THEORETICAL ANALYSIS OF THE MAGNETIZATION BEHAVIOR AS A FUNCTION OF THE APPLIED MAGNETIC FIELD IN ONE DOMAIN THIN FILM SAMPLES

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Abstract

A method for determining of the magnetization direction in thin one-domain films with uniaxial anisotropy in the film plane has been introduced. It is based on the Stoner–Wohlfarth model and propounds mathematical procedure, leading to polynomial representation in the angle of magnetization. Bearing in mind that the analytical methods are rather cumbrous than handy, the proposed method could be useful with its good numerical correctness and its clear relation with the physical behavior of the system. Propositions about extensions of the method for more complicated cases are made.

1. INTRODUCTION

The reaction of magnetization in certain medium as function of the applied external magnetic field gives basic information either about the crystal and magnetic structure, or about the macro effects as a magnetoresistance, Hall effect and so on. In the thin films with one-domain structure the magnetization lies in the film plain and its behavior is described by various methods, based on Stoner–Wohlfarth model [1,2,3]. According this model the full energy E per unit volume is the sum of the anisotropy energy E_k, needed to divert the vector of magnetization $\mathbf{M}_s$ to some angle φ from the easy axis, and the energy of interaction E_H between the magnetization and the external field **H**.

$$E = E_k + E_H = K \sin^2 \varphi - \mathbf{M}_s \cdot \mathbf{H}, \qquad (1)$$

where K is the constant of the magnetic anisotropy, H and θ are the magnitude and the angle with respect to the easy axis of the external field. The vector of magnetization is supposed to have fixed size, depending on the material structure and corresponding to the anisotropy field $H_a = 2K/M_s$. Therefore the aim is to determine the angle φ between the magnetization vector and the easy axis. It could be found from the

conditions for local minimum of the full energy, i.e. for stable stationary position of the magnetization:

$$\frac{\partial E}{\partial \varphi} = 0 \tag{2}$$

$$\frac{\partial^2 E}{\partial \varphi^2} > 0 \tag{3}$$

It is convenient to express **H** in non–dimensional parameters **h** = **H**/H_a . Than Eq. (3) turns to [3,4]

$$h \sin\theta \cos\varphi - h \cos\theta \sin\varphi - \sin\varphi \cos\varphi = 0. \tag{4}$$

From Eq. (4) without difficulties the relation θ(φ) could be obtained, i.e. the angle of the external field could be found, under which the magnetization takes given direction. It comes out to be

$$\theta = \varphi + \arcsin\left(\frac{\sin 2\varphi}{2h}\right) \tag{5a}$$

$$\theta = \pi + \varphi - \arcsin\left(\frac{\sin 2\varphi}{2h}\right) \tag{5b}$$

These equations are solutions of the problem, which is opposite to the main goal – to find φ(θ). Doubling of the direction of magnetization for fields with $h < 1$ is well known physical effect. It follows from (2) and (3), because when $h < 1$ the total energy stands to have two local minima for θ varying over (0,π/2) [3,4,5]. For such files, the angular momentum of anisotropy, which trends the domain towards the easy axis, and the angular momentum of the dipole magnetic interaction, are compensated in stable stationary at two positions of φ. Further only fields, situated in I quadrant will be considered, because of the evident symmetry the situation possess. Due to the mentioned doubling, some supplementary condition $\theta_0 = \theta(\varphi_0)$ must be known for to point which of two possible values of magnetization is realized in regarded system, and so to have an ability to define inverse relation φ(θ).

2. THEORETICAL BACKGROUND

Algebraic treating of Eq. (4) leads to quatric equation in cos φ, with some additional restrictions. A general solution is possible [5], but it appears to be not only very troublesome to derive, but also, the obtained result would be not adapted for convenient numerical calculations. By reason of this φ(θ) has been expressed in a Taylor series about given (initial) point θ_0, at which related value φ_0 is known. As it was said above, such condition is also necessary for the system to be uniquely determined, i.e. to be acquainted whether (5a) or (5b) holds. It must be kept in mind, that there might be discontinuances in the relations (5), and extending around such meanings should be avoided. Physically this may be determined from history of the magnetization process [4]. Here it is assumed to be (5a), which also has been expanded in Taylor series. Thus, when both θ(φ) and φ(θ) are represented in local terms, the relation between their Taylor coefficients will be find. So

$$\varphi(\theta) = \varphi(\theta_0) + \varphi'(\theta_0)(\theta - \theta_0) + \frac{1}{2!}\varphi''(\theta_0)(\theta - \theta_0)^2 + \ldots$$
$$.. + \frac{1}{n!}\varphi^{(n)}(\theta_0)(\theta - \theta_0)^n + \ldots \tag{6}$$

The main idea is to express the Taylor coefficients of φ(θ) in terms of θ(φ), applying rules for differentiating of the composite functions. Representing φ(θ) as implicit $\varphi(\theta(\varphi)) = \varphi$ and starting with the trivial

$$\varphi' = \frac{1}{\theta'} \tag{7}$$

differentiating with respect to φ both sides, stage by stage it has been found

$$\varphi''\theta' = \frac{d}{d\varphi}\frac{1}{\theta'} = -\frac{\theta''}{\theta'^2}, \text{ or } \varphi'' = -\frac{\theta''}{\theta'^3} \tag{8}$$

$$\varphi'''\theta' = -\frac{d}{d\varphi}\frac{\theta''}{\theta'^3} = \frac{3\theta''^2 - \theta'''\theta'}{\theta'^4}, \text{ so it appears } \varphi''' = \frac{3\theta''^2 - \theta'''\theta'}{\theta'^5} \tag{9}$$

Here sign $(')$ to each parameter φ or θ, means derivative of it with respect to the other one. This way recurrently the *n*-th derivative could be obtained. In the adopted denotations

$$\varphi^{(n)}(\theta) = \frac{1}{\theta'}\frac{d}{d\varphi}\varphi^{(n-1)}(\theta(\varphi)) \tag{10}$$

Thus $\varphi^{(IV)}$ is found to be

$$\varphi^{(IV)} = \frac{10\,\theta'''\theta''\theta' - \theta^{(IV)}\theta'^2 - 15\,\theta''^3}{\theta'^7} \tag{11}$$

and so on. Now derivatives of θ with respect to φ should be found from (5a). It is ordinary explicit function, so consecutively differentiating gives

$$\theta' = 1 + \frac{2\cos 2\varphi}{\sqrt{4h^2 - \sin^2 2\varphi}}$$

$$\theta'' = \frac{4\left(1 - 4h^2\right)\sin 2\varphi}{\left(4h^2 - \sin^2 2\varphi\right)^{\frac{3}{2}}}$$

$$\theta''' = \frac{8\left(4h^2 - 1\right)\left(\cos 4\varphi - 4h^2 - 1\right)\cos 2\varphi}{\left(4h^2 - \sin^2 2\varphi\right)^{\frac{5}{2}}} \tag{12}$$

$$\theta^{(IV)} = \frac{8\left(32h^4 - 32h^2 - 3 - \left(40h^2 - 2\right)\cos(4\varphi) + \cos(8\varphi)\right)\left(4h^2 - 1\right)\sin(2\varphi)}{\left(4h^2 - \sin^2(2\varphi)\right)^{\frac{7}{2}}}$$

and so on. It is important, that here it is taken an interest meanings of this expressions only at point φ_0, which corresponds to center θ_0 of series (6). Thus, knowing the direction of the magnetization for given direction of the field, to describe the magnetization behavior when the field varies round, get able. In resumption, the steps to be followed are: calculating from the relations (12) the derivatives $\theta'...\theta^{(n)}$ in θ_0, replacing them in (8), (9), (10)...for to find φ', φ''... $\varphi^{(n)}$ and then to put them as coefficients in power series (6).

We applied the method to estimate the magnetization angle φ as a function of the angle θ of the magnetic field near the zero, when at the initial physical state the magnetization and the magnetic field are disposed in parallel. This is the condition,

which points out that from relations (5a) and (5a) must be chosen (5a), because it gives $\theta = 0$ in $\varphi = 0$. If the field and the magnetization were antiparallel at the initial state, we had to choose (5b) [4]. Computing $\theta^{(n)}$ from formulae (12) at $\varphi = 0$, we obtained

$$\theta'(0) = 1 + \frac{1}{h}, \text{ further } \theta''(0) = 0 \qquad \theta'''(0) = 0 \qquad \theta^{(IV)} = 0 \tag{13}$$

up to $\theta^{(V)}(0) \neq 0$, which is

$$\theta^{(V)}(0) = \frac{9 - 40h^2 + 16h^4}{h^5}, \tag{14}$$

and $\theta^{(VI)}(0) = 0$ again. Now we could compute $\varphi^{(n)}$ from (8), (9)...(11)

$$\begin{gathered} \varphi' = \frac{h}{h+1} \\ \varphi'' = 0 \quad \varphi''' = 0 \quad \varphi^{(IV)} = 0 \\ \varphi^{(V)} = -\frac{h\left(9 - 40h^2 + 16h^4\right)}{(h+1)^6} \\ \varphi^{(VI)} = 0 \end{gathered} \tag{15}$$

With a precision of sixth order the angle of magnetization φ depends on the angle of the external field θ as

$$\varphi = \frac{h}{h+1}\theta - \frac{h\left(9 - 40h^2 + 16h^4\right)}{120(h+1)^6}\theta^5 + 0 + \ldots \tag{16}$$

This formula gives a possibility to observe the behavior of the magnetization when the external field rotates. We were interested to trace back how the nonlinear term depends on the relative value h of the magnetic field. Fig.1 gives the relative weight of the coefficients in two terms in (16) for $0 \leq h \leq 4$ (which are common physical values). Fig.1a presents the behavior of the linear term and fig.1b – for the fifth order term. It was found that the contribution of the fifth order term may be neglect in comparison with the linear term. Thus for a magnetic field, applied on the small angle about the easy axis, the angle of magnetization depends on the angle of the field near linearly.

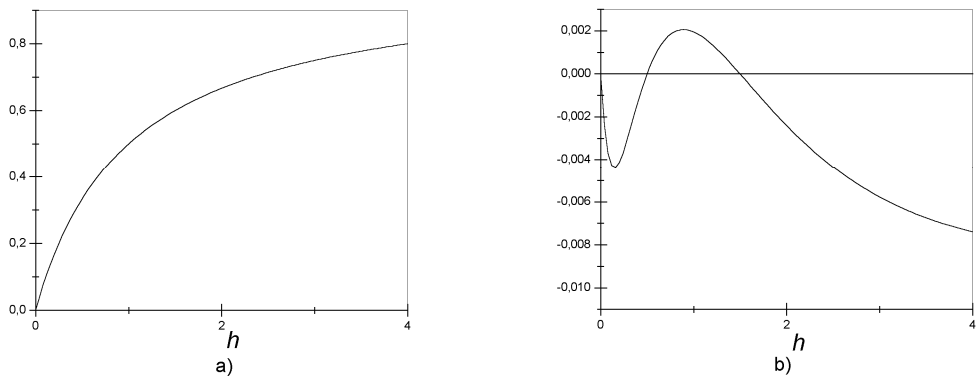

Figure 1. The relative weights of the coefficients for a)linear terms and b) fifth order term in (16).

For $h < 1$, or $\theta < 57^{o}$, the contribution of the terms with higher orders is very small, taking into account of the coefficient $1/n!$. For the relatively wide interval of θ the formula (16) determines the magnetization direction with a good precision if the magnetization and the applied field are collinear in the initial state.

Such technique could be used in large area of physical topics for to find inverse relations in local measures, when it is difficult to express them explicitly. The attention should be paid to the intervals of varying and the power, to which the Taylor series is truncated, for it to match out the original function closely enough.

3. CONCLUSIONS

One domain model plays a basic part in the description of the magnetic behavior in more complicated crystal structures. For the massive magnetic bodies the magnetization position is determined from two angles but to find them on use the same relations (2) and (3). The appearance of new physical objects (domain wall, oblique anisotropy etc.) modifies the energy of the anisotropy and the new components are present at a superposition with the components of the one domain model. One might say that the details of the complicated models are built on the base of the one domain model. Therefore to find the dependence of the magnetization as a function of the applied magnetic field is the basic problem in the magnetic investigations. The general methods are very complicated and abstract [5]. The essential priority of the obtained

formulae is the possibility to compute (with determined accuracy) the angle of magnetization when the applied field rotates and in the other hand they give a possibility to trace out the qualitative reaction of the system at the defined field parameters.

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