CHAOTIC OSCILLATIONS IN FERRORESONANCE CIRCUITS

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Abstract

The paper presents a continuous wavelet analysis as an approach to the understanding of solutions the nonlinear equations of Duffing type. The wavelet transformation has been used for local analysis of ferroresonance system's behavior. Some of Wavelet Toolbox Matlab properties have been demonstrated.

1. INTRODUCTION

Ferroresonance is an interesting subject that many engineers find difficult to analyze. The results are sometimes unpredictable and certainly arduous to visualize. Recently there has been a renewal of the interest for this problem both from theoretical and experimental point of view.

One of the systems that generates chaotic oscillations is a dynamic system described with a Duffing equation [2, 10]. Examples of that system can be observed in the power electronics. Such are the nonlinear phenomena, due to the ferromagnetic properties of iron core choke (transformer) and radio – electronics these are radio – electronic systems such as ferro – electric resonators, ferromagnetic circuits, etc.

2. A FERRORESONANCE SYSTEM GENERATING CHAOTIC OSCILLATIONS

One of the numerous existing systems generating chaotic oscillations considered here is presents a series RLC circuit with nonlinear inductance scheme to which harmonic voltage E (t) has been applied (see Fig. 1). The inductive voltage can be found from the following equation:

$$U_L(t) + U_C(t) + U_R(t) = E(t) \tag{1}$$

where :

$E(t) = E\sin(\omega t)$ - the outer source voltage,

$U_L(t) = \dfrac{d\psi(t)}{dt}$ - voltage of inductance L,

$U_C(t) = \dfrac{1}{C}\int_0^T i(t)dt + U_C(0)$ - voltage of capacitor C.

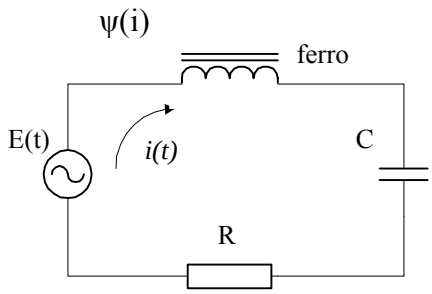


Figure 1. Simple series RLC circuit for explaining ferroresonance.

Leakage inductance L_S and active losses are neglected (1). The magnetic flow $\psi(t)$ can be given by the following cubic approximation :

$$\begin{aligned} \psi(t) &= L(t).i(t) \\ i(t) &= c_1\psi(t) + c_0\psi^3(t) \end{aligned} \tag{2}$$

A detailed description of ferroresonance is shown in [9]. Inductance voltage $v = U_L(t)$, as shown in Figure 1 is

$$v = \dot{\psi}(t) = E - U_C(t) - RI(t) \cong E - U_C(t) \ . \tag{3}$$

It is obvious that the analytic expression is more complex:

$$v(t) = \dot{\psi}(t) = (c_1 E + \frac{3}{4}c_0 E^3)\omega\cos\omega t - \frac{3}{4}c_0 E^3 \omega\cos 3\omega t \tag{4}$$

The equation that described flux in a series RLC circuit with nonlinear inductance can be written as follow:

$$\ddot{\psi}(t) + 2\delta\dot{\psi}(t) + \omega_0\psi(1 + \alpha\psi^2) = E\cos 3\omega t \tag{5}$$

where :

ω_0 – resonance system's frequency ($\psi \to 0$),

α – coefficient of nonlinearity,

E – outer source amplitude,

ω – outer source frequency.

Fig. 2 below shows a diagram of the process of ferroresonance chaotic oscillations occurrence. If we increase the voltage slightly (see Fig. 2), the capacitor line will move upwards, eliminating the solution at intersection point 1. The solution would then jump to the third quadrant. The resulting current will be so strong that it will make the voltage drop again and the solution point will start to fluctuate between points 1 and 3. Actually, phenomena like this can be observed when there is ferroresonance. The voltage and current appear to vary randomly and unpredictably [9].

More detailed theoretical descriptions can be found in a number of reference books [2], [9].

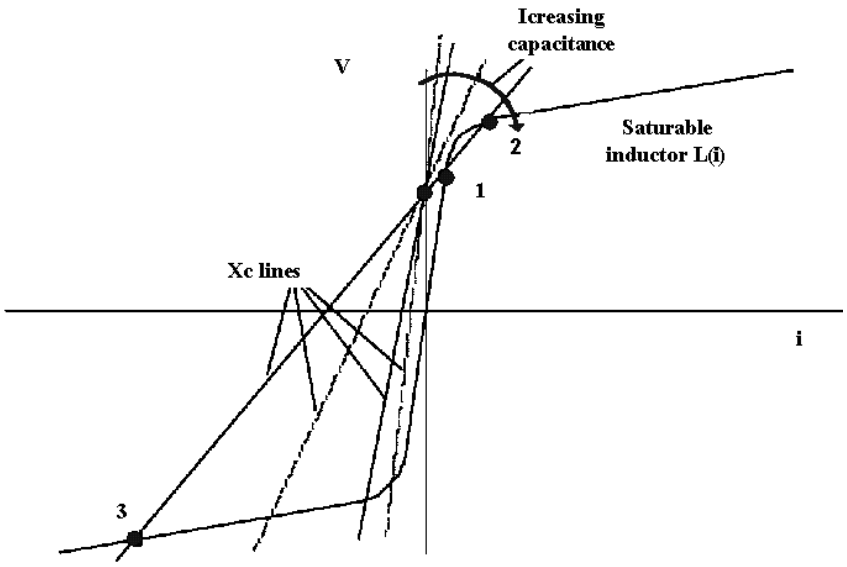


Figure 2 Graphic presentation of ferroresonance

3. DUFFING'S EQUATION

In 1918 G. Duffing described some fundamental results of his research of nonlinear second order equations (with nonlinear restoration force) in his paper [6]. A table, summarizing the above mentioned results concerning autonomous and forced oscillations is shown in [2], p. 121.

It is well – known that an analytical solution of Duffing's equation (6) does not exist [2], [3].

$$\ddot{x} + \delta \dot{x} + x^3 - x = \gamma \sin \omega t \quad , \tag{6}$$

where δ, γ, ω - const.

An alternative expression of the equation is presented by the following system of equations (a nonlinear oscillator):

$$\left| \begin{array}{l} \dot{x}_1 = \delta x_2 \\ \dot{x}_2 = -x_2^{\,3} + x_2 - \gamma \sin(\varpi t) \end{array} \right. \quad . \tag{7}$$

The following characteristics of nonlinear fluxional equations are known:

- excitation of high harmonics (angular frequencies $2\omega, 3\omega, \ldots$),
- invalidity of the superposition rule,
- appearance of different solutions depending on the initial conditions.

Different asymptotic methods can be used to study the nonlinear system behavior, e. g. the small parameter method, the harmonic balance method, etc.

3. SIMULATION OF DUFFING'S EQUATION

In this section a nonlinear oscillator has been simulated in Simulink, Matlab.

3.1. Duffing equation simulation (one harmonic)

The nonlinear nonautonomous system, described by equation (6) was simulated to produce a chaotic oscillation array, using a standard Simulink blocks

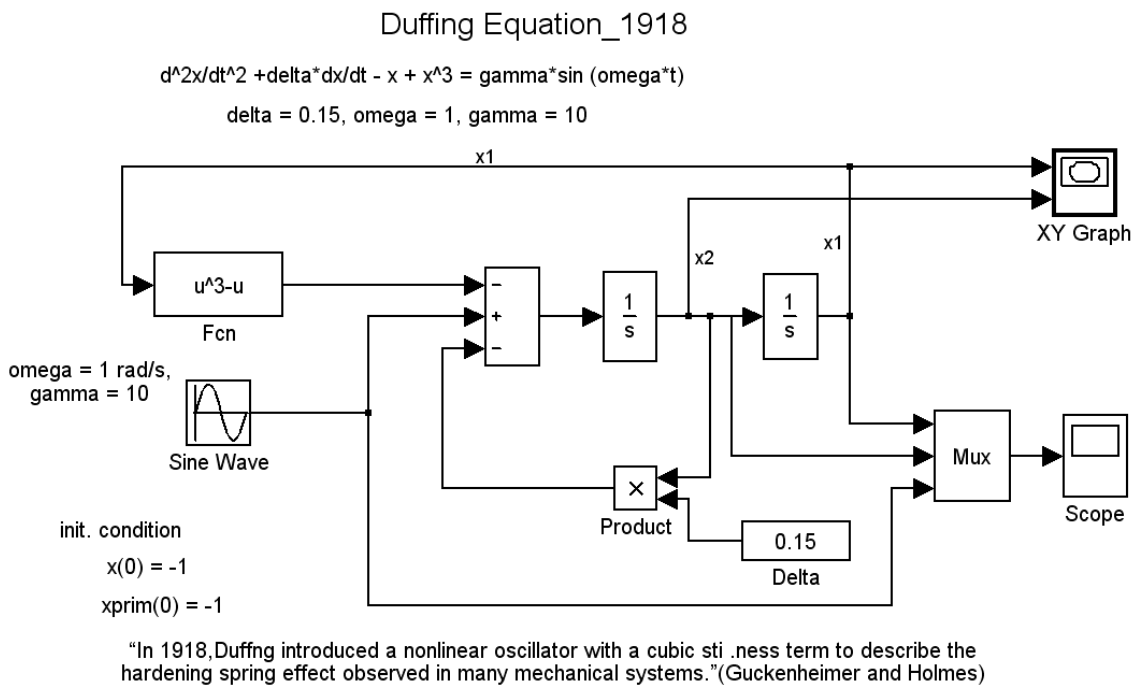


Figure 3**.** The block – diagram of the Duffing oscillator as it modeled in Simulink, Matlab.

Fig. 4, 5 represent calculated chaotic oscillations from a Duffing equation simulation and phase portraits for concrete initial conditions ($x(0)=-1, \dot{x}(0)=-1$). Simulation time is set to: t = 100 (Fig. 5 a) and t = 10000 (Fig. 5 b), respectively. The integration method has a variable step fourth order Runge – Kutta, Dormand – Prince ($RK_{[4,5]}$).

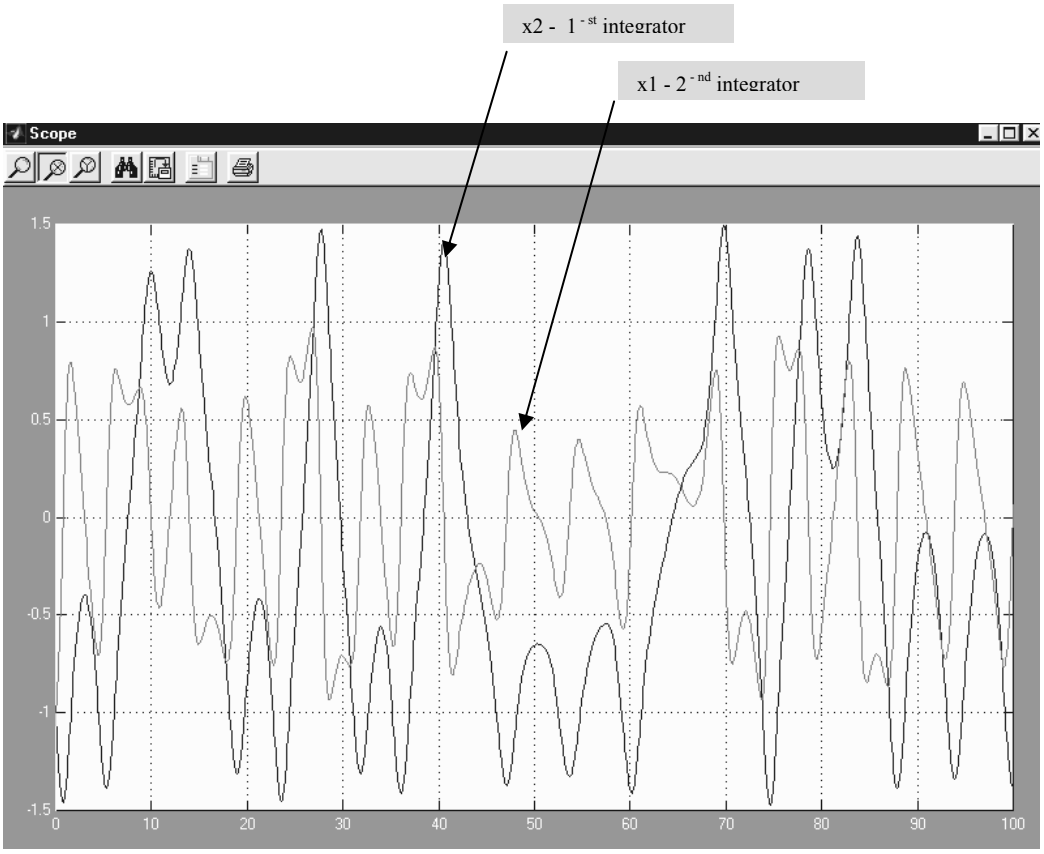


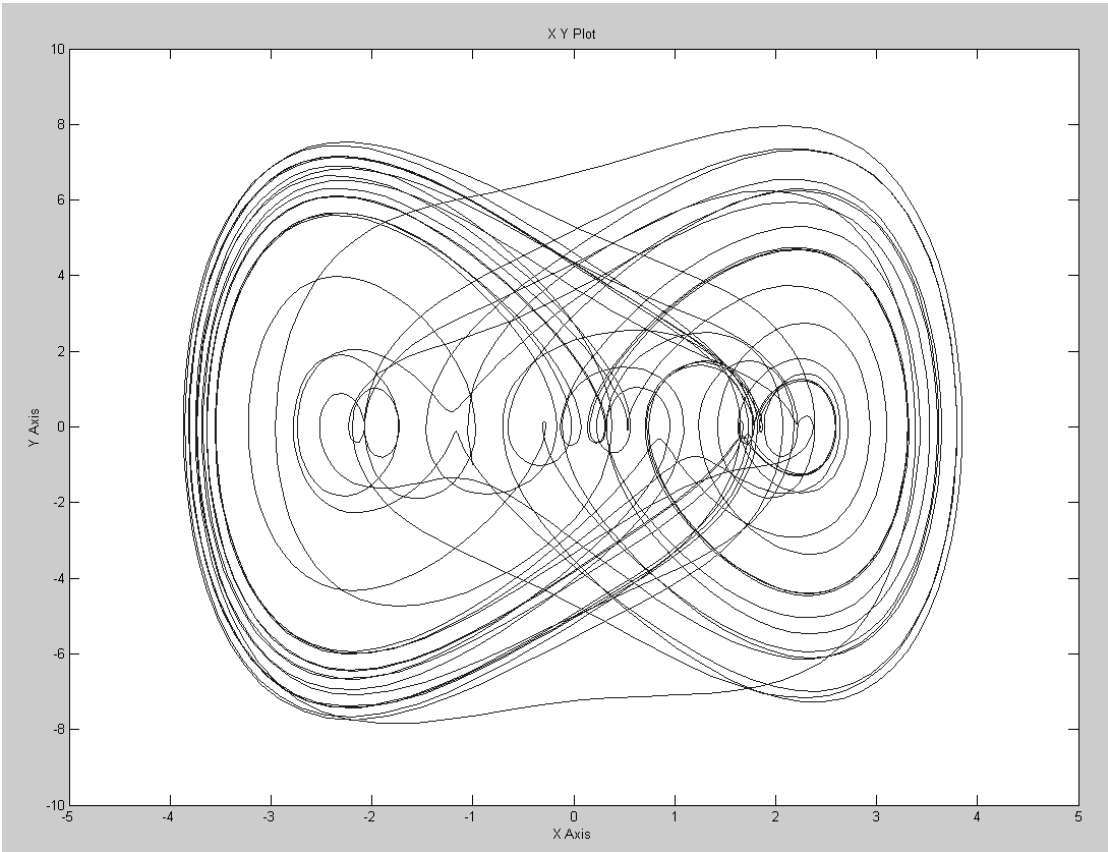
Figure 4. Calculated chaotic oscillations of Duffing equation simulation

3.2. Duffing – like equation simulation (polyharmonic)

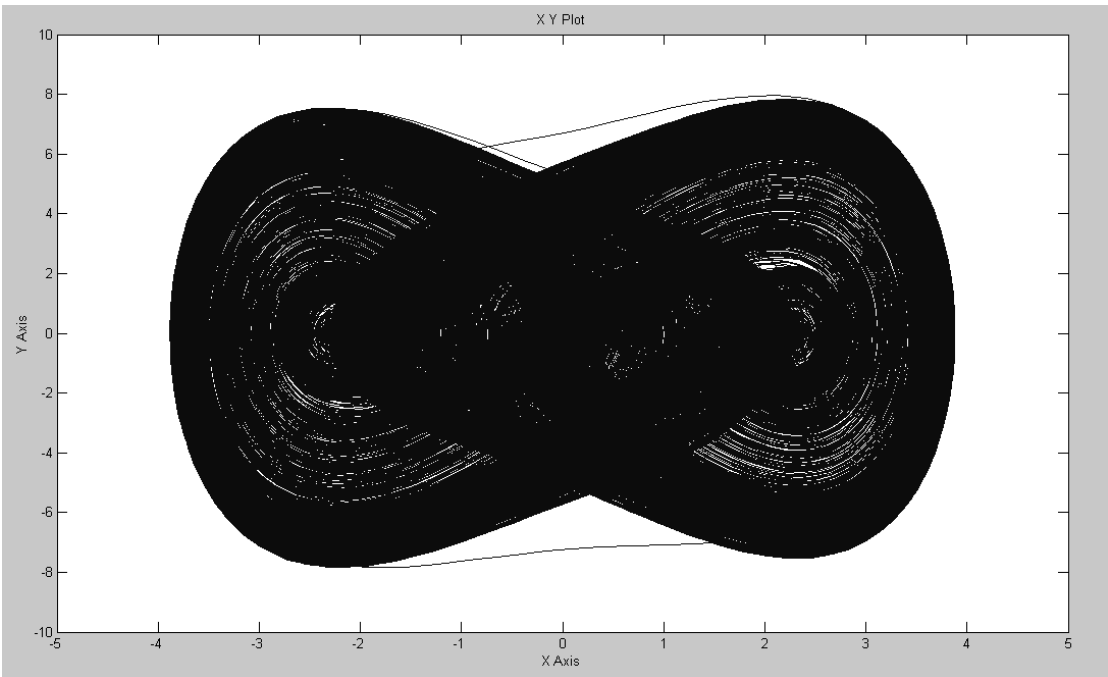
The nonlinear system with polyharmonic forced function can be described by the following equation [2]:

$$\ddot{x}+\delta\,\dot{x}+x^3-x=\gamma_1\cos\omega_1 t+\gamma_2\cos\omega_2 t. \tag{8}$$

We assume that $\omega_1=1\,rad/\sec, \omega_2=\sqrt{2}\ rad/\sec$. In Fig. 6. the block – diagram of the Duffing – like oscillator model for concrete initial conditions is presented ($x(0)=-1, \dot{x}(0)=-1$). The integration method is variable step is $RK_{(4,5)}$ i.e.“ode45” in Matlab.



a)



b)

Figure 5. Phase portraits of the simulated system for initial conditions, $x(0)=-1, \dot{x}(0)=-1$ (a) simulation time t = 100, (b) t = 10000. Integration method: $RK_{(4,5)}$ i.e.ode45 in Simulink.

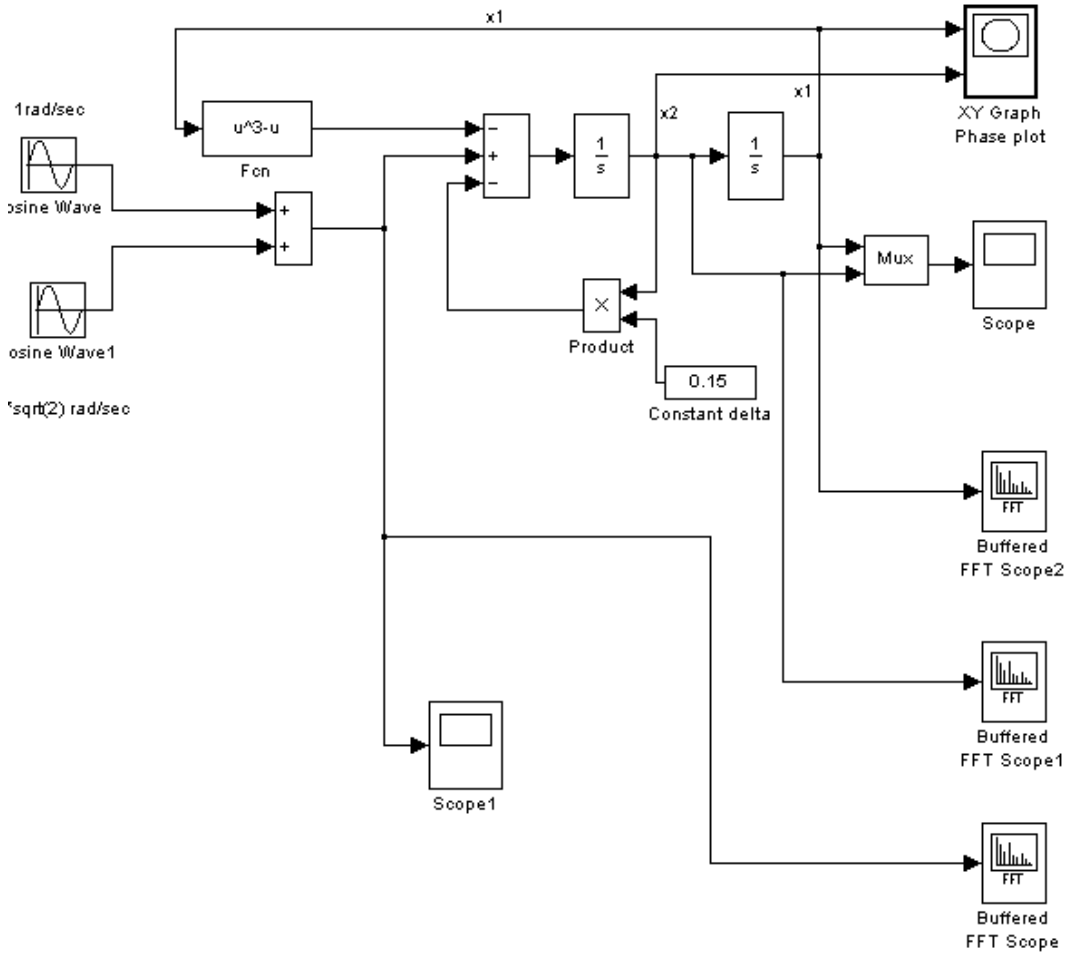


Figure 6. The block – diagram of the Duffing – like oscillator

Calculated waveforms and phase portrait of this polyharmonic force model are presented in Fig. 7. (See boxes labeled Scope XY Graph Phase plot in Fig. 6.).

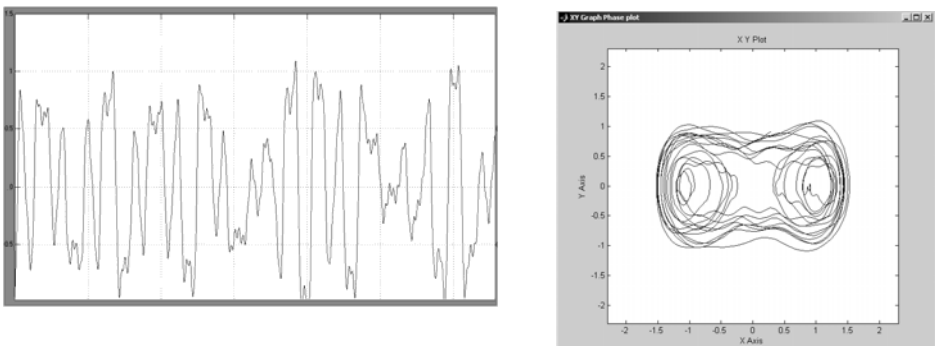


Figure 7. Waveforms (Scope) and phase portrait (XY Graph) of Duffing – like equation simulation.

Fig. 8 illustrates the Fast Fourier Transform (FFT) spectra with Hanning window, the point of scope 1 and signal $x1$ in model Fig. 6. As seen from Fig. 8 b) two harmonics were transformed in chaotic peaks.

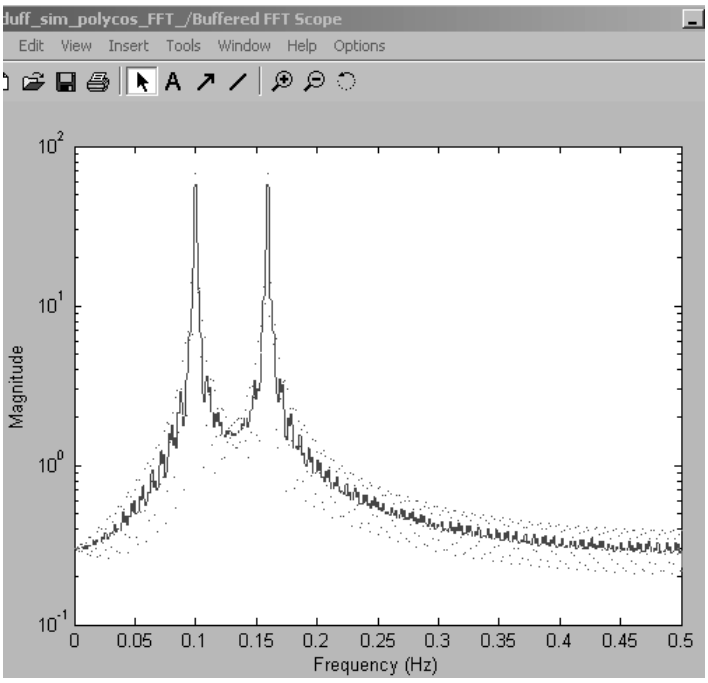


Figure 8 a). FFT spectra of forced signal from Duffing – like equation simulation – "Scope1"

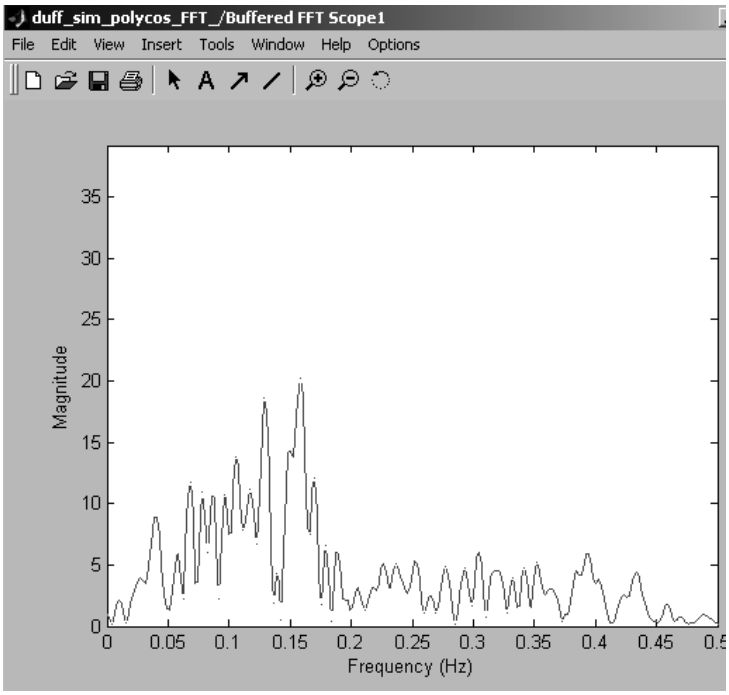


Figure 8 b). FFT spectra of signal x1 from Duffing – like equation simulation, "buffered FFT scope 1"

4. WAVELET ANALYSIS

Fundamentally, wavelets are a new type of functions which provide an excellent orthonormal basis for functions of one or more variables. They form a local basis and can represent any square – integral functions ($\forall\ f \in L^2(R)$) in a locally finite manner.

Some of the basic concepts of the wavelets theory used in this paper are listed below [1], [4], [6]:

· Each wavelet system is generated from a scaling function and a wavelet function by rescaling and translations, and the wavelet system is an orthonormal basis.

· Original time series can be reconstructed exactly from the averaged time series and the coefficients, generated by an orthogonal (orthonormal) wavelet transformations.

· Each element in a given system has compact support in the time and frequency domain.

· There are fast algorithms for computing the coefficients of wavelet transformations.

4.1. Continuous wavelet transform (CWT)

The spaces $L^2(R)$ consist of continuous wavelets. Here values of parameters s - scale of transformation and τ - shift state are arbitrary real numbers:

$$\psi(\tau,s) = |s|^{-\frac{1}{2}}\psi\left(\frac{t-\tau}{s}\right), \tag{9}$$

where $\tau, s \in R$, $\psi \in L^2(R)$.

Taking into account this expression, the continuous wavelet transform (CWT) can be written as follows [6]:

$$CWT_x^{\psi}(\tau,s) = |s|^{-\frac{1}{2}}\int x(t)\psi^*\left(\frac{t-\tau}{s}\right)dt \tag{10}$$

This transformation shows how a function of the x(t) type can be expanded into its components combined with basic functions, named wavelets. Variables s and τ from formula (9), are the scale and shift respectively and they may be connected with Short – Time Fourier Transformation (STFT) parameters. For additional information, concerning this analysis method see references [1], [3], [7].

Analogically to the Fourier transformation, expansion coefficients $c_{j,k} = \langle x, \psi_{j,k} \rangle$ of the function $x(t)$ in the wavelet series, have been determined with integral wavelet transform:

$$c_{j,k} = CWT_x^{\psi}\left(\frac{1}{2^j}, \frac{k}{2^j}\right) \tag{11}$$

One of the typical examples of orthogonal wavelets is the Haar function [6]. When analyzing the structure of the signals, it is convenient to represent it as a CWT [7], and to use scalograms together with local maxima.

4.2. Advantages and disadvantages of the wavelet analysis

There are different variants of algorithms for wavelet analysis, including Haar wavelets [8], Daubechies wavelets [4], Morlet wavelets [7], Gauss wavelets[7], etc. The last two algorithms have the advantage of possessing better solution options for relatively proportionally changing temporary rows. Of all the above mentioned algorithms Haar wavelets require comparatively few calculations in respect to Morlet and Gauss algorithms.

Basis scalograms developed through complex wavelets and particularly Gauss wavelets, are especially expressive. They represent the transformation amplitude:

$$\sqrt{\mathrm{Re}[CWT(\tau,s)]^2 + \mathrm{Im}[CWT(\tau,s)]^2} \quad . \tag{12}$$

Since Gauss (Morlet) wavelets are products of natural and imaginary exponents, the resulting scales can be represented by whole numbers as well as by fractions.

A conclusion can be made that the wavelet analysis locates excellently the scale variations of the amplitude of the signal.

4.3. CWT analysis of numerical solution of Duffing equation

With the help of the Simulink model, two vectors have been determined - ‘cosout’ and ‘duffout’ - standing for the output signal ‘sine wave’, and for the second integrator output signal (x2), respectively (see Fig. 3 and the Matlab Toolbox commands). In fact, ‘duffout’ is the numerical expression of the equation solution (6).

Scalograms and local maxima coefficients of the above mentioned vectors have been determined as Step 2, using the continuous wavelet transformation at Matlab Wavelet Toolbox. Figures 9 and 10 show the resulting skeletons of local maxima for ‘cosout’ and ‘duffout’ respectively. The local characteristics of the ’duffout’ signal can be clearly seen.

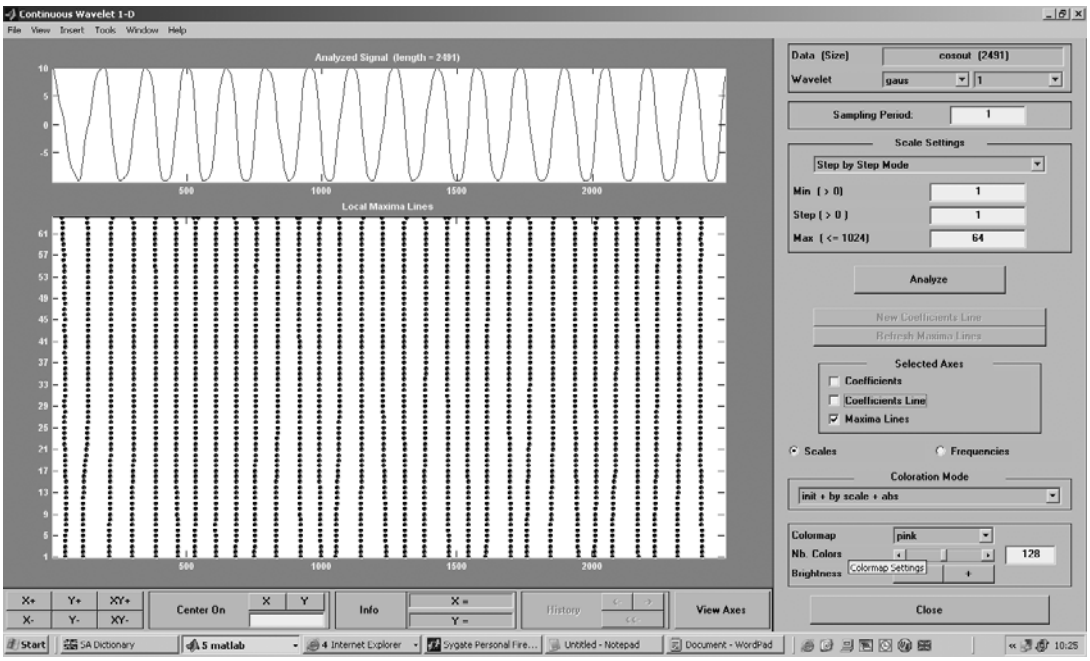


Figure 9**.** Local maximum coefficients (Gauss basis) for ‘cosout’ array.

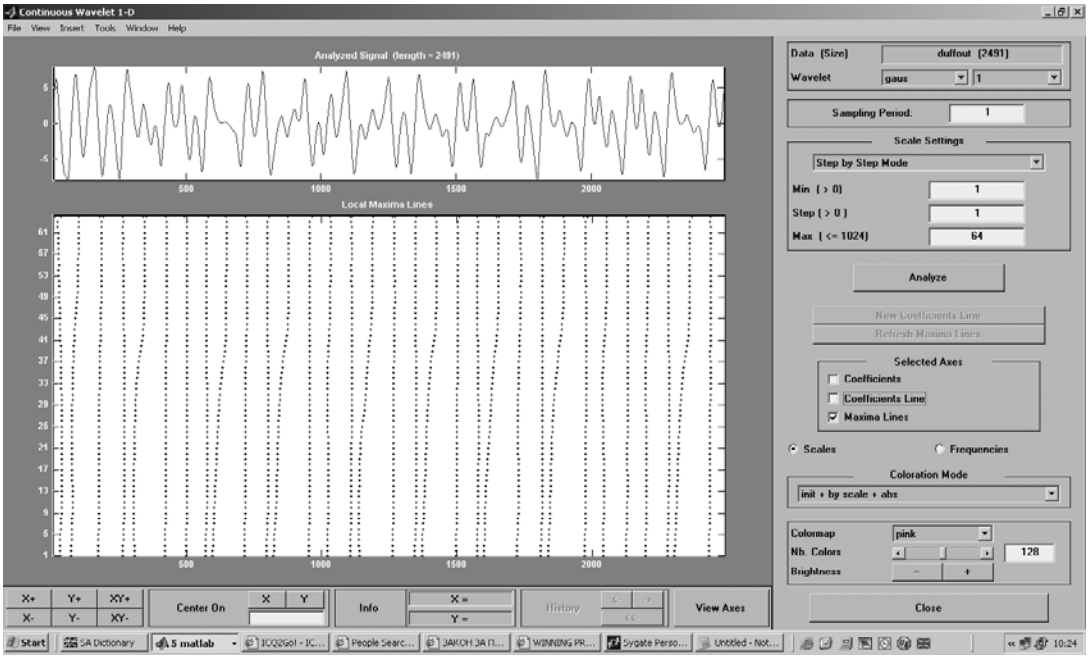


Figure 10. Local maximum coefficients (Gauss basis) for array ‘duffout’.

Analogically, for the case of the polyharmonic force model (see Fig. 6) the vectors ‘cosout_p’ and ‘ss4’ have been saved - standing for the output signal ‘sine wave’, and for the second integrator output signal (x2), respectively. Figures 11 and 12 show the resulting scalogram and skeletons of local maxima for the Duffing – like oscillator (polyharmonic).

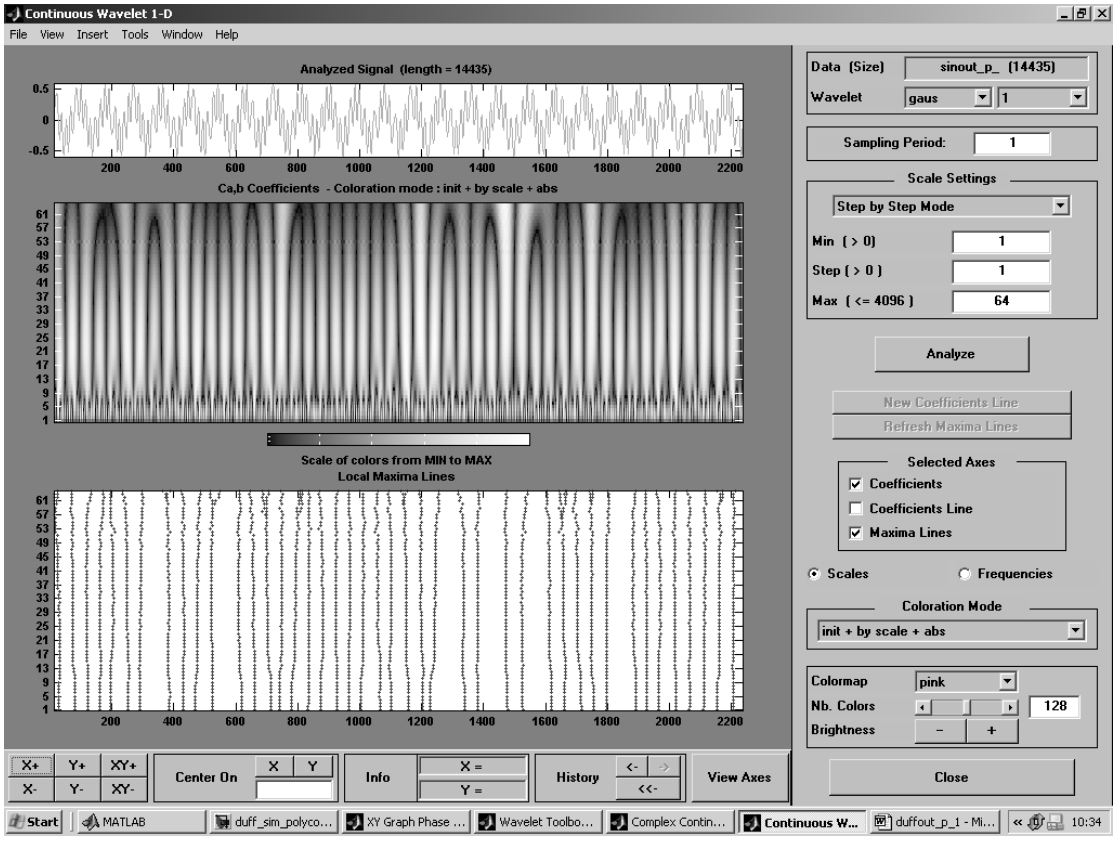


Figure 11. Local maximum coefficients (Gauss basis) for ‘cosout_p’ array.

As it seen from Figure 12 the resulting scalogram present subharmonic’s local behavior .

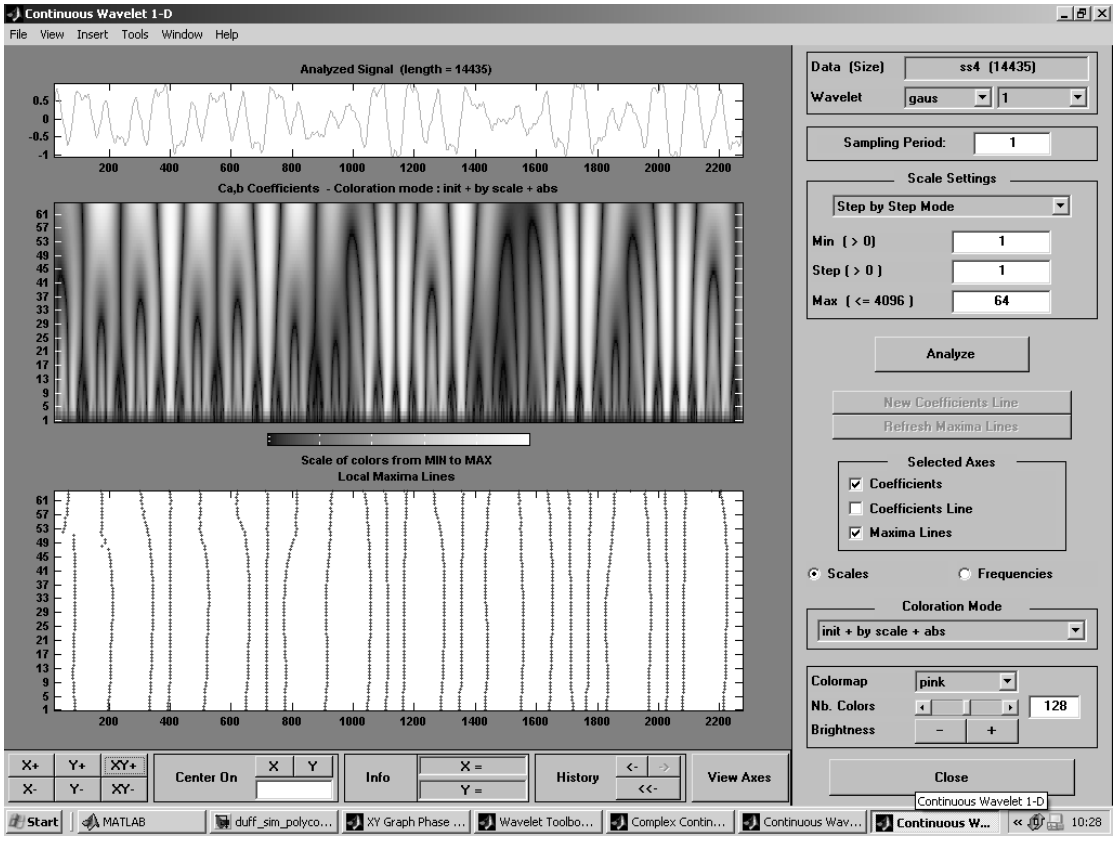


Figure 12. Local maximum coefficients (Gauss basis) for array ‘ss4’.

5. CONCLUSIONS

The numerical experiments prove the potentialities of wavelet analysis to reveal the local peculiarities of chaotic oscillations in singularity points, which create problems for the traditional harmonic analysis.

As a result of the numerical experiments carried out, a conclusion is drawn that simulation of nonlinear equations initiating chaotic oscillations can be analyzed successfully through the various methods of wavelet analysis using a Wavelet Toolbox and Simulink from the Matlab package.

Wavelet techniques are suitable for studying periodic and chaotic behavior of highly nonlinear dynamic system models.

They facilitate the study of the process of transition of the oscillation system from regular to chaotic oscillations in order to determine the zones for efficient work of the respective ferroresonance system.

In the future the focus will shift to the cases when the system represents a more complex forced function.

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