STABILIZING PERMANENT MAGNETIC LEVITATION BY DIAMAGNETIC MATERIALS

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Abstract

The Earnshaw's theorem prohibits the stable static levitation of permanent magnets in stationary fields with no energy input. However, by a proper placement of a diamagnetic material, the levitation is stabilized and two distinct suspension arrays (vertical and radial) are obtained. This compact implementations works without superconductors and may require no energy input. For the vertical stabilized configuration, this paper establishes the equilibrium and stability condition using an analytical procedure of magnetic field computation. An experimental model based on strong permanent magnets (NdFeB) and pyrolytic graphite validates these analytical results.

1. INTRODUCTION

As Earnshaw's theorem shows, [1], the stable levitation phenomena cannot take place in stationary fields. Due to their negative magnetic susceptibility, the diamagnetic materials behave differently [2]. They are repelled by static magnetic fields and can even stable levitate if the repulsion is strong enough to balance gravity [2, 3]. These substances can stabilize the unstable equilibrium of a permanent magnet suspension [4, 5]. A small permanent magnet can be stable levitated by a magnetic field source (which can also be a permanent magnet) using a diamagnetic sheet (vertical stabilization) or a hollow diamagnetic cylinder (radial stabilization) [5].

The application of this permanent magnet levitation can be found in very high-sensitive gravity sensors or in designing frictionless suspensions whose parameters (such rigidity) can by controlled by adjusting the field profile.

This paper analyzes both theoretically and experimentally the vertical stabilization of levitation for a particular setting that uses only permanent magnets. The study has a general character and can be extended for any other magnetic field source and levitated magnet. Section 2 bellows deals with theoretical issues regarding the stabilization of equilibrium while Section 3 shows the experimental results.

2. STABILIZING LEVITATION

If we place a permanent magnet within the field of a fixed lifter magnet, the balance of magnetic and gravity forces settles the equilibrium position. The classical result of Earnshaw [1] says this equilibrium is unstable.

If a **M** dipole magnet is imbedded in a field **B,** and **M** and **B** are parallel, then the potential energy U is:

$$U = -\mathbf{MB} + mgz = -MB + mgz, \tag{1}$$

where mgz is the gravitational energy. The magnet will align with the local field direction because of the torques and therefore the energy is only dependent on the magnitude B of the magnetic field. For a circularly symmetric field $\mathbf{B}(r, z)$, the equilibrium points will be on z-axis of the symmetry. Then the condition that $(z_0, 0)$ be an equilibrium point is:

$$\mathbf{F} = -\nabla U\big|_{\substack{z=z_0\\ r=0}} \Rightarrow \nabla B\big|_{\substack{z=z_0\\ r=0}} = -mg/M\,, \tag{2}$$

where **F** is the total force.

Our approach toward stabilizing this equilibrium is to place a diamagnetic material beneath the lifted magnet. In this case, in the relation for potential energy (1) a new term Cz^2 is added, which takes into account the influence of diamagnetic material. The stability conditions ask for a minimum value of energy, which means positive curvature of the energy function in every direction:

$$\partial^2 U/\partial z^2 > 0; \quad \text{vertical stability} \quad \partial^2 U/\partial z^2 > 0. \quad \text{radial stability} \tag{3}$$

To complete the problem, we express the magnitude of the magnetic field B in terms of its z-component $B_z(0,z)$ only. Taking into account that $\nabla\mathbf{B} = 0$ and $\nabla\times\mathbf{B}=0$, the following expression of $B(r, z)$ around B_0 derive [4]:

$$B(r,z) = B_0 + B_0^{'}(z - z_0) + \frac{1}{2}B_0^{''}(z - z_0)^2 + + \frac{1}{4}\left[\frac{B_0^{'2}}{(2B_0)} - B_0^{''}\right]r^2 + \cdots.$$

$$\text{Here } B_0^{'} = \left.\frac{\partial B_z(0,z)}{\partial z}\right|_{\substack{z=z_0\\r=0}}; B_0^{''} = \left.\frac{\partial^2 B_z(0,z)}{\partial z^2}\right|_{\substack{z=z_0\\r=0}}. \tag{4}$$

The potential energy can be now expanded using (4) as:

$$U = -M\left[B_0 + \left\{B_0^{'} - \frac{mg}{M}\right\}(z - z_0) + \frac{1}{2}B_0^{''}(z - z_0)^2 + \frac{1}{4}\left\{\frac{B_0^{'}}{(2B_0)} - B_0^{''}\right\}r^2 + \cdots\right] + C(z - z_0)^2. \tag{5}$$

From the equilibrium condition required by (2), we obtain the magnetic field gradient $B^{'}$ equal to mg/M; this means that the quantity in the first curly branches of (5) must go to zero. The conditions for vertical and horizontal stability asked by (3) relations are:

$$\frac{\partial^2 U}{\partial z^2} > 0 \Rightarrow D_v = C - \frac{MB_0^{''}}{2} > 0$$
$$\frac{\partial^2 U}{\partial r^2} > 0 \Rightarrow D_h = \frac{M}{4}\left\{B_0^{''} - \frac{B_0^{''2}}{2B_0}\right\} > 0. \tag{6}$$

For achieving a stable static levitation of the floating magnet, the above condition, called discriminants of stability D_v and D_h, must be simultaneous satisfied. Without diamagnetic materials, setting $C = 0$, we see that if the curvature of flux density of the field is positive and large enough to create horizontal stability: $B^{''} > (mg)^2/(2M^2B_0)$; $D_h > 0$, the magnet is unstable vertically: $D_v < 0$.

If we consider now the case when $B^{''} > 0$, and it is large enough to create horizontal stability ($D_h > 0$), adding a diamagnetic material in vertical direction a *vertical stabilized configuration* can be obtained.

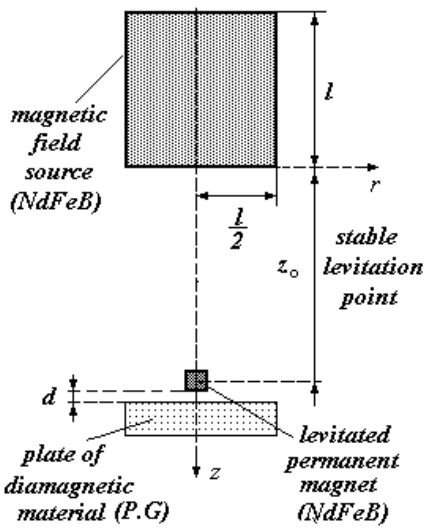

Figure 1. A compact implementation for the vertically stabilised levitation system.

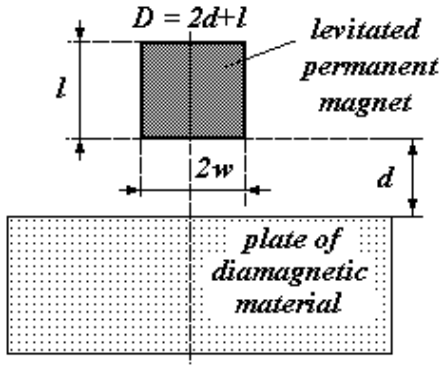

Fig.ure 2. Levitated magnet above the diamagnetic sheet.

In Fig. 1 a simple vertical configuration that uses a permanent magnet as the field source is shown. Fig. 2 magnifies the cylindrical levitated magnet of height l and radius w near the diamagnetic material. The fixed magnet is of strong magnetic materials i.e. with very high value for remanent magnetization (rare-earth materials). The diamagnetic material can be made from graphite, bismuth or pyrolytic graphite [7] (materials with great absolute values for magnetic susceptibility). For the floating magnet one uses rare-earth materials such as mixture of $Nd_2Fe_{14}B$, which could reach 1.2 T for remanent magnetization. All this restrictions are necessary to satisfy the equilibrium condition (2) and to fulfil the stability requirement (6) in the equilibrium point as well. Combining conditions for vertical stability using (6) we can write:

$$2C/M > B_0'' > m^2 g^2 / (2B_0 M^2). \tag{7}$$

The C term is proportional to the diamagnetic susceptibility (χ) and gets smaller if the gap d between the levitated magnet and diamagnetic body increases (Fig. 2). Its value can be determined by using the dipole approximation [4]: $C = 3\chi\mu_0 M^2/\pi D^5$, where $D = 2d + l$. We can see that a large gap or a weaker diamagnetic material requires a larger field B in the levitation position. The limits for separation d can be achieved taking into account (7) and the vertical dimension l of the floating magnet:

$$l < D < \left\{12\mu_0 B_0 M^3 |\chi| / \left[\pi (mg)^2\right]\right\}^{1/5}. \tag{8}$$

3. EXPERIMENTAL RESULTS

To verify the analytical results above studied, we chose for the magnetic field source and the levitated magnet the same material ($Nd_2Fe_{14}B$) with $B_{rem} = 1.2$ T and the following geometrical data: $L = 12$ mm and $l = 2w = 4$ mm. As a diamagnetic material we use pyrolytic graphite ($\chi = 450 \cdot 10^{-6}$), which is placed at $d = 0.5$ mm bellow the levitated magnet. This will lead us to $C = 3\chi\mu_0 M^2/\pi D^5 = 0.228$. The whole configuration is presented in Fig. 3. Fig.4 shows the levitated magnet in detail.

The magnetic field variation on the symmetric z-axis (Fig.1) outside the lifter magnet is given in [6]. Assuming a cubical geometry for the magnetic field source with remanent magnetization B_{rem} we have:

$$B(0,z) = \frac{B_{rem}}{2}\left(\frac{z+L}{\sqrt{(z+L)^2 + (L/2)^2}} - \frac{z}{\sqrt{z^2 + (L/2)^2}}\right). \tag{9}$$

The equilibrium equation $B' = -mg/M$ has one solution in this case, namely $z_0 = 50.4$ mm. In this point the stability conditions (6). The position of equilibrium point and the z-dependence of discriminants of stability D_v, D_h are presented in Fig. 5.

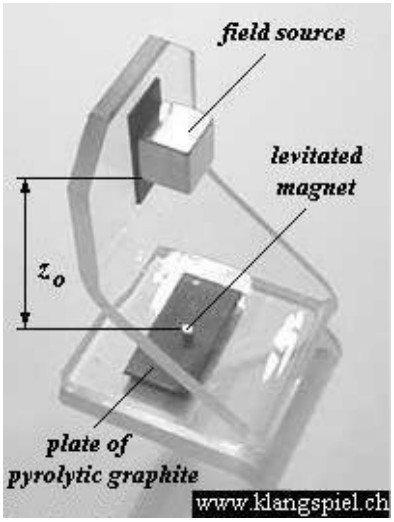

Figure 3. Vertically stabilised levitation configuration.

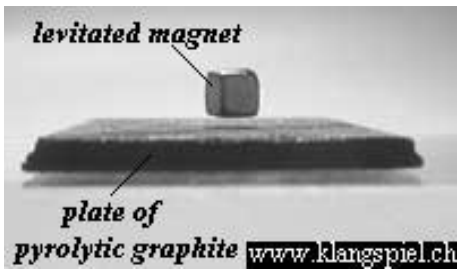

Figure 4. Detail of the levitated magnet.

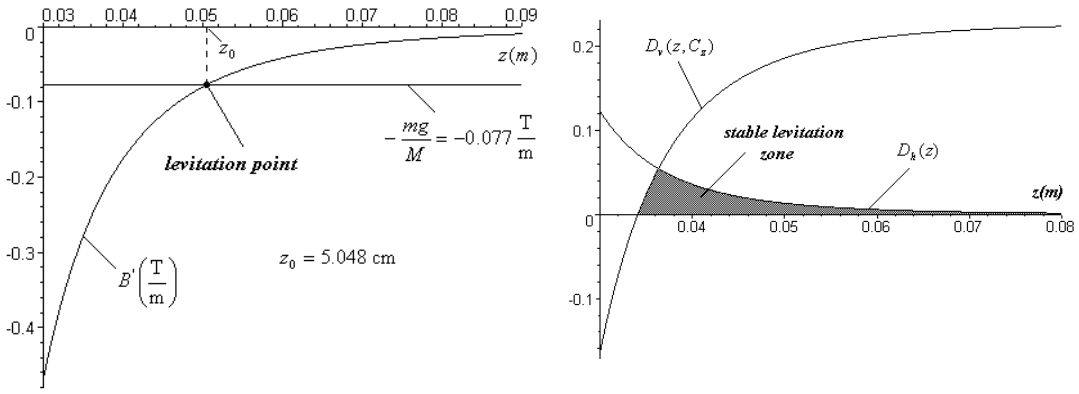

Figure 5. The equilibrium point (left) and discriminants variation for d = 0.5 mm (right).

From (8), the maximum value of the gap is d_{max} = 1.22 mm. This corresponds to C = 0.04, which gives a positive value for the discriminant D_v at levitation point z_0. Thus, the stability area is restricted by $d \in (0 - 1.22)$ mm. Using two diamagnetic materials one below and the other above the levitated magnet, this relatively narrow zone can be enlarged 1.1 units. The coordinate z_0, for the configuration given by Fig. 3, is 51.5 mm. This value is close enough to the one given by numerical computation 50.4 mm. This result validates our analytical study.

3. CONCLUSIONS

The paper shows that properly placed diamagnetic materials can stabilize vertically the levitation of a permanent magnet by a stationary field.

The stability area around the equilibrium position is theoretically inferred. We set up an experiment that used the pyrolytic graphite as a diamagnetic material as a permanent magnet as a lifter. The measurements compare well with the theoretical predictions. The levitation performances can be increased by using either diamagnetic material with higher absolute permeability, or permanent magnets with high value of remanent magnetization.

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