CALCULATION OF FORCE BETWEN TWO COAXIAL DISK-CONDUCTORS BY CALCULATING FUNCTIONS

M. Braneshi*, P. Cipo*, L. Ahma, E. Hamiti****

*Electrical Department, Electric Engineering Faculty, Polytechnic Universitety of Tirana
Sheshi Nënë Tereza, Nr. 4, Tirana, Albania
e-mail: mbraneshi@hotmail.com, piro_cipo@postmark.net
**University of Prishtina, Faculty of Electrical Engineering
Bregu i diellit p.n. Prishtinë, Kosova
e-mail: luan.ahma@fiek.uni-pr.edu, ehamiti@postmark.net

Abstract

This article analyzes two approaches for calculation of the axial force between two coaxial disk-conductors with non-uniform distribution (Bitter's distribution) of the current density in iron-free media. The first approach is based on application of the calculating function of the force for the given configuration of conductors. The calculating function is brought in an appropriate form where its further calculation can be performed through standard procedures of the numerical integration. The tabulation of the calculating function increases velocity of calculation and decreases its costs. The second approach is based on equivalent filaments where conductors are presented by an entity of current-carrying filaments. The results obtained by two approaches are in very good agreement with each other.

1. INTRODUCTION

Circular coils are widely used in various electromagnetic devices. They are encountered in the tubular linear motors, transformers and reactors, in plasma physics, MRI, antennas etc. This is the reason that in various papers, these coils are considered from different point of views, in accordance to the problem that has to be solved. One of them is the calculation of the force between different coils. The traditional way, used for this purpose, consists in two steps [1-3]. As the first step, it is calculated the magnetic field in the domain with current. The calculation of force is performed as a second step. For coils placed in an iron-free media, the use of Biot-Savart law reduces these steps in one, the calculation of a define integral.

In this paper, we present a simple and fast procedure for the calculation of force between two circular coaxial disk-conductors placed in an iron-free media. It consists in using the calculating function of force that, according to [4], represents a normous primitive of the force integral. The calculating function is studied by generating its calculating expression that consists in a form of a single integral. To calculate this integral, one of the standard procedures of the numerical integration can be used,

since its kernel is a continuous function. Due to the fact, that the calculating function depends on two variables, it is performed its tabulating, thus making possible the table calculation of force. This increases, very sensitively, the computation velocity and decreases its cost. Such procedure is encountered in the inductance calculation [5, 6] where, in our opinion, the calculating methods are highly structured rather than those of calculating the force. To show the efficiency and accuracy of the proposed method, the obtained results are compared with the ones obtained by the equivalent filaments. Both results are in very good agreement with each other. We would like to stress that the method is suitable for the direct current but it can be a very good approach for the low frequency sinusoidal current.

2. BASIC EXPRESSION

The derivation of the general force formula for circular coaxial current-carring disks starts with the force between two circular coaxial filaments. Due to cylindrical symmetry only the axial component of the force exists. In integral form, it expression is as follows:

$$F_z = \frac{1}{2}\mu_0 I_R I_r \int_0^{2\pi} \frac{RrH\cos\theta d\theta}{\left(R^2 + r^2 + H^2 - 2Rr\cos\theta\right)^{3/2}} \tag{1}$$

where μ_0 is the permeability of free space, I_R and I_r are currents of filaments with radii R and r respectively and H is the distance between filament planes as shown in figure 1.

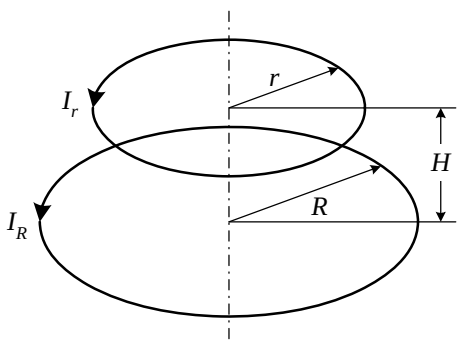

Figure 1. Circular coaxial filaments

Force (1) is analytically expressed in terms of elliptic integrals [7]:

$$F_z = \frac{\mu_0 I_R I_r H}{\sqrt{(R+r)^2 + H^2}} \left[\frac{R^2 + r^2 + H^2}{(R-r)^2 + H^2} \mathbf{E}(k) - \mathbf{K}(k) \right] \tag{2}$$

where $\mathbf{K}(k)$ and $\mathbf{E}(k)$ are complete elliptic integrals of first and second kind respectively with the module

$$k^2 = \frac{4Rr}{(R+r)^2 + H^2} \tag{3}$$

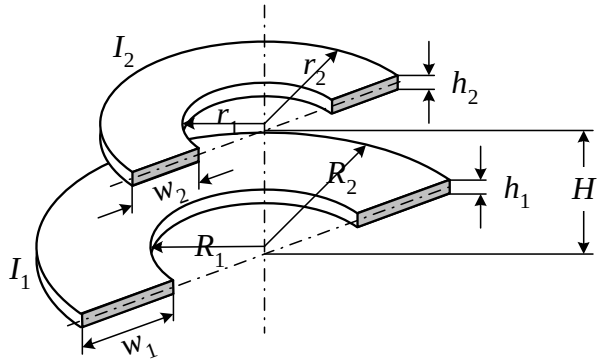

Figure 2. Circular coaxial disks with rectangular cross-section.

Figure 2 shows the dimensions and arrangement of the two coaxial disk-conductors. In practice, these conductors could represent coil windings, where its spiral arrangement can be modeled by circular coaxial conductors. The calculation of the force between them is performed by integrating the expression (1) over conductor cross-sections. In the first approximation, the distribution of the current density in the conductor cross-section is assumed constant. Such approximation provides a good accuracy when the ratio between width *w* and height *h* of the conductor cross-section approaches 1. However to increase mechanical stability of the coils used for providing strong magnetic fields [8], or in planar structures [9], this ratio goes from 5:1 up to 50:1. In such cases, current path in the inner side of conductor is shorter than in the outer side, therefore inner side resistance is lower and the current density higher. So, there is an inverse relation between current density and conductor radius, which in our case is:

$$J_1(R) = \frac{J_1}{R} \qquad J_2(r) = \frac{J_2}{r} \tag{4}$$

where J_1 and J_2 constants are related to currents and geometrical dimensions of conductors by the following expressions:

$$J_1 = \frac{I_1}{h_1 \ln(R_2/R_1)} \quad J_2 = \frac{I_2}{h_2 \ln(r_2/r_1)} \tag{5}$$

The above-mentioned distribution is accurate for the direct current and it can be used as the first approximation in sinusoidal current for the considered type of the conductors. Due to the small sizes in axial direction, current distribution is almost constant in that direction, so we can consider the currents as concentrated in the conductor central planes. Taking into account (4) and (5) the force expression in the integral form is

$$F = \frac{\mu_0}{4\pi} \frac{I_1 I_2}{\ln(R_2/R_1)\ln(r_2/r_1)} \int_{R_1}^{R_2} dR \int_{r_1}^{r_2} dr \int_0^{2\pi} \frac{2\pi H \cos\theta d\theta}{\left(R^2 + r^2 + H^2 - 2Rr\cos\theta\right)^{3/2}} \tag{6}$$

3. PROPOSED METHOD

The proposed method for the calculation of force consists in using its calculating function. This function depends on two variables, making possible its tabulation and thus increasing calculating speed. It can be studied and, for particular regions of its variables, approximate expressions can be determined.

3.1. DETERMINATION OF CALCULATING FUNCTION

Among the calculating functions of the force between two circular coaxial disk-conductors we have chosen the following function:

$$\mathbf{F}\,(r,z) = \int_1^{\infty} dR \int_0^{r} dt \int_0^{2\pi} \frac{2\pi z \cos\theta d\theta}{\left(R^2 + t^2 + z^2 - 2Rt\cos\theta\right)^{3/2}} \tag{7}$$

This is an odd function with respect to variable z

$$\mathbf{F}(r, -z) = -\mathbf{F}(r, z) \tag{8}$$

It also satisfies the following relation according r

$$\mathbf{F}\left(\frac{1}{r}, z\right) = \mathbf{F}\,(r, rz) + 4\pi^2 \ln \frac{z + \sqrt{(1/r)^2 + z^2}}{z + \sqrt{1 + z^2}} \tag{9}$$

The proof of (9) is given in Appendix A attached to this paper.

3.2. FORCE EXPRESSION USING CALCULATING FUNCTION

Force (6) is expressed by means of its calculating function (7) as follows

$$F = \frac{\mu}{4\pi} \frac{I_1 I_2}{\ln(R_2/R_1)\ln(r_2/r_1)} \sum_{i=1}^{2} \sum_{j=1}^{2} (-1)^{i+j+1} \mathbf{F}\left(\frac{r_j}{R_i}, \frac{H}{R_i}\right) \tag{10}$$

3.3. EXPRESSION FOR THE CALCULATING FUNCTION F

Integration according to variables R and t provides the following result [10]

$$\mathbf{F}\,(r,z) = -2\pi \int_0^{2\pi} \frac{\cos\theta}{\sin\theta}\left[\operatorname{arctg}\frac{r\sin^2\theta + z^2\cos\theta}{z\sin\theta\sqrt{1+r^2+z^2-2r\cos\theta}} - \operatorname{arctg}\frac{z\cos\theta}{\sin\theta\sqrt{1+z^2}}\right] d\theta \tag{11}$$

Such function cannot be furthermore integrated in the framework of the elementary functions or special ones already known from us. The kernel of integral is a continuous function in points sin θ = 0, therefore it can be used one of the standard procedures of the numerical integration for its integration. In these points its values are:

$$\frac{1}{z}\left[\sqrt{1+z^2} - \sqrt{1+r^2+z^2-2r\cos\theta}\right] \tag{12}$$

In Appendix B, in table B1 are given the values of function **F** for $0 < r \le 1$ and $0 \le z \le 1$, meanwhile in table B2, values for $0 < r \le 1$ and $z \ge 1$ ($1 \ge 1/z > 0$). Values of **F** for $z < 0$ can be calculated by taking into consideration (8), while for $r > 1$ by using expression (9). Fourth degree Newton-Cotes procedure is being used to compute the tables [11].

3.4. APPROXIMATION EXPRESSION

For the calculation of the calculating function in particular domains of r and z variables approximate expressions can be used. Following we will give an expression that is valid for $z \to \infty$ ($0 < r \le 1$). For $r > 1$ relation (9) it is recommended.

$$\mathbf{F}\,(r,z) \cong 4\pi^2 \ln\frac{z+\sqrt{1+z^2}}{2z} - \frac{\pi^2 r^2}{2z^4}\left[3 - \frac{15}{4}\frac{A_2}{z^2} + \frac{35}{8}\frac{A_4}{z^4} - \frac{315}{64}\frac{A_6}{z^6}\right] \tag{13}$$

where

$$A_2 = 1 + r^2, \qquad A_4 = 1 + {}^9/_4 r^2 + r^4, \qquad A_6 = 1 + 4r^2 + 4r^4 + r^6 \tag{14}$$

Using of the first two terms of (13) for z ≥ 10 yields an accuracy up to 4 digits with the precise value, which increases up 6 digits for z ≥ 20. The first three terms of (13) give an accuracy up to 4 digits for z ≥ 5 while for $z \geq 10$ this goes up to 6 digits. The whole expression (13) gives an accuracy up to 4 digits for $z \geq 5$, which goes up to 8 digits for $z \geq 10$.

4. EQUIVALENT FILAMENT METHOD

According to this method, each conductor is divided in a determined number of smaller conductors, where each of them is represented by current-carrying filaments. The force between two initial conductors is the sum of forces between respective filaments.

$$F = \sum_{i=1}^{N_1} \sum_{j=1}^{N_2} F_{ij} \tag{15}$$

Here N_1 and N_2 is the number of filaments of the first and second conductor respectively, while F_{ij} represents the force between the i-th filament of the first conductor and j-th filament of the second conductor. Based on (2), it is

$$F_{ij} = \frac{\mu H I_{1i} I_{2j}}{\sqrt{(R_{Ki} + r_{Kj})^2 + H^2}} \left[\frac{R_{Ki}^2 + r_{Kj}^2 + H^2}{(R_{Ki} - r_{Kj})^2 + H^2} \mathbf{E}(k_{ij}) - \mathbf{K}(k_{ij}) \right] \tag{16}$$

where module of the elliptic integrals k_{ij}, according to (3) results

$$k_{ij}^2 = \frac{4 R_{Ki} r_{Kj}}{(R_{Ki} + r_{Kj})^2 + H^2} \tag{17}$$

Current I_{1i}, I_{2j} and filament radii R_{Ki}, r_{Kj} depends upon the way of conductor division. Here are compared two sorts of divisions: the division of cross-section where elementary conductors have equal area of cross-section and the division of current where elementary conductors have the equal current. Current of each elementary conductor and its geometrical sizes are calculated as follows:

– for the division of cross-section

$$\Delta R = (R_2 - R_1)/N_1 \qquad \Delta r = (r_2 - r_1)/N_2$$

$$R_{1i} = R_1 + (i-1)\Delta R \qquad r_{1j} = r_1 + (j-1)\Delta r$$

$$R_{2i} = R_1 + i\,\Delta R \qquad r_{2j} = r_1 + j\,\Delta r \tag{18}$$

$$I_{1i} = I_1 \frac{\ln(R_{2i}/R_{1i})}{\ln(R_2/R_1)} \qquad I_{2j} = I_2 \frac{\ln(r_{2j}/r_{1j})}{\ln(r_2/r_1)}$$

– for the division of current

$$I_{1i} = I_1/N_1 \qquad I_{2j} = I_2/N_2$$

$$C_1 = \frac{1}{N_1}\ln\frac{R_2}{R_1} \qquad C_2 = \frac{1}{N_2}\ln\frac{r_2}{r_1} \tag{19}$$

$$R_{1i} = R_1 e^{(i-1)C_1} \qquad r_{1j} = r_1 e^{(j-1)C_2}$$

$$R_{2i} = R_1 e^{iC_1} \qquad r_{2j} = r_1 e^{jC_2}$$

In both cases the two approximations [9] of the placement of the filament are being analyzed

– the filament is placed at the center of the section (arithmetical approximation)

$$R_{Ki} = \frac{1}{2}(R_{1i} + R_{2i}) \qquad r_{Kj} = \frac{1}{2}(r_{1j} + r_{2j}) \tag{20}$$

– the filament is placed at the geometrical center of the section (geometrical approximation)

$$R_{Ki} = \sqrt{R_{1i}R_{2i}} \qquad r_{Kj} = \sqrt{r_{1j}r_{2j}} \tag{21}$$

5. A NUMERICAL EXAMPLE

To illustrate the above proposed method it is applied to calculate the force between two disk-conductors with the following data: R_1 = 15 cm, R_2 = 30 cm, r_1 = 10 cm, r_2 = 25 cm, $I_1 = I_2$ = 10 kA, where their planes are H = 5 cm distant from each other.

According to these data

$$\frac{\mu_0}{4\pi}\frac{I_1 I_2}{\ln(R_2/R_1)\ln(r_2/r_1)} = 15.744949$$

Values of function **F**, as required by expression (10), are calculated by numerically integrating (11) and making use of the calculating tables. In case of values not present in the table a four-point interpolation [11] is used

$$\mathbf{F}(r, z) = A_1\mathbf{F}(r_1, z_1) + A_2\mathbf{F}(r_2, z_1) + A_3\mathbf{F}(r_2, z_2) + A_4\mathbf{F}(r_1, z_2) \tag{22}$$

where $r_1 < r < r_1 + h_r = r_2$ and $z_1 < z < z_1 + h_z = z_2$ are the nearest table values of the given point, and h_r and h_z are the steps of table value, according to r and z respectively. A_i constants are calculated by means of these expressions

$$A_1 = \frac{r_2 - r}{h_r}\frac{z_2 - z}{h_z} \quad A_2 = \frac{r - r_1}{h_r}\frac{z_2 - z}{h_z} \quad A_3 = \frac{r - r_1}{h_r}\frac{z - z_1}{h_z} \quad A_4 = \frac{r_2 - r}{h_r}\frac{z - z_1}{h_z} \tag{23}$$

The results are shown in Table I.

TABLE I

$\frac{r_j}{R_i}$	$\frac{H}{R_i}$	**F**(r, z) according to (11)	**F**(r, z) according to (22)	Error [%]
2/3	1/3	1.533761	1.534562	–0.052
*5/3 0.6	*1/3 0.2	*15.908518 0.825174	*15.908518 0.825174	0
1/3	1/6	0.186355	0.187055	–0.376
5/6	1/6	1.803473	1.802500	0.054
F [N]		200.868375	200.882640	–0.007

Values marked with (*) are calculated by the expression (9).

It is obvious that the precise value, as well as the interpolated one are in very good agreement with each other.

TABLE II

Number of divisions	Division of cross-section				Division of current			
	Arithmetical approx.		Geometrical approx.		Arithmetical approx.		Geometrical approx.	
	Force [N]	Error [%]	Force [N]	Error [%]	Force [N]	Error [%]	Force [N]	Error [%]
1	222.7269	-10.882	185.1665	7.817	222.7269	-10.882	185.1665	7.817
2	202.1723	-0.649	193.5163	3.660	204.2752	-1.696	200.6808	0.093
3	206.8828	-2.994	203.2569	-1.189	206.5135	-2.810	204.2511	-1.684
4	203.6470	-1.383	201.5932	-0.361	203.8505	-1.485	202.5840	-0.854
5	202.6498	-0.887	201.3377	-0.234	202.7465	-0.935	201.9478	-0.537
10	201.3229	-0.226	200.9959	-0.063	201.3468	-0.238	201.1500	-0.140
20	200.9823	-0.057	200.9006	-0.016	200.9884	-0.060	200.9394	-0.035
50	200.8866	-0.009	200.8735	-0.003	200.8876	-0.010	200.8798	-0.006
100	200.8729	-0.002	200.8697	-0.001	200.8732	-0.002	200.8712	-0.001

Table II shows the values of force calculated according to the equivalent filament method for different numbers of filaments. Two sorts of divisions are being

compared, according to their cross-section and the current, for both filament radius approximations, arithmetical and geometrical one. This table gives the error in computing the force. It is considered as an accurate value the one given by formula (11).

6. CONCLUSIONS

In this paper a simple and quick approach of calculation of the force between two coaxial disk-conductors, with Bitter's distribution of current density, is being proposed. It consists in using calculating function of force for the given configuration. Its expression can be integrated with one of the standard procedures of numerical integration due to the continuous kernel. Tabulating of the calculating function increases calculating speed and decreases its cost. The obtained results are in very good agreement with the equivalent filament method. Based on the results obtained from the considered example, we could come to a conclusion that the uniform division of cross-section with the geometrical mean approximation of the filament radius gives much more precise results.

APPENDIX A.

To prove formula (9) let's note

$$\mathbf{F}_{\infty}(r,z) = \int_0^{\infty} dR \int_0^{r} dt \int_0^{2\pi} \frac{2\pi z \cos\theta d\theta}{\left(R^2 + t^2 + z^2 - 2Rt\cos\theta\right)^{3/2}} \tag{A1}$$

$$\mathbf{F}_{1}(r,z) = \int_0^{1} dR \int_0^{r} dt \int_0^{2\pi} \frac{2\pi z \cos\theta d\theta}{\left(R^2 + t^2 + z^2 - 2Rt\cos\theta\right)^{3/2}} \tag{A2}$$

According to (A7)

$$\mathbf{F}_{\infty}(r,z) = 4\pi^2 \ln \frac{z + \sqrt{r^2 + z^2}}{2z} \tag{A3}$$

By direct manner can be shown that function $\mathbf{F}_1(r, z)$ satisfies the below relation

$$\mathbf{F}_1\left(\frac{1}{r}, z\right) = \mathbf{F}_1(r, rz) \tag{A4}$$

From the structure of functions $\mathbf{F}_1$, $\mathbf{F}$ and $\mathbf{F}_\infty$ can be written

$$\mathbf{F}_1(r, z) + \mathbf{F}(r, z) = \mathbf{F}_\infty(r, z) \tag{A5}$$

from which could note

$$\mathbf{F}_1\left(\frac{1}{r}, z\right) + \mathbf{F}\left(\frac{1}{r}, z\right) = 4\pi^2 \ln \frac{z + \sqrt{(1/r)^2 + z^2}}{2z}$$

$$\mathbf{F}_1(r, rz) + \mathbf{F}(r, rz) = 4\pi^2 \ln \frac{z + \sqrt{1 + z^2}}{2z}$$

The subtraction of above relations and referring to (A4) is obtained

$$\mathbf{F}\left(\frac{1}{r}, z\right) - \mathbf{F}(r, rz) = 4\pi^2 \ln \frac{z + \sqrt{(1/r)^2 + z^2}}{z + \sqrt{1 + z^2}}$$

from which the relation (9) is obtained.

A1. The derivation of (A3)

Integrating with variable R of (A1) yields [10]

$$\mathbf{F}_\infty(r, z) = 2\pi z \int_0^r dt \int_0^{2\pi} \frac{t \cos^2 \theta d\theta}{\left(t^2 \sin^2 \theta + z^2\right)\sqrt{t^2 + z^2}}$$

The result of integration according to variable t is

$$\mathbf{F}_\infty(r, z) = 2\pi \int_0^{2\pi} \frac{\cos\theta}{\sin\theta} arctg\left(\frac{\sqrt{t^2 + z^2}}{z} tg\theta\right) d\theta \Bigg|_{t=0}^{t=r} \tag{A6}$$

To get the integration according to θ, the substitution $u = \mathrm{tg}\ \theta$ is performed. The above expression is transformed as below:

$$\mathbf{F}_\infty(r, z) = 8\pi \int_0^\infty \frac{arctg\left(\frac{\sqrt{t^2 + z^2}}{z} u\right)}{u\left(1 + u^2\right)} du \Bigg|_{t=0}^{t=r}$$

and its integration gives [12]

$$\mathbf{F}_{\infty}(r,z) = 4\pi^2 \ln\left(1+\frac{\sqrt{t^2+z^2}}{z}\right)\Bigg|_{t=0}^{t=r} \tag{A7}$$

where (A3) is obtained.

APPENDIX B.

TABLE B1. THE VALUES OF FUNCTION **F** FOR $0 < r \leq 1$ AND $0 \leq z \leq 1$

r	*z*										
	0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
0.00	0	0	0	0	0	0	0	0	0	0	0
0.05	0	0.001231	0.002434	0.003584	0.004659	0.005639	0.006512	0.007269	0.007907	0.008427	0.008834
0.10	0	0.004944	0.009777	0.014395	0.018706	0.022637	0.026135	0.029166	0.031717	0.033795	0.035417
0.15	0	0.011203	0.022151	0.032606	0.042359	0.051242	0.059134	0.065962	0.071700	0.076359	0.079987
0.20	0	0.020117	0.039769	0.058520	0.075990	0.091876	0.105962	0.118121	0.128310	0.136557	0.142950
0.25	0	0.031849	0.062943	0.092577	0.120143	0.145156	0.167276	0.186311	0.202202	0.215007	0.224879
0.30	0	0.046620	0.092102	0.135386	0.175562	0.211918	0.243962	0.271425	0.294244	0.312529	0.326522
0.35	0	0.064731	0.127821	0.187752	0.243231	0.293259	0.337167	0.374612	0.405543	0.430152	0.448816
0.40	0	0.086575	0.170856	0.250731	0.324421	0.390583	0.448349	0.497311	0.537465	0.569136	0.592889
0.45	0	0.112676	0.222200	0.325694	0.420771	0.505677	0.579332	0.641295	0.691670	0.730987	0.760070
0.50	0	0.143727	0.283165	0.414436	0.534392	0.640800	0.732383	0.808729	0.870146	0.917471	0.951894
0.55	0	0.180664	0.355503	0.519318	0.668009	0.798812	0.910304	1.002227	1.075239	1.130633	1.170093
0.60	0	0.224780	0.441601	0.643492	0.825174	0.983336	1.116548	1.224918	1.309679	1.372787	1.416586
0.65	0	0.277904	0.544782	0.791238	1.010554	1.198971	1.355342	1.480501	1.576590	1.646500	1.693437
0.70	0	0.342734	0.669827	0.968484	1.230347	1.451548	1.631811	1.773275	1.879463	1.954536	2.002794
0.75	0	0.423452	0.823876	1.183642	1.492833	1.748413	1.952059	2.108102	2.222062	2.299753	2.346786
0.80	0	0.526995	1.018109	1.448897	1.809072	2.098641	2.323116	2.490247	2.608242	2.684927	2.727382
0.85	0	0.666005	1.271005	1.782137	2.193542	2.512979	2.752621	2.925015	3.041637	3.112510	3.146207
0.90	0	0.866875	1.614716	2.209172	2.664084	3.003085	3.248037	3.417138	3.525210	3.584321	3.604325
0.95	0	1.196409	2.105273	2.763726	3.239649	3.579542	3.815288	3.969900	4.060717	4.101204	4.102033
1.00	0	1.843901	2.820991	3.477900	3.934158	4.248542	4.457014	4.584212	4.648191	4.662743	4.638688

r	*z*									
	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00
0.00	0	0	0	0	0	0	0	0	0	0
0.05	0.009135	0.009340	0.009458	0.009500	0.009478	0.009401	0.009280	0.009121	0.008934	0.008725
0.10	0.036614	0.037424	0.037888	0.038050	0.037954	0.037639	0.037145	0.036506	0.035752	0.034911
0.15	0.082654	0.084446	0.085460	0.085795	0.085549	0.084815	0.083679	0.082219	0.080504	0.078594
0.20	0.147622	0.150733	0.152457	0.152975	0.152465	0.151093	0.149013	0.146364	0.143268	0.139833
0.25	0.232035	0.236739	0.239273	0.239927	0.238981	0.236701	0.233329	0.229083	0.224153	0.218705
0.30	0.336564	0.343053	0.346413	0.347074	0.345449	0.341926	0.336855	0.330550	0.323287	0.315303
0.35	0.462038	0.470394	0.474489	0.474929	0.472288	0.467102	0.459853	0.450968	0.440821	0.429730
0.40	0.609445	0.619609	0.624215	0.624079	0.619975	0.612608	0.602616	0.590556	0.576911	0.562095
0.45	0.779932	0.791670	0.796395	0.795183	0.789033	0.778855	0.765457	0.749543	0.731720	0.712501
0.50	0.974802	0.987658	0.991914	0.988946	0.980019	0.966268	0.948693	0.928157	0.905398	0.881039
0.55	1.195494	1.208750	1.211713	1.206110	1.193502	1.175274	1.152632	1.126609	1.098081	1.067781
0.60	1.443566	1.456185	1.456763	1.447417	1.430038	1.406276	1.377553	1.345079	1.309867	1.272763
0.65	1.720647	1.731227	1.728023	1.713584	1.690144	1.659632	1.623689	1.583699	1.540812	1.495981
0.70	2.028377	2.035104	2.026397	2.005254	1.974262	1.935626	1.891201	1.842536	1.790910	1.737371
0.75	2.368316	2.368939	2.352669	2.322953	2.282722	2.234439	2.180157	2.121573	2.060080	1.996809
0.80	2.741834	2.733658	2.707437	2.667039	2.615703	2.556119	2.490508	2.420693	2.348154	2.274089
0.85	3.149972	3.129891	3.091043	3.037648	2.973194	2.900552	2.822071	2.739660	2.654866	2.568926
0.90	3.593300	3.557876	3.503501	3.434643	3.354962	3.267441	3.174505	3.078113	2.979839	2.880938
0.95	4.071775	4.017365	3.944440	3.857584	3.760524	3.656285	3.547306	3.435551	3.322587	3.209653
1.00	4.584651	4.507572	4.413070	4.305696	4.189137	4.066373	3.939800	3.811336	3.682504	3.554502

TABLE B2. THE VALUES OF FUNCTION **F** FOR $0 < r \leq 1$ AND $z \geq 1$ ($1 \geq 1/z > 0$)

r	1/*z*									
	1.00	0.95	0.90	0.85	0.80	0.75	0.70	0.65	0.60	0.55
0.00	0	0	0	0	0	0	0	0	0	0
0.05	0.008725	0.008487	0.008208	0.007886	0.007519	0.007106	0.006647	0.006144	0.005600	0.005021
0.10	0.034911	0.033955	0.032836	0.031545	0.030075	0.028421	0.026584	0.024571	0.022395	0.020077
0.15	0.078594	0.076429	0.073899	0.070983	0.067664	0.063935	0.059797	0.055265	0.050367	0.045153
0.20	0.139833	0.135949	0.131419	0.126205	0.120282	0.113634	0.106265	0.098199	0.089489	0.080220
0.25	0.218705	0.212566	0.205423	0.197221	0.187918	0.177494	0.165952	0.153334	0.139719	0.125240
0.30	0.315303	0.306339	0.295942	0.284034	0.270557	0.255482	0.238817	0.220620	0.201005	0.180162
0.35	0.429730	0.417335	0.403006	0.386644	0.368173	0.347555	0.324802	0.299993	0.273283	0.244925
0.40	0.562095	0.545615	0.526640	0.505043	0.480730	0.453656	0.423838	0.391376	0.356474	0.319455
0.45	0.712501	0.691236	0.666856	0.639207	0.608178	0.573713	0.535838	0.494678	0.450486	0.403666
0.50	0.881039	0.854237	0.823651	0.789097	0.750447	0.707637	0.660700	0.609791	0.555215	0.497460
0.55	1.067781	1.034638	0.996997	0.954650	0.907445	0.855316	0.798302	0.736589	0.670538	0.600727
0.60	1.272763	1.232424	1.186839	1.135773	1.079057	1.016616	0.948502	0.874931	0.796321	0.713345
0.65	1.495981	1.447540	1.393081	1.332341	1.265134	1.191378	1.111136	1.024655	0.932413	0.835177
0.70	1.737371	1.679882	1.615585	1.544191	1.465496	1.379414	1.286016	1.185580	1.078646	0.966076
0.75	1.996809	1.929288	1.854161	1.771115	1.679927	1.580505	1.472927	1.357506	1.234838	1.105880
0.80	2.274089	2.195529	2.108566	2.012860	1.908171	1.794399	1.671633	1.540210	1.400788	1.254416
0.85	2.568926	2.478305	2.378492	2.269120	2.149931	2.020814	1.881866	1.733451	1.576281	1.411499
0.90	2.880938	2.777239	2.663571	2.539540	2.404867	2.259432	2.103335	1.936965	1.761086	1.576929
0.95	3.209653	3.091876	2.963368	2.823710	2.672598	2.509902	2.335722	2.150469	1.954956	1.750499
1.00	3.554502	3.421680	3.277385	3.121167	2.952700	2.771839	2.578684	2.373662	2.157630	1.931987

r	1/*z*										
	0.50	0.45	0.40	0.35	0.30	0.25	0.20	0.15	0.10	0.05	0.00
0.00	0	0	0	0	0	0	0	0	0	0	0
0.05	0.004413	0.003789	0.003160	0.002541	0.001951	0.001408	0.000931	0.000537	0.000243	0.000062	0
0.10	0.017649	0.015151	0.012635	0.010163	0.007804	0.005631	0.003722	0.002148	0.000972	0.000246	0
0.15	0.039691	0.034074	0.028417	0.022859	0.017553	0.012667	0.008373	0.004832	0.002188	0.000553	0
0.20	0.070515	0.060537	0.050490	0.040616	0.031192	0.022512	0.014881	0.008588	0.003889	0.000983	0
0.25	0.110087	0.094511	0.078830	0.063420	0.048710	0.035161	0.023245	0.013417	0.006076	0.001536	0
0.30	0.158360	0.135959	0.113410	0.091251	0.070095	0.050605	0.033461	0.019316	0.008748	0.002212	0
0.35	0.215282	0.184836	0.154194	0.124083	0.095331	0.068837	0.045525	0.026285	0.011906	0.003011	0
0.40	0.280785	0.241086	0.201141	0.161887	0.124400	0.089847	0.059433	0.034322	0.015548	0.003933	0
0.45	0.354796	0.304649	0.254203	0.204631	0.157280	0.113622	0.075178	0.043424	0.019675	0.004977	0
0.50	0.437228	0.375455	0.313327	0.252276	0.193948	0.140149	0.092755	0.053590	0.024287	0.006144	0
0.55	0.527986	0.453425	0.378454	0.304781	0.234377	0.169415	0.112158	0.064817	0.029381	0.007434	0
0.60	0.626965	0.538474	0.449519	0.362099	0.278538	0.201401	0.133377	0.077103	0.034959	0.008847	0
0.65	0.734048	0.630508	0.526449	0.424182	0.326400	0.236092	0.156407	0.090446	0.041019	0.010382	0
0.70	0.849110	0.729425	0.609169	0.490976	0.377928	0.273468	0.181237	0.104841	0.047560	0.012040	0
0.75	0.972016	0.835118	0.697597	0.562422	0.433086	0.313508	0.207858	0.120285	0.054583	0.013820	0
0.80	1.102620	0.947470	0.791646	0.638461	0.491835	0.356192	0.236261	0.136776	0.062086	0.015723	0
0.85	1.240767	1.066359	0.891223	0.719028	0.554135	0.401495	0.266434	0.154309	0.070068	0.017748	0
0.90	1.386295	1.191655	0.996232	0.804054	0.619941	0.449395	0.298366	0.172880	0.078529	0.019896	0
0.95	1.539032	1.323223	1.106571	0.893470	0.689210	0.499865	0.332046	0.192485	0.087468	0.022167	0
1.00	1.698797	1.460921	1.222133	0.987201	0.761893	0.552878	0.367460	0.213120	0.096884	0.024559	0

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