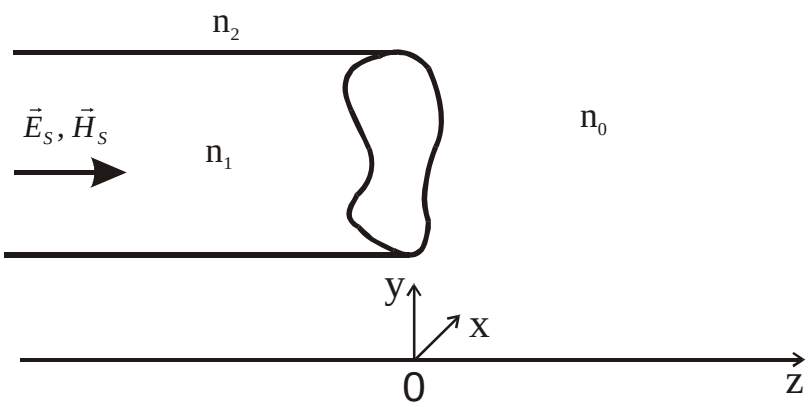


Fig. 1. Abrupt end of a dielectric waveguide: geometry of the problem

Guided mode of the dielectric waveguide is propagating from the left side to the facet, and then scattering on the facet. The wave partially radiates to the right semi-space, partially reflects back to the waveguide, and partially radiates to the envelope of the waveguide. The aim of our analysis is to find the reflection coefficient of the guided mode from the facet.

Full electric field on the left from the facet $z \le 0$ can be expressed as following

$$\vec{E}(r) = \vec{E}_S^{+}(1) + \rho\vec{E}_S^{-}(1) + \sum_{m,q} C_{m,q}^{-}(1)\vec{E}_{m,q}^{-}(1) + \sum_{m}\int_0^{\infty} C_{m\kappa}^{-}(1)\vec{E}_{m\kappa}^{-}(1)d\kappa\,, \qquad (1)$$

where $\vec{E}_S^{+}(1)$ is the field of incident guided mode, $\vec{E}_S^{-}(1)$ is the field of reflected guided mode, ρ is the reflection coefficient, $\vec{E}_{m,q}^{-}(1)$ are the fields of other guided modes that may appear as a result of scattering, $\vec{E}_{m\kappa}^{-}(1)$ are the fields of radiating modes of continuous spectrum, $C_{m,q}^{-}$ and $C_{m\kappa}^{-}$ are the amplitude coefficients. Full electric field on the right from the facet $z \ge 0$ is a sum of only radiating modes

$$\vec{E}(r) = \sum_{m}\int_0^{\infty} C_{m\kappa}^{+}(2)\vec{E}_{m\kappa}^{+}(2)d\kappa\,. \qquad (2)$$

At the plane $z=0$ both expressions (1) and (2) must be equal to each other and from this condition we can obtain the following expression [1]

$$\frac{1-\rho}{1+\rho}=2\left\langle \vec{E}_S,\vec{H}_S\right\rangle^{-1}\sum_m^{\infty}\int_0^{\infty}\left\langle \vec{E}_S,\vec{H}_{m\kappa}^{(0)}\right\rangle^2\left[D_m^{(0)}(\kappa)\right]^{-1}d\kappa \tag{3}$$

where $\left\langle \vec{E}_S,\vec{H}_S\right\rangle=\iint_{z=0}\vec{E}_S\times\vec{H}_S\vec{e}_z dx\,dy$, $\vec{E}_S,\vec{H}_S$ are the electric and magnetic fields of incident guided mode, $\vec{H}_{m\kappa}^{(0)}$ is the magnetic field of radiating mode of right semi-space, $D_m^{(0)}(\kappa)$ is the norm coefficient for the mode $\vec{H}_{m\kappa}^{(0)}$.

For the calculation of the reflection coefficient ρ using formula (3) it is necessary to know the fields of the guided mode in the cross section of the waveguide. All other values in (3) can be computed analytically. The field of guided mode can be computed numerically using two dimensional vector formulation of the finite element method.

If the problem under consideration is analyzed under weak guidance condition $|n_1-n_2|<<n_1$, then the field of guided mode has almost linear polarization in the cross section of the waveguide and can be described by one scalar function of the transverse coordinates x,y. It is convenient to choose the electric field in the cross section of the waveguide $E(x,y)$ for this scalar function.

Under conditions $|n_1-n_2|<<n_1$, $|n_1-n_0|<<n_1$ formula (3) becomes the following

$$\frac{\rho}{1-\rho^2}=\frac{k^2(n_1^2-n_2^2)}{hN}\sum_{m=0}^{\infty}\int_0^{\infty}\frac{Q_m^2(\kappa)d\kappa}{h_\kappa D_m(\kappa)(p^2+\kappa^2)}, \tag{4}$$

where $k=2\pi/\lambda$, h is propagation constant of guided mode, $p=\sqrt{h^2-k^2n_2^2}$ is transverse wavenumber in the envelope, $N=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}E^2(x,y)dxdy$ is the norm of the surface mode, $h_\kappa=\sqrt{k^2n_0^2-\kappa^2}$ is the propagation constant of radiating mode, $Q_m(\kappa)=\iint_{z=0}E(x,y)E_{m,\kappa}(x,y)dxdy$, $E_{m,\kappa}(x,y)=2J_m(\kappa r)\cos(m\varphi)$ is the electric field of the radiating mode of continuous spectrum, $D_m(\kappa)$ is norm of the radiating mode of continuous spectrum. In the expression (4) all values except the electric field of guided

mode $E(x, y)$ can be computed analytically. The field $E(x, y)$ can be calculated numerically using the finite element method in two dimensional scalar formulation.

3. PRACTICAL IMPLEMENTATION OF THE ALGORITHM

Now consider some questions of practical implementation of the reflection coefficient computation algorithm using formulas (3) and (4).

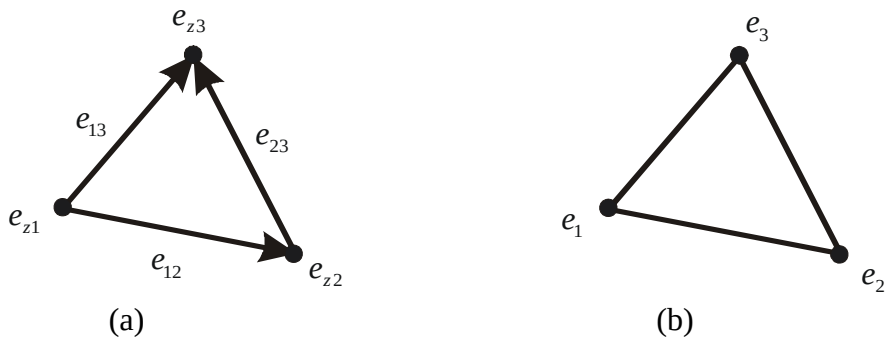


Fig. 2. Types of finite element used. (a) mixed order triangular element,

(b) nodal triangular element

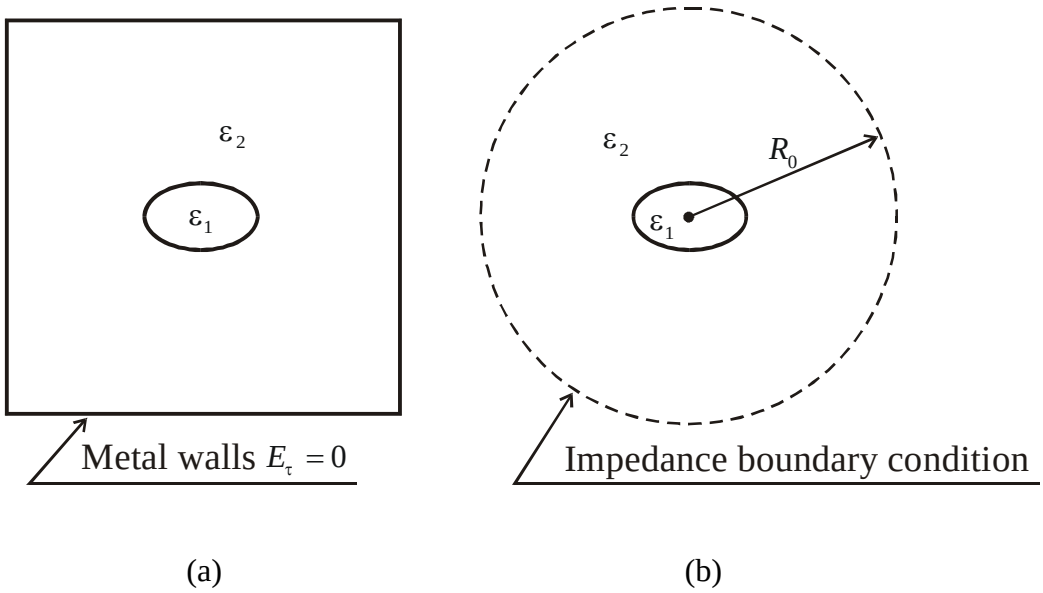


Fig. 3. Boundary conditions used. (a) metall walls for the vector cases,

(b) impedance boundary condition for the scalar cases.

The first point is the type of finite elements used. For the vector cases, when the refractive indeces of the core, envelope, and right semi-space differ significantly, we used mixed order triangular elements (see Fig. 2 (a)). The field in the cross section of the waveguide is decomposed on vector functions, associated with the edges of the

mesh, and the longitudinal component of the field is decomposed on scalar functions, associated with the nodes of the mesh

$$\vec{E}_{\perp} = \sum_{\substack{i,j \\ i<j}} e_{i,j} \vec{w}_{i,j} \ , \quad E_z = \sum_i e_{zi} \lambda_i \tag{5}$$

For the scalar cases when the weak guidance condition is satisfied we used nodal triangular elements (see Fig. 2 (b))

$$E = \sum_i e_i \lambda_i \ . \tag{6}$$

The second point is the boundary condition on the outer boundary of the computational domain. For the vector cases we set simply metallic walls as an outer boundary (see Fig. 3 (a)). For the scalar cases we used impedance boundary conditions [2], taking into account the eigenmode field distribution on the outer boundary (see Fig. 3 (b))

$$\left(E - Z \frac{\partial E}{\partial n} \right) \Bigg|_{r=R_0} = 0 , \quad Z = -p \frac{K_0(pR_0)}{K_1(pR_0)} . \tag{7}$$

Next question is the calculation of the sum and integral in the formulas (3), (4) for the reflection coefficient. The sum is calculated upon the radiating modes of continuous spectrum. The index m is changed from 0 to ∞. Integrating is performed upon the transverse wavenumber κ from 0 to ∞. In practice for the high order radiating modes the integral is close to zero and its value decreases with increasing of the index m. Therefore we can bound the sum by some value $m_{\max}$

$$\sum_m^{\infty} \ldots \cong \sum_m^{m_{\max}} \ldots$$

The expression under integral is reducing rapidly with increasing of κ. Therefore for the practical calculations we can set some finite value $\kappa_{\max}$ as an up limit of the integral

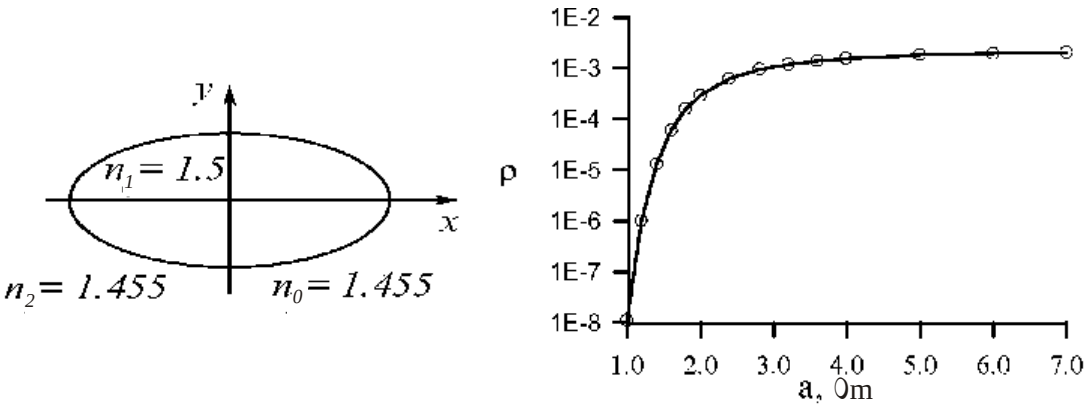
$$\int_0^{\infty} \ldots d\kappa \cong \int_0^{\kappa_{\max}} \ldots d\kappa \ .$$

The values $m_{\max}$ and $\kappa_{\max}$, sufficient to obtain desired precision, we determined from the numerical experiments.

4. NUMERICAL RESULTS

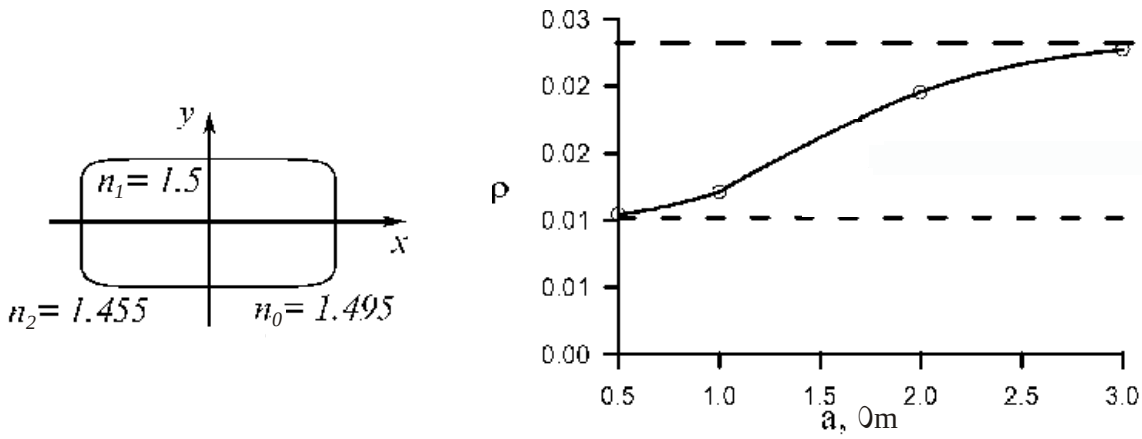
Consider some numerical results of the guided mode reflection coefficient calculation using our approach in scalar and vector problems.

1. The first case is the abrupt end of the dielectric waveguide with elliptical cross section (see Fig. 4 (a)). Refractive indeces of core $n_1 = 1.5$, envelope $n_2 = 1.455$, and right semi-space $n_0 = 1.455$ are close to each other so the weak guidance condition is valid and scalar formula (4) is applicable. Wavelength is $\lambda = 1.3\,\mu$m, and axis ratio is $a/b = 2$. The reflection coefficient versus the size of large semi-axis a of the ellipse is presented in Fig. 4 (b). Solid line is constructed using finite element method. Circles correspond to the results obtained by integral equation method [3]. Due to the application of the impedance boundary condition at the outer boundary of the computational domain the finite element method provides high accuracy for all values of the parameter a.



(a) (b)

Fig. 4. Abrupt end of the dielectric waveguide with elliptical cross section.(a) geometry of the waveguide cross section, (b) reflection coefficient versus large semi-axis



(a) (b)

Fig. 5. Abrupt end of the dielectric waveguide with super ellipse cross section.

(a) geometry of the waveguide cross section, (b) reflection coefficient versus large semi-axis

2. The second case is the abrupt end of the dielectric waveguide with the super ellipse shape (see Fig. 5 (a))

$$(x/a)^{2N} + (y/b)^{2N} = 1, \quad a/b = 2, \quad N = 4, \quad \lambda = 1.3\,\mu\text{m}.$$

Refractive indeces are $n_1 = 1.5$, $n_2 = 1.455$, $n_0 = 1.495$. This problem also was solved in scalar formulation. The results obtained by the finite element method (solid line) and the integral equation method (circles) are presented in Fig. 5 (b).

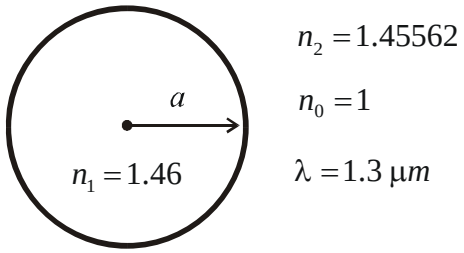


Fig. 6. Abrupt end of the dielectric waveguide with circular cross section.

3. The third case was the test problem for the vector modification of the method. Consider the abrupt end of the dielectric waveguide with the circular cross section (Fig. 6). Refractive indeces of the core $n_1 = 1.46$ and envelope $n_2 = 1.45562$ are close to each other, but the refractive index of semi-space $n_0 = 1$. Wavelength is $\lambda = 1.3\,\mu$m. The results obtained by the finite element method in vector formulation and results, calculated using formula (3) with substituting of the analytical expressions for the eigenmode field in it are presented in the Table 1. Relative error in the last column of the Table 1 was calculated according to the formula

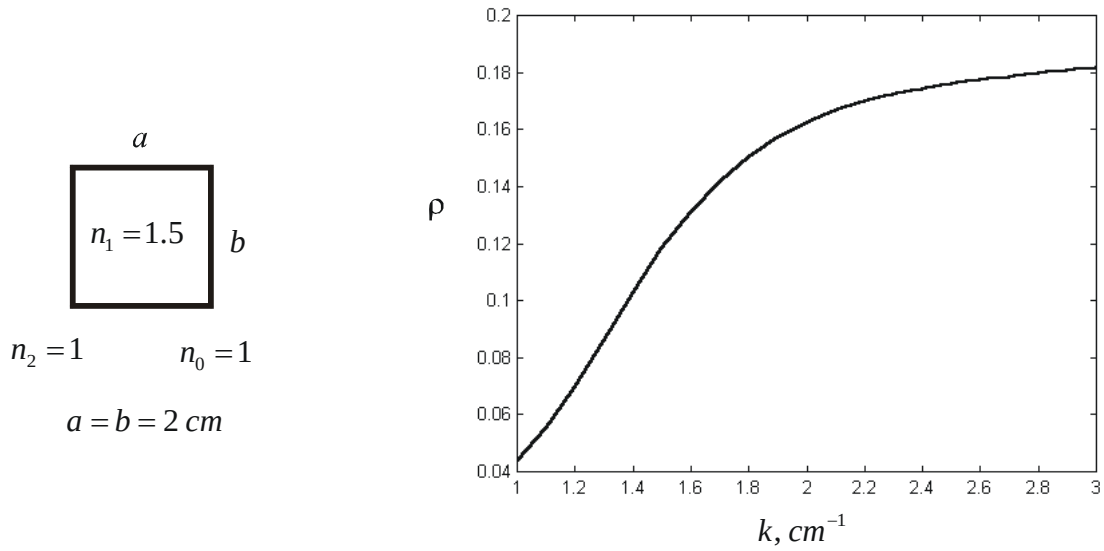
$$\delta = (\rho(FEM) - \rho(analytical)) / \rho(analytical) \cdot 100\%.$$

a, μm	ρ (FEM)	ρ (analytical)	δ, %
3.0	0.18740	0.18710	0.1403
3.5	0.18709	0.18720	-0.0555
4.0	0.18716	0.18725	-0.0459
4.5	0.18731	0.18727	0.0213
5.0	0.18726	0.18727	-0.0042
6.0	0.18741	0.18725	0.0871

7.0	0.18742	0.18723	0.1036
8.0	0.18741	0.18721	0.1047

Table 1

4. The forth case is the abrupt end of the dielectric waveguide with rectangular cross section and significantly differed refractive indeces of core $n_1 = 1.5$ and envelope $n_2 = 1$ (see Fig. 7 (a)). Refractive index of semi-space is $n_0 = 1$, and the sizes of the waveguide are $a = b = 2$ cm. This problem cannot be solved in scalar formulation, so we should use vector formula (3) and vector modification of the finite element method. The reflection coefficient versus the wavenumber $k = 2\pi / \lambda$ is presented in Fig. 7 (b).



(a) (b)

Fig. 7. Abrupt end of the dielectric waveguide with rectangular cross section.

(a) geometry of the waveguide cross section, (b) reflection coefficient versus wavenumber

5. CONCLUSION

We have proposed a new method for the calculation of scattering parameters of the guided mode on the facet of a dielectric waveguide with arbitrary shape of its cross section. The method is based on the combination of the variational formula and two dimensional finite element method. There are two variants of the method: for vector cases and for scalar cases.

Scalar modification of the algorithm works only under conditions $|n_1 - n_2| << n_1$, $|n_1 - n_0| << n_1$. The method allows to compute the waveguides even near cut-off frequency due to the application of the impedance boundary condition.

Vector variant of the algorithm works with arbitrary values of the refractive indeces of core and envelope of the waveguide andof the semi-space after the facet.

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REFERENCES

[1] A. B. Manenkov “Accuracy of approximation for fibre discontinuity analysis”, *Optical and Quantum Electron.*, vol. 23, no. 1, pp. 81-90, 1991.

[2] A. B. Manenkov, A. G. Rozhnev “Optical dielectric waveguide analysis, based on the modified finite element and integral equation method”, *Optical and Quantum Electron.*, vol. 30, no. 1, pp. 61-70, 1998.

[3] A. B. Manenkov “Step discontinuities in dielectric waveguides (fibres)”, *Optical and Quantum Electron.*, vol. 22, no. 1, pp. 65-76, 1990.