ELECTROMAGNETIC FIELDS GENERATED BY A DIPOLE ANTENNA IMPLANTED INSIDE BIOLOGICAL TISSUE

Christos Marmalidis*

*Hellenic Navy Research Office (GETEN)
Ministry of Defense – Greece

Abstract

In this article a canonical problem of radiation of an infinite dipole antenna placed inside a tissue medium is analyzed by using a semi-analytical approach. A Fourier analysis is pursued to solve analytically the boundary value while numerical integration algorithm are utilized to compute the electrical fields inside the tissue medium. Interesting results clarifying the power deposition mechanisms inside the tissue media are observed.

1. INTRODUCTION

Dipole type insulated antennas are being used in clinical hyperthermia. Several authors in the past have analyzed the radiation properties of insulated antennas, at various frequencies, irradiating tissue medium. R. W. P. King et al. [1], [2] have analyzed this phenomenon by using assumed current distributions, while other authors have used numerical methods [3], [4], [5]. In all these works the importance of the "source problem" have been emphasized. This problem is associated with the excessive absorption of electromagnetic radiation near the dipole antenna feeding point where there is a strong storing of reactive power. In order to analyze in detail this phenomenon in this article a canonical model of dipole element consisting of an infinite insulated line excited by a voltage gap generator is analyzed. In the following analysis an exp(+jωt) time dependence is assumed for the field quantities and is suppressed through the analysis.

2. DEFINITION OF THE PROBLEM

Consider a dipole antenna consisting of an infinite length cylindrical conductor lying between $+\infty, -\infty$. The antenna is fed at z=0 point, as shown in figure 1.

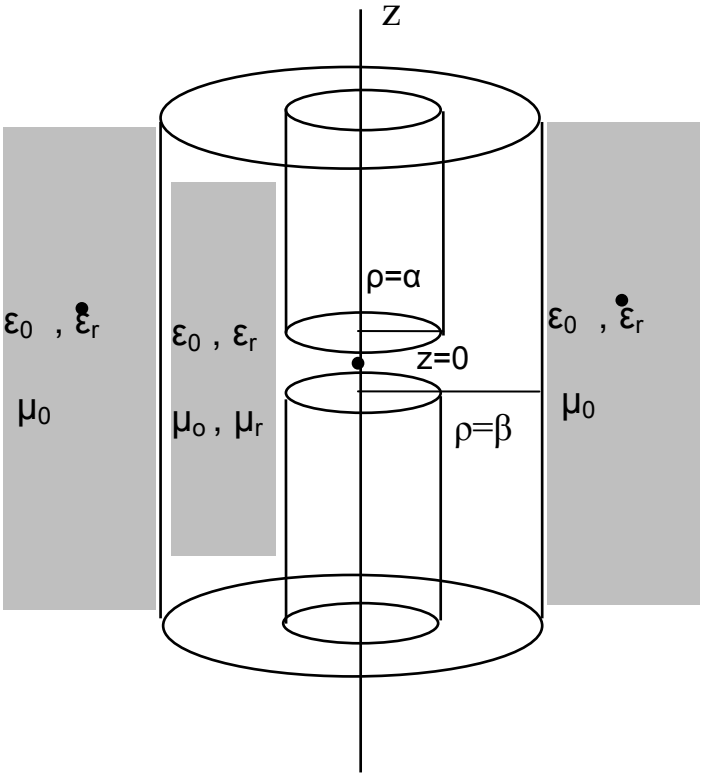

Figure 1. Dipole antenna

In order to describe the field problem, cylindrical co-ordinates ρ,φ,z, will be used, as dictated from the cylindrical symmetry. The z axis is chosen to coincide with the symmetry axis of the cylinder, having the effect the inside surface of the cylinder to be defined by the simple expression ρ=α. For α<ρ<b the antenna is surrounded with cylindrical shaped material having relative dielectric permittivity ε_r and relative magnetic permeability μ_r.

Finally, in the space ρ> b the relative dielectric permittivity is ε_r. The material in space α<ρ<b is able to change (e.g. by changing μ_r) in order to permit theoretical and experimental study of the changes in antenna's behavior and obtain results. The space for ρ> b simulated the human tissue and the relative dielectric permittivity in this space is complex. In the following analysis, the biological tissue is considered homogenous and isotropic as for its electrical properties.

3. ANALYSIS OF THE STRUCTURE

Considering that because of the existing symmetry $\frac{\partial}{\partial \phi} = 0$, the corresponding field equations for this problem are expressed in cylindrical co-ordinates in the form given below:

$$\frac{\partial e_z}{\partial \rho} - \frac{\partial e_\rho}{\partial z} = j\omega\mu h_\varphi \tag{1}$$

$$-\frac{\partial h_\varphi}{\partial z} = j\omega\varepsilon e_\rho \tag{2}$$

$$\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho h_\varphi\right) = j\omega\varepsilon e_z \tag{3}$$

Solving (3) for e_z, (2) for e_ρ and substituting in (1) :

$$\frac{1}{j\omega\varepsilon}\frac{\partial}{\partial \rho}\left(\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho h_\varphi\right)\right) + \frac{1}{j\omega\varepsilon}\frac{\partial}{\partial z}\left(\frac{\partial h_\varphi}{\partial z}\right) = j\omega\mu h_\varphi$$

$$\frac{\partial}{\partial \rho}\left(\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho h_\varphi\right)\right) + \frac{\partial^2 h_\varphi}{\partial z^2} + \omega^2\varepsilon\mu h_\varphi = 0$$

$$\frac{1}{\rho}\frac{\partial^2}{\partial \rho^2}\left(\rho h_\varphi\right) + \left(-\frac{1}{\rho^2}\right)\frac{\partial}{\partial \rho}\left(\rho h_\varphi\right) + \frac{\partial^2 h_\varphi}{\partial z^2} + \omega^2\varepsilon\mu h_\varphi = 0$$

$$\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(h_\varphi + \rho\frac{\partial h_\varphi}{\partial \rho}\right) - \frac{1}{\rho^2}\left(h_\varphi + \rho\frac{\partial h_\varphi}{\partial \rho}\right) + \frac{\partial^2 h_\varphi}{\partial z^2} + \omega^2\varepsilon\mu h_\varphi = 0$$

$$\frac{1}{\rho}\frac{\partial h_\varphi}{\partial \rho} + \frac{1}{\rho}\frac{\partial h_\varphi}{\partial \rho} + \frac{\rho}{\rho}\frac{\partial^2 h_\varphi}{\partial \rho^2} - \frac{h_\varphi}{\rho^2} - \frac{1}{\rho}\frac{\partial h_\varphi}{\partial \rho} + \frac{\partial^2 h_\varphi}{\partial z^2} + \omega^2\varepsilon\mu h_\varphi = 0$$

$$\frac{\partial^2 h_\varphi}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial h_\varphi}{\partial \rho} - \frac{h_\varphi}{\rho^2} + \frac{\partial^2 h_\varphi}{\partial z^2} + \omega^2\varepsilon\mu h_\varphi = 0 \tag{4}$$

using

$$h_{\varphi} = RZ \tag{5}$$

where R,Z are corresponding functions of ρ,z (4) looks like:

$$\frac{\partial^2}{\partial \rho^2}\left(RZ\right) + \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(RZ\right) - \frac{RZ}{\rho^2} + \frac{\partial^2}{\partial z^2}\left(RZ\right) + \omega^2 \varepsilon \mu RZ = 0$$

$$Z\frac{d^2 R}{d\rho^2} + \frac{1}{\rho} Z \frac{dR}{d\rho} - \frac{RZ}{\rho^2} + R\frac{d^2 Z}{dz^2} + \omega^2 \varepsilon \mu RZ = 0$$

$$\frac{Z}{ZR}\frac{d^2 R}{d\rho^2} + \frac{Z}{\rho ZR}\frac{dR}{d\rho} - \frac{RZ}{\rho^2 RZ} + \frac{R}{ZR}\frac{d^2 Z}{dz^2} + \frac{\omega^2 \varepsilon\mu RZ}{ZR} = 0$$

$$\frac{1}{R}\left(\frac{d^2 R}{d^2 \rho} + \frac{1}{\rho}\frac{d}{d}\frac{R}{\rho} - \frac{R}{\rho^2}\right) + \frac{1}{Z}\frac{d^2 Z}{dz^2} + \omega^2 \varepsilon \mu = 0 \tag{6}$$

Using

$$\frac{1}{Z}\frac{d^2 Z}{dz^2} = -k^2 \Rightarrow Z = e^{jkz} A \tag{7}$$

where A is an arbitrary constant (A=1 can be considered without impairing generality) then (6) with the help of (7) becomes:

$$\rho^2 \frac{d^2 R}{d\rho^2} + \rho^2 \frac{1}{\rho}\frac{d}{d}\frac{R}{\rho} - \frac{R}{\rho^2}\rho^2 + \rho^2\left(\omega^2 \varepsilon\mu - k^2\right) R = 0$$

$$\rho^2 R'' + \rho R' + \left(\left(\omega^2 \varepsilon \mu - k^2\right)\rho^2 - 1\right) R = 0$$

The general solution of the above D.E. is written as a combination of Bessel function J1 and Neumann function Y1.

$$R = A J_1\left(\sqrt{k_i^2 - k^2}\right) + B Y_1\left(\sqrt{k_i^2 - k^2}\right) \tag{8}$$

where

$$k_i = \omega\sqrt{\varepsilon\mu}$$

Taking into consideration (3),(5),(7),(8) Hφ, Ez are expressed as follows:

$$H_{\varphi} = \int_{-\infty}^{+\infty} dk\, e^{jkz} \left(A J_1\left(\sqrt{k_i^2 - k^2}\,\rho\right) + B Y_1\left(\sqrt{k_i^2 - k^2}\,\rho\right) \right) \tag{9}$$

$$E_z = \frac{1}{j\omega\varepsilon} \frac{1}{\rho} \frac{\partial}{\partial\rho}\left(\rho\right) H_{\varphi} \tag{10}$$

Because:

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho J_1\left(\sqrt{k_i^2 - k^2}\,\rho\right)\right) = \frac{1}{\rho}\rho J_1'\left(\sqrt{k_i^2 - k^2}\,\rho\right) + \frac{1}{\rho} J_1\left(\sqrt{k_i^2 - k^2}\,\rho\right) \tag{11}$$

$$= J_1'\left(\xi\right) + J_1\left(\xi\right)\frac{1}{\rho}$$

where $\xi = \left(\sqrt{k_i^2 - k^2}\rho\right)$ was substituted

Bessel and Neumann functions satisfy the recursive relationships [6]:

$$m = 1 \qquad J_0 - J_2 = 2 J_1' \tag{12}$$

$$J_0 + J_2 = \frac{2}{x} J_1$$

$$2 J_0 = 2\left(\frac{J_1}{x} + J_1'\right)$$

Similar relationships exist for Neumman functions. Based on (12), (11) can be expressed as:

$$\frac{J_1\left(\sqrt{k_i^2 - k^2}\,\rho\right)}{\rho} + \frac{d}{d\rho} J_1\left(\sqrt{k_i^2 - k^2}\,\rho\right) = \sqrt{k_i^2 - k^2}\left(\frac{J_1\left(\xi\right)}{\xi} + J'_1\left(\xi\right)\right) = \sqrt{k_i^2 - k^2}\, J_0 \tag{13}$$

From (11), (12) and (13), (10) for Ez is expressed as:

$$E_Z = \frac{1}{j\omega\varepsilon} \int_{-\infty}^{+\infty} dk\, e^{jkz} \sqrt{k_i^2 - k^2}\left(A J_0\left(\sqrt{k_i^2 - k^2}\,\rho\right) + B Y_0\left(\sqrt{k_i^2 - k^2}\,\rho\right) \right) \tag{14}$$

For <u>ρ>b</u> as the space expands to infinity, the fields should have decaying characteristics. In order to achieve this a suitable combination of Bessel and Neumann functions should be chosen.

This suitable combination is $J_\nu\left(x\right) - jY_\nu\left(x\right) = H_\nu^{(2)}\left(x\right)$, where $H_\nu^{(2)}\left(x\right)$ is called Hankel function of second type, and the condition Henkel radiation condition [7]. For this region the following expressions for Hφ,Ez exist:

$$H_\varphi^{(II)} = \int_{-\infty}^{+\infty} d\ k\, e^{jkz}\ C(k)\ H_1^{(2)}\left(\sqrt{k_i^2 - k^2}\,\rho\right) \tag{15}$$

$$E_Z^{(II)} = \frac{1}{j\omega\varepsilon}\int_{-\infty}^{+\infty} d\ k\sqrt{k_i^2-k^2}\ e^{jkz}\ C(k)\ H_0^{(2)}\left(\sqrt{k_i^2-k^2}\,\rho\right) \tag{16}$$

For ρ=α :

$$E_Z = \delta\left(z\right)$$

$$E_Z = \frac{1}{j\omega\varepsilon}\int_{-\infty}^{+\infty} dk\ e^{jkz}\sqrt{k_i^2-k^2}\left(A(k)J_0\left(\sqrt{k_i^2-k^2}\,\rho\right) + B(k)Y_0\left(\sqrt{k_i^2-k^2}\,\rho\right)\right)$$

$$\delta\left(z\right) = \frac{1}{j\omega\varepsilon}\int_{-\infty}^{+\infty} dk\ e^{jkz}\sqrt{k_i^2-k^2}\left(A(k)\ J_0\left(\sqrt{k_i^2-k^2}\,\alpha\right) + B(k)Y_0\left(\sqrt{k_i^2-k^2}\,\alpha\right)\right)$$

$$\int_{-\infty}^{+\infty} dz\frac{1}{j\omega\varepsilon}\int_{-\infty}^{+\infty} dk\ e^{jkz}e^{-jk'z}\sqrt{k_i^2-k^2}\left(A(k)\ J_0\left(\sqrt{k_i^2-k^2}\,\alpha\right) + B(k)Y_0\left(\sqrt{k_i^2-k^2}\,\alpha\right)\right) = \int_{-\infty}^{+\infty} dz e^{-jk'z}\ \delta\left(z\right)$$

but

$$\int_{-\infty}^{+\infty} dz\ e^{j(k-k')\ z} = 2\pi\,\delta(k-k')$$

$$\frac{1}{j\omega\varepsilon}\int_{-\infty}^{+\infty} dk 2\pi\,\delta(k-k')\sqrt{k_i^2-k^2}\left(A(k)\ J_0\left(\sqrt{k_i^2-k^2}\,\alpha\right) + B(k)Y_0\left(\sqrt{k_i^2-k^2}\,\alpha\right)\right) = 1$$

$$\frac{1}{j\omega\varepsilon}2\pi\sqrt{k_i^2-k'^2}\left(A(k')\ J_0\left(\sqrt{k_i^2-k'^2}\,\rho\right) + B(k')Y_0\left(\sqrt{k_i^2-k'^2}\,\rho\right)\right) = 1$$

$k' \to k$

$$\frac{1}{j\omega\varepsilon}\sqrt{k_i^2-k^2}\left(A(k)\ J_0\left(\sqrt{k_i^2-k^2}\,\alpha\right)+B(k)\,Y_0\left(\sqrt{k_i^2-k^2}\,\alpha\right)\right)=\frac{1}{2\pi} \tag{17}$$

where k_i : is the propagation constant of the dielectric, ε : is the dielectric permittivity of the dielectric.

For <u>ρ=b</u> Hφ continuous and Ez continuous

For Hφ= continuous :

$$\int_{-\infty}^{+\infty} dk\ e^{jkz}\ C\left(k\right)H_1^{(2)}\left(\sqrt{k_0^2-k^2}\,b\right)=$$

$$=\int_{-\infty}^{+\infty} dk\ e^{jkz}\left(A(k)J_1\left(\sqrt{k_i^2-k^2}\,b\right)+B(k)Y_1\left(\sqrt{k_i^2-k^2}\,b\right)\right)\Rightarrow$$

$$\int_{-\infty}^{+\infty} dz\int_{-\infty}^{+\infty} dk\ e^{jkz}\ e^{-jk'z}C\left(k\right)H_1^{(2)}\left(\sqrt{k_0^2-k^2}\,b\right)=$$

$$\int_{-\infty}^{+\infty} dz\int_{-\infty}^{+\infty} dk\ e^{jkz}\ e^{-jk'z}\left(A(k)J_1\left(\sqrt{k_i^2-k^2}\,b\right)+B(k)Y_1\left(\sqrt{k_i^2-k^2}\,b\right)\right)$$

It is known that:

$$\int_{-\infty}^{+\infty} dz\ e^{j(k-k')\ z}=2\pi\ \delta\left(k-k'\right)$$

Taking this into consideration the expression above can be written as:

$$\int_{-\infty}^{+\infty} dk\ 2\pi\ \delta(k-k')C\left(k\right)H_1^{(2)}\left(\sqrt{k_0^2-k^2}\,b\right)=$$

$$\int_{-\infty}^{+\infty} dk\ 2\pi\ \delta(k-k')\left(A(k)J_1\left(\sqrt{k_i^2-k^2}\,b\right)+B(k)Y_1\left(\sqrt{k_i^2-k^2}\,b\right)\right)\Rightarrow$$

$$2\pi\ C\left(k'\right)H_1^{(2)}\left(\sqrt{k_0^2-k'^2}\,b\right)=$$

$$2\pi\left(A(k')J_1\left(\sqrt{k_i^2-k'^2}\,b\right)+B(k')Y_1\left(\sqrt{k_i^2-k'^2}\,b\right)\right)$$

where the also known property was used:

$$\int_{-\infty}^{+\infty} \delta\left(k-k'\right) dk = 1$$

Putting $k' \to k$, the expression above can be written as:

$$C\left(k\right) H_1^{(2)}\left(\sqrt{k_0^2-k^2}\, b\right) = A\left(k\right) J_1\left(\sqrt{k_i^2-k^2}\, b\right) + B\left(k\right) Y_1\left(\sqrt{k_i^2-k^2}\, b\right) \tag{18}$$

Working similarly with expressions (14) and(16) and for Ez= continuous:

$$\frac{\varepsilon_{r\delta}}{\varepsilon_{ri}} C(k)\left(\sqrt{k_0^2-k^2}\right) H_0^{(2)}\left(\sqrt{k_0^2-k^2}\, b\right) = \left(A\left(k\right) J_0\left(\sqrt{k_i^2-k^2}\, b\right) + B\left(k\right) Y_0\left(\sqrt{k_i^2-k^2}\, b\right)\right)\sqrt{k_i^2-k^2} \tag{19}$$

where $\varepsilon_{r\delta}$ is the relative dielectric permittivity of the dielectric, ε_{ri} is the relative dielectric permittivity of the tissue

(17) =>

$$A\left(k\right) = \frac{1}{2\pi} \frac{j\omega\varepsilon}{\sqrt{k_i^2-k^2}} \frac{1}{J_0\left(\sqrt{k_i^2-k^2}\, a\right)} - B\left(k\right) \frac{Y_0\left(\sqrt{k_i^2-k^2}\, a\right)}{J_0\left(\sqrt{k_i^2-k^2}\, a\right)} \tag{20}$$

(18), (20) =>

$$\begin{aligned} C(k) H_1^{(2)}\left(\sqrt{k_0^2-k^2}\, b\right) = {} & \frac{1}{2\pi} \frac{j\omega\varepsilon}{\sqrt{k_i^2-k^2}} \frac{J_1\left(\sqrt{k_i^2-k^2}\, b\right)}{J_0\left(\sqrt{k_i^2-k^2}\, a\right)} + \\ & + B\left(k\right)\left(Y_1\left(\sqrt{k_i^2-k^2}\, b\right) - \frac{Y_0\left(\sqrt{k_i^2-k^2}\, a\right)}{J_0\left(\sqrt{k_i^2-k^2}\, a\right)} J_1\left(\sqrt{k_i^2-k^2}\, b\right)\right) \end{aligned} \tag{21}$$

(19),(20) =>

$$\begin{aligned} C(k)\left(\sqrt{k_0^2-k^2}\right) H_0^{(2)}\left(\sqrt{k_0^2-k^2}\, b\right) = {} & \sqrt{k_i^2-k^2}\, \frac{1}{2\pi} \frac{j\omega\varepsilon}{\sqrt{k_i^2-k^2}} \frac{J_0\left(\sqrt{k_i^2-k^2}\, b\right)}{J_0\left(\sqrt{k_i^2-k^2}\, a\right)} + \\ & + \sqrt{k_i^2-k^2}\, B\left(k\right)\left(Y_0\left(\sqrt{k_i^2-k^2}\, b\right) - \frac{Y_0\left(\sqrt{k_i^2-k^2}\, a\right)}{J_0\left(\sqrt{k_i^2-k^2}\, a\right)} J_0\left(\sqrt{k_i^2-k^2}\, b\right)\right) \end{aligned} \tag{22}$$

$$C(k)=\frac{j\omega\varepsilon}{2\pi J_0\left(\sqrt{k_i^2-k^2}\,a\right)}\left\{\frac{-J_0\left(\sqrt{k_i^2-k^2}\,b\right)\xi(k)+J_1\left(\sqrt{k_i^2-k^2}\,b\right)}{\sqrt{k_i^2-k^2}\,H_1^{(2)}\left(\sqrt{k_0^2-k^2}\,b\right)-\sqrt{k_0^2-k^2}\,\xi(k)\frac{\varepsilon r\delta}{\varepsilon ri}H_0^{(2)}\left(\sqrt{k_0^2-k^2}\,b\right)}\right\} \tag{23}$$

where

$$\xi(k)=\frac{Y_1\left(\sqrt{k_i^2-k^2}\,b\right)J_0\left(\sqrt{k_i^2-k^2}\,a\right)-Y_0\left(\sqrt{k_i^2-k^2}\,a\right)J_1\left(\sqrt{k_i^2-k^2}\,b\right)}{\left(Y_0\left(\sqrt{k_i^2-k^2}\,b\right)J_0\left(\sqrt{k_i^2-k^2}\,a\right)-Y_0\left(\sqrt{k_i^2-k^2}\,a\right)J_0\left(\sqrt{k_i^2-k^2}\,b\right)\right)} \tag{22a}$$

with

$$H_1^{(2)}\left(\sqrt{k_0^2-k^2}\,b\right)=J_1\left(\sqrt{k_0^2-k^2}\,b\right)-jY_1\left(\sqrt{k_0^2-k^2}\,b\right)$$

$$H_0^{(2)}\left(\sqrt{k_0^2-k^2}\,b\right)=J_0\left(\sqrt{k_0^2-k^2}\,b\right)-jY_0\left(\sqrt{k_0^2-k^2}\,b\right)$$

and ξ(k) derived from (22α).

In (22α) and (23) in other words for C(k), ξ(k):

ko : is the complex propagation constant of the tissue

kι : is the propagation constant of the dielectric

ε : is the dielectric permitivity of the dielectric (ε was derived from (17)).

Knowing C(k), Ez and Hφ can be calculated from (16) and (15) correspondingly for the space region with ρ>b.

$$E_z^{II}(z,\rho)=\frac{1}{j\omega\varepsilon}\int_{-\infty}^{+\infty}dk\left(\sqrt{k_i^2-k^2}\right)e^{jkz}C(k)\,H_0^{(2)}\left(\sqrt{k_i^2-k^2}\,\rho\right)=$$

$$=\frac{1}{j\omega\varepsilon}\int_{-\infty}^{+\infty}dk\left(\sqrt{k_i^2-k^2}\right)\left(\cos(kz)+j\sin(kz)\right)C(k)\,H_0^{(2)}\left(\sqrt{k_i^2-k^2}\,\rho\right)\Rightarrow \tag{24}$$

$$E_z^{II}(z,\rho)=\frac{2}{j\omega\varepsilon}\int_{0}^{+\infty}dk\left(\sqrt{k_i^2-k^2}\right)\cos(kz)\,C(k)\,H_0^{(2)}\left(\sqrt{k_i^2-k^2}\,\rho\right)$$

Similarly

$$H_{\varphi}^{II}\left(z,\rho\right) = 2\int_{0}^{+\infty} dk \quad \cos\left(kz\right) H_{1}^{(2)}\left(\sqrt{k_i^2 - k^2}\,\rho\right) \tag{25}$$

In order to calculate Eρ, (2) should be solved for Eρ.

$$E_{\rho}^{II}\left(z,\rho\right) = -\frac{1}{j\omega\varepsilon}\frac{\partial H_{\varphi}}{\partial z}$$

In the expression above Hφ is substituted using (16).

$$E_{\rho}^{II}\left(z,\rho\right) = -\frac{1}{\omega\varepsilon}\int_{-\infty}^{+\infty} k\, dk\; e^{jkz}\, C\left(k\right) H_{1}^{(2)}\left(\sqrt{k_i^2 - k^2}\,\rho\right)$$

and because C(-k)=C(k) the above integral becomes:

$$E_{\rho}^{II}\left(z,\rho\right) = -\frac{2j}{\omega\varepsilon}\int_{0}^{+\infty} k\, dk\; \sin\left(kz\right) C\left(k\right) H_{1}^{(2)}\left(\sqrt{k_i^2 - k^2}\,\rho\right) \tag{26}$$

In (24) , (25) and (26) above :

ki is the complex propagation constant of the tissue, ε is the dielectric permittivity of the tissue

Having calculated the fields in the various space regions, the complex input admittance of the antenna can be calculated. At point ρ=α and z=0 the antenna is fed with voltage V=1Volt. Using:

$$I_z = 2\pi\alpha\; H_{\varphi}$$

For **V**=1Volt **:**

$$Y_{in} = \frac{I_z}{V} = 2\pi\alpha\; H_{\varphi}$$

In the above expression Hφ is substituted from (9) with ρ=α .

$$Y_{in} = 2\pi\alpha\int_{-\infty}^{+\infty} dk\left(A(k)J_1\left(\sqrt{k_i^2-k^2}\,\alpha\right) + B(k)Y_1\left(\sqrt{k_i^2-k^2}\,\alpha\right)\right)e^{jkz} \tag{27}$$

where ki is the propagation constant of the dielectric

Substituting A(k) from (20) to (18) :

$$C\left(k\right)H_1^2\left(\sqrt{k_0^2-k^2}\,b\right)=\frac{1}{2\pi}\frac{j\omega\varepsilon}{\sqrt{k_i^2-k^2}}\frac{J_1\left(\sqrt{k_i^2-k^2}\,b\right)}{J_0\left(\sqrt{k_i^2-k^2}\,a\right)}+$$

$$B\left(k\right)\left(Y_1\left(\sqrt{k_i^2-k^2}\,b\right)-\frac{Y_0\left(\sqrt{k_i^2-k^2}\,a\right)J_1\left(\sqrt{k_i^2-k^2}\,b\right)}{J_0\left(\sqrt{k_i^2-k^2}\,a\right)}\right)\Rightarrow$$

$$C\left(k\right)H_1^2\left(\sqrt{k_0^2-k^2}\,b\right)J_0\left(\sqrt{k_i^2-k^2}\,a\right)=\frac{1}{2\pi}\frac{j\omega\varepsilon}{\sqrt{k_i^2-k^2}}J_1\left(\sqrt{k_i^2-k^2}\,b\right)+$$

$$+B\left(k\right)\left(Y_1\left(\sqrt{k_i^2-k^2}\,b\right)J_0\left(\sqrt{k_i^2-k^2}\,a\right)-Y_0\left(\sqrt{k_i^2-k^2}\,a\right)J_1\left(\sqrt{k_i^2-k^2}\,b\right)\right)\Rightarrow$$

$$B\left(k\right)=\frac{C\left(k\right)H_1^2\left(\sqrt{k_0^2-k^2}\,b\right)J_0\left(\sqrt{k_i^2-k^2}\,a\right)-\frac{1}{2\pi}\frac{j\omega\varepsilon}{\sqrt{k_i^2-k^2}}J_1\left(\sqrt{k_i^2-k^2}\,b\right)}{Y_1\left(\sqrt{k_i^2-k^2}\,b\right)J_0\left(\sqrt{k_i^2-k^2}\,a\right)-Y_0\left(\sqrt{k_i^2-k^2}\,a\right)J_1\left(\sqrt{k_i^2-k^2}\,b\right)} \tag{28}$$

Expression (27) can be written as:

$$Y_{in}=4\pi\alpha\int_0^{+\infty}dk\left(A(k)J_1\left(\sqrt{k_i^2-k^2}\,a\right)+B(k)Y_1\left(\sqrt{k_i^2-k^2}\,a\right)\right)\cos(kz) \tag{29}$$

where **B(k)** is derived from (28) and **A(k)** from (20).

For this problem the following considerations are taken: (a) the space region for ρ>b includes the biological tissue and (b) in the space region for α<ρ<b the relative dielectric permittivity and the relative magnetic permeability can be changed by changing the material.

It is of interest to find how the fields Ez(z,ρ) and Eρ(z,ρ) are varied for different values of z,ρ inside the biological tissue. The variation of Ez(z,ρ) and Eρ(z,ρ) fields will also be examined by increasing the corresponding relative values of the dielectric permittivity and the relative magnetic permeability.

4. SOFTWARE DEVELOPMENT

For the calculation of the integrals of expressions (24) and (26) numerical integration is used with the aid of Simpson complex rule.

A computer software program was developed in Fortran language. The required input data for the program execution are the relative dielectric permittivity of the dielectric $å_r$ (for á<ñ<b) and the relative magnetic permeability $ì_r$ (for á<ñ<b) .

For $E_z(z,\rho)$ calculation, the program calculates function under integration for value of z given by the user, with constant ρ (ρ=4b) and for 12.287 values of k (k is varied with small discrete steps). For every k the real and imaginary part of the function is calculated. Results are written in a file, the name of which has been given by the user. The integration of the function follows using the Simpson's method, with limits from 0 to 30ko (ko is the propagation constant of the dielectric) , while z still has the value set before. For every ρ from b to 4b (with step 0,05), the corresponding value of $E_z(z,\rho)$ is obtained. Results are again written in a file, the name of which has been given by the user.

The calculation of the Bessel and Neumman functions the polynomial approach has been used [8].

Similarly, values for $E_\rho(z,\rho)$ are obtained.

[1] Program variables values.

The values of the program variables are chosen carefully, in order to ensure adequate precision of the results.

For space region with ρ>b (tissue), the electrical characteristics of the biological tissue are set according to the complex dielectric permittivity:

$\varepsilon=\varepsilon_o(\varepsilon'-j\varepsilon'')= \varepsilon_o(\varepsilon'-j\,\sigma/\omega\,\varepsilon_o)$ where

$\omega=2\pi f$ and f the frequency ($f=432\ 10^{-6}$ HZ)

ε: the complex dielectric permittivity (F/m).

ε_o : the dielectric constant of free space [$10^{-9}/36\pi$ (F/m)].

σ: the electric conductivity of the tissue (S/m).

ε′: the relative dielectric constant of the tissue.

The conductivity and the dielectric constant depend on the frequency. In the table below (Table 1) the values of relative dielectric constant and conductivity are given for two types of tissue and for frequency of 432MHZ.

Table 1

Frequency (MHZ)	relative dielectric constant	Conductivity (S/m)	Tissue type
432	45	0.60	high water concentration
	5	0.24	low water concentration

In calculations $\varepsilon'=41$ and $\varepsilon''=21$ was used. Because $\sigma/\omega\varepsilon' \ll 1$, the tissue behavior is a dielectric. The magnetic permeability μ (H/m) for all practical applications is constant within the tissue and has the value of free space [$\mu o=4\pi 10^{-7}$ (H/m)]. The complex propagation constant of the tissue k_{tissue} is:

$$k_{tissue} = \omega \sqrt{\varepsilon \mu_o}$$

For the space region with $a<\rho<b$, dielectric region, the propagation constant

$$k_{\delta\iota\eta\lambda} = \omega \sqrt{\varepsilon_o \mu_o \varepsilon_r \mu_r}$$

where ε_r , μ_r can take values from 1 to 10 respectively, values which are preset by the user.

5. NUMERICAL RESULTS

In the figures given at the end of this paper the electric field $E_z(z,\rho)$ and $E_\rho(z,\rho)$ of a dipole antenna implanted in a tissue is graphically presented. All diagrams were constructed using data from the corresponding output files of the program.

[1] Variation for $E_z(z,\rho)$

For $E_z(z,\rho)$ there are two types of diagrams. The first refers to the change of the real and imaginary parts of the function FUNC(Ez) under integration with k, for various z and for $\rho=4b$. Function FUNC(Ez) (as it is shown from relation 24) is given as follows:

$$FUNC\ (E_z) = \frac{2}{j\omega\varepsilon}\cos\left(kz\right)\left(\sqrt{k_i^2 - k^2}\right)C\left(k\right)H_0^{(2)}\left(\sqrt{k_i^2 - k^2}\,\rho\right)$$

The second type of diagrams show the change of the real, imaginary part and the magnitude of the electric field $E_z(z,\rho)$ (as given by relation 24), for various z (entered as inputs to the program) with ρ taking values from b up to 10b with step of 0,05. Graphical presentations where made for various εr and μr (entered as inputs to the program) for the space region for α<ρ< b.

The variation of function FUNC(Ez) with k με το k showed a certain oscillation so an increase of the integration span and a decrease in integration step was dictated in order to achieve the required accuracy.

Comparing the diagrams of the function with the same z but with different ε_r and μ_r, of the space region for α<ρ< b a change in place of the maxima and minima of the real and imaginary parts of the function was observed. This is expected as the values of k for which maxima and minima of the real and imaginary parts of the function occur depends on ko which changes in every case, changing also the electrical properties of the dielectric (diagrams 1,2).

Function FUNC(Ez) is even as results from the analytical process and is confirmed by comparing diagrams.

[2] Variation of $E_z(z,\rho)$ diagrams

Comparing diagrams with the same ε_r and μ_r but different z it was observed that going away from the feed-point (z=0) of the antenna the magnitude of the electric $E_z(z,\rho)$ decreases as ρ increses.

In all diagrams the magnitude of the electric field $E_z(z,\rho)$ decreases exponentially as the radial distance from the antenna is increases (diagrams 2,3).

Comparing diagrams having the same εr and μr it was observed that the magnitude of the electric field $E_z(z,\rho)$ for z equals that for with -z, for every corresponding value of ρ.

Increasing the values of εr or and μr, it was observed that the magnitude of the electric field $E_z(z,\rho)$ also increases for every ρ (comparison of diagrams of the same z) (diagrams 3,5,6). This is important as it gives the possibility of constructing antennas with high μ_r value and testing them under real conditions, possibly replacing the existing conventional ones.

[3] Variation of $E_\rho(z,\rho)$

For Eρ(z,ρ) there are two types of diagrams. The first refers to the change of the real and imaginary parts of the function FUNC(Eρ) under integration with k, for various z and for ρ=4b. Function FUNC(Eρ (as it is shown from relation 26) is given as follows:

$$FUNC\ (E_\rho) = -\frac{2j}{\omega\varepsilon}\sin\left(kz\right)(k)C\left(k\right)H_1^{(2)}\left(\sqrt{k_i^2 - k^2}\,\rho\right)$$

The second type of diagrams show the change of the real, imaginary part and the magnitude of the electric field $E_\rho(z,\rho$ (as given by relation 26), for various z (entered as inputs to the program) with ρ taking values from b up to 10b with step of 0,05. Graphical presentations where made for various εr and μr (entered as inputs to the program) for the space region for α<ρ< b.

[4] FUNC(Eρ) diagrams

If in the function FUNC(Eρ) z is set to zero (z=0) , then the function also goes to zero. Function FUNC(Eρ) is odd as results from the analytical process and is confirmed by comparing the appropriate diagrams. In all the rest, the same things apply for FUNC(Eρ) as for FUNC(Ez).

For $E_\rho(z,\rho)$ diagrams, similar observations were made as for $E_z(z,\rho)$.

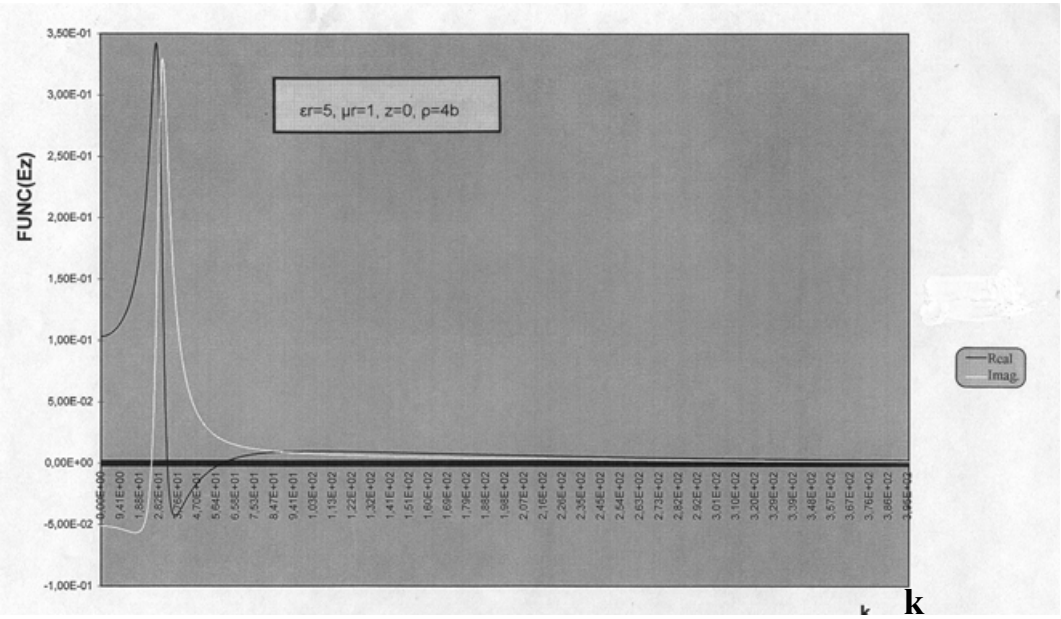

Diagram 1.

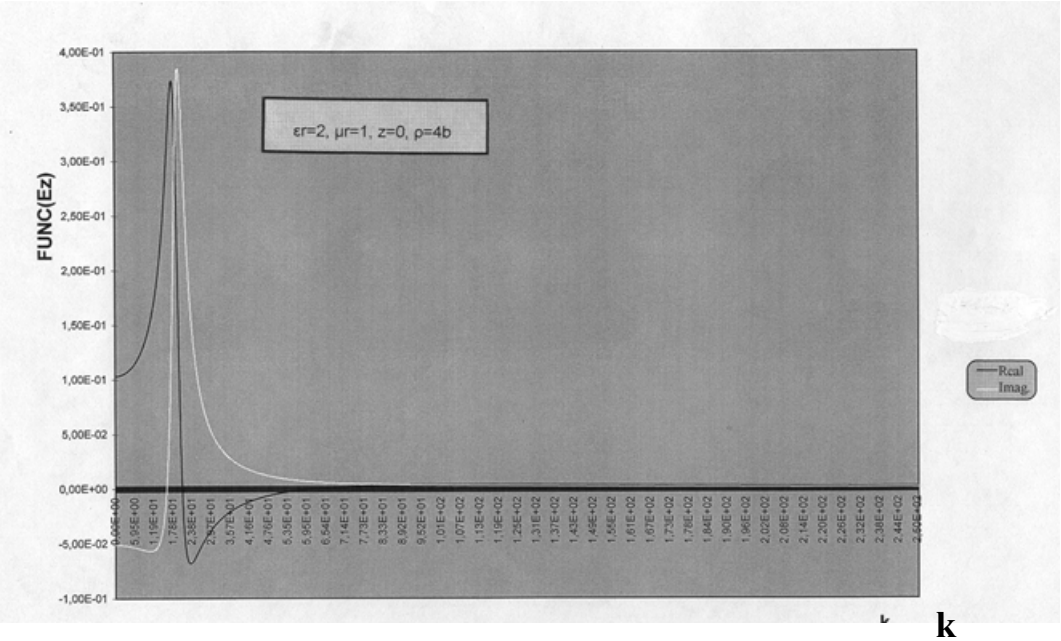

Diagram 2.

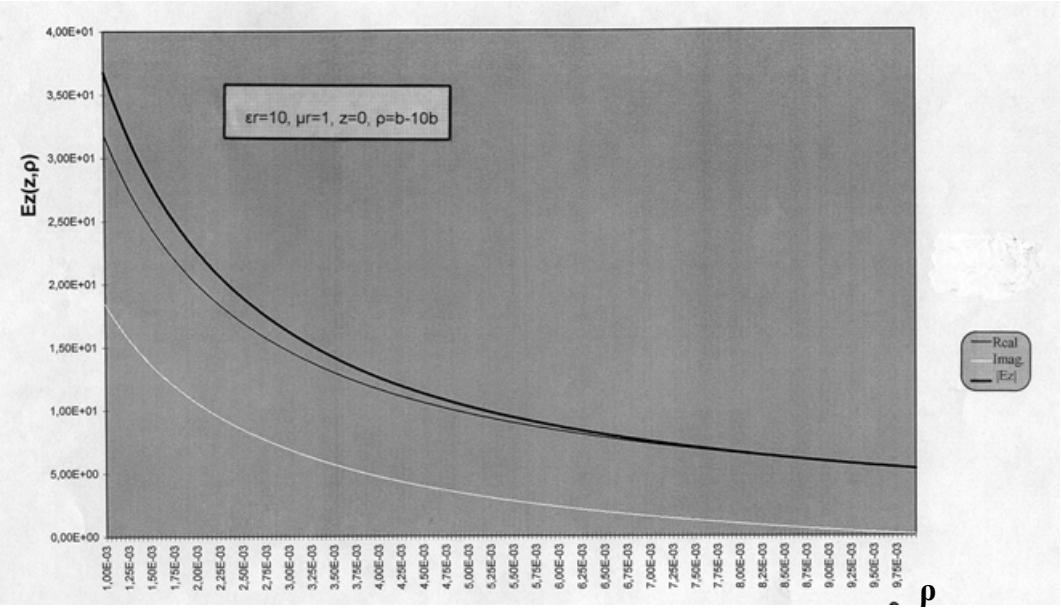

Diagram 3

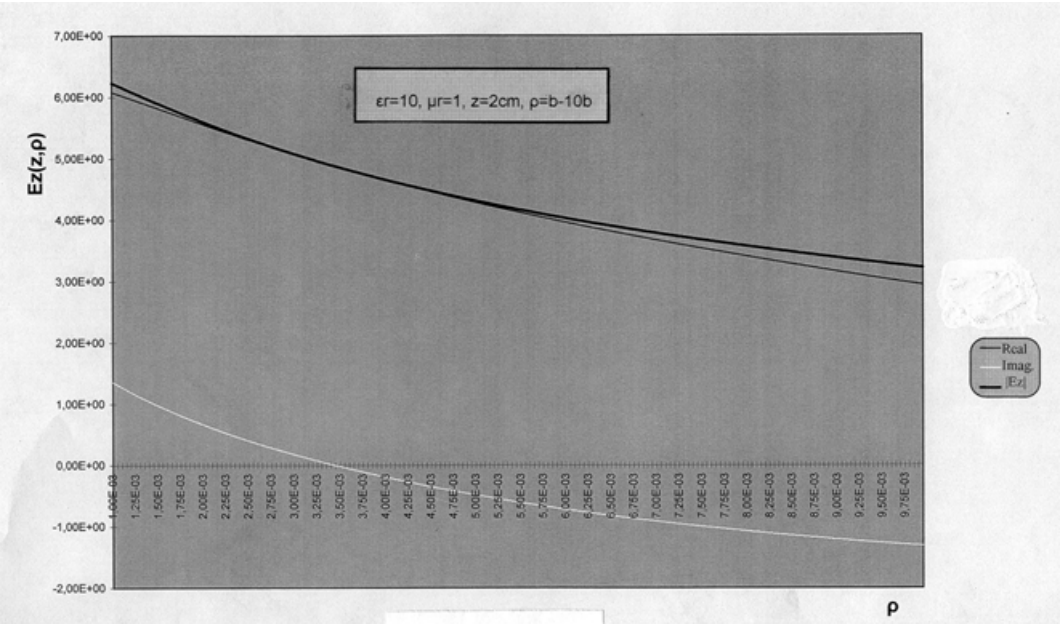

Diagram 4

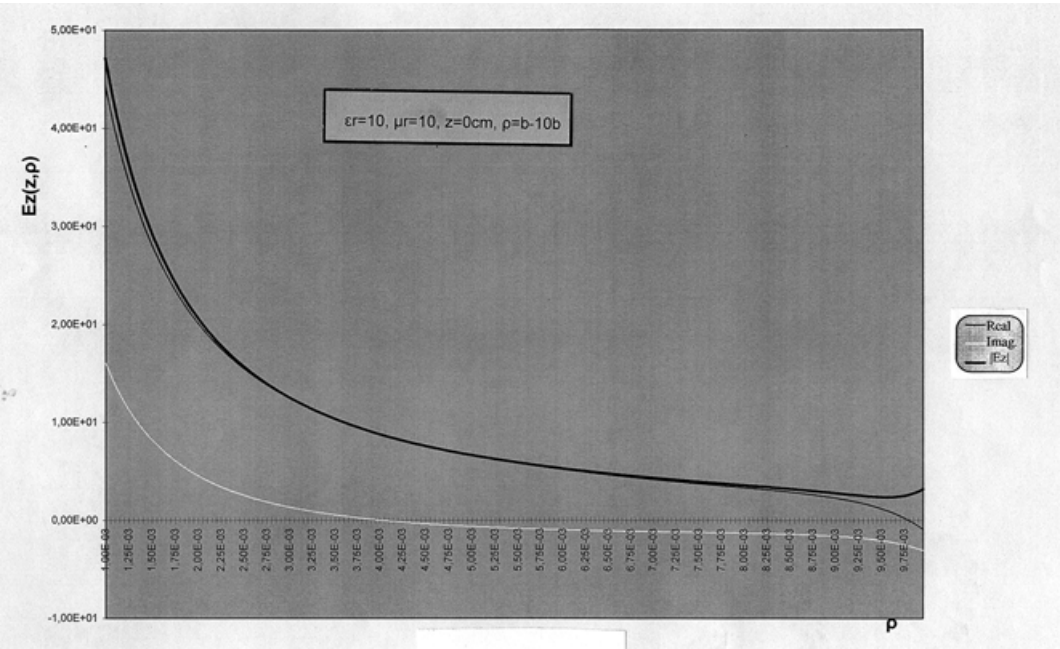

Diagram 5.

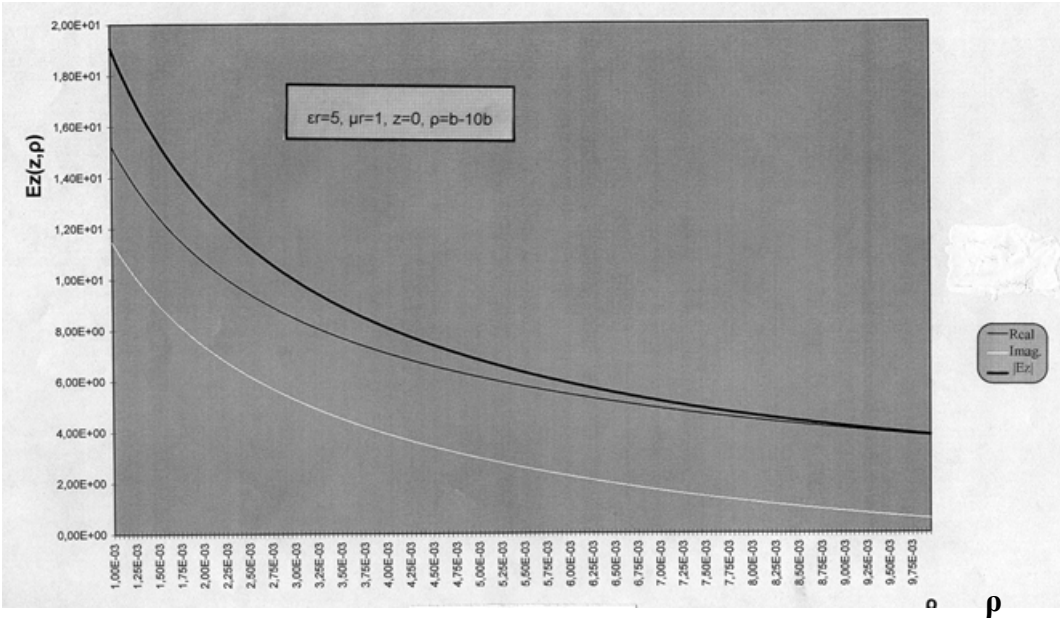

Diagram 6

6. CALCULATION OF PENETRATION OF THE ELECTRIC FIELDS $E_z(z,\rho)$, $E_\rho(z,\rho)$

A Fortran program was developed to calculate the inclination of $E_z(z,\rho)$ at the points 0.5Eo and 0.7Eo, where Eo is the value of $E_z(z,\rho)$ for z=0 and ρ=b. The inclination of the field $E_z(z,\rho)$ is calculated by the program taking into account about 250 points around 0.5Eo and 0.7Eo for ρ varying with step 0.0002. User is asked to input to the program εr, μr and z. Results are kept in a file. Exactly the same process is followed for $E_\rho(z,\rho)$. Results are shown in tables 2 and 3.

Table 2. Inclination of $E_z(z,\rho)$ field

ε_r	μ_r	z (cm)	field inclination 0.5Eo	$E_z(z,\rho)$ at 0.7Eo points
15	2	0	1.532E+04	2.720E+04
15	5	0	2.124E+04	3.836E+04
15	10	0	2.533E+04	4.555E+04
15	15	0	2.708E+04	4.862E+04

Table 3. Inclination of $E_\rho(z,\rho)$ field

ε_r	μ_r	z (cm)	field inclination 0.5Eo	$E_\rho(z,\rho)$ at 0.7Eo points
15	2	1	1.408E+04	2.337E+04
15	5	1	1.671E+04	2.614E+04
15	10	1	3.281E+04	5.399E+04
15	15	1	6.135E+04	8.676E+04

From tables 2, 3 it is observed that as μ_r increases, so does the inclination of the fields $E_z(z,\rho)$ and $E_\rho(z,\rho)$, which means that their penetration is decreasing,

[1] Antenna Complex Input Admittance

Finally the complex input admittance of the dipole antenna has to be calculated. In order to calculate the integral (expression 29) numerical integration is used (Simpson complex rule). The required input data for the program execution are the relative dielectric permitivity of the dielectric ε_r and the relative magnetic permeability of the dielectric μ_r

Results of the magnitude of the complex input admittance of the dipole antenna implanted in a tissue for various ε_r , μ_r are shown in table 4.

Table 4. Complex input admittance of the antenna

ε_r (dielectric)	μ_r (dielectric)	Yin=G+jB
15	15	0.928137E-3-j0.320848E-2
15	10	0.848461E-3-j0.380694E-2
15	2	0.204005E-3-j0.683924E-2
10	5	0.383646E-3-j0.422465E-2

7. CONCLUSIONS

The radiation of an infinite length dipole placed inside a lossy medium has been studied theoretically using an analytically approach. It is shown that the penetration depth of the radiation inside the tissue medium could be increased by using a high magnetic permeability sleeve placed around the dipole element antenna.

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