CONVERGENT GALERKIN MoM SOLVER TO THE EFIE FOR 2-D SMOOTH SCREENS

A. G. Tyzhnenko

Department of Mathematics
Kharkiv State Economic University
Prospect Lenina 9A
61059, Kharkiv, Ukraine
tyzhnenko@ksue.edu.ua

Abstract

The convergence of the Galerkin method of moments (GMoM) is proved in the familiar Hilbert space L_2 *for the electric field integral equation (EFIE) solver for two-dimensional smooth screens. Completeness of the orthogonal basis functions used permits one to obtain an exact relationship between residual and actual errors. A Fredholm MoM equation is obtained making use of a regularization procedure applied to the traditional MoM equation in the space of sequences. This issue permits one to increase substantially the accuracy of this new GMoM compared to the traditional one, and consider the adequacy problem for the traditional GMoM.*

1. INTRODUCTION

The Dirichlet problem can be solved with the aid of different methods. In this work, we consider the integral equation formulation of this problem making use of the single-layer potential representation of scattered field, which permits one to account automatically for the boundary conditions at the infinity. Such a statement leads to the necessity of solving the EFIE. To solve it, we use the GMoM on a complete orthogonal entire-domain basis. The main problem herein is that the convergence of the GMoM algorithm for this equation is not proven to now in the familiar Hilbert space (L_2). Due to this issue, one cannot guarantee the tending of approximate solution to an exact one when the residual error is diminished. This one raises many critical remarks addressed to the MoM [1-3]. However, despite the not proven convergence any appropriate MoM solution to the EFIE for screens is evidently adequate. To clarify this situation, we consider in this paper the convergence of the Galerkin MoM solver making use of transforming the traditional GMoM equation to a Fredholm 2nd kind one in the space of sequences. To this end, we use the special parameterization of a boundary curve, which permits us to use the original change of variable proposed first in [4] for eliminating the known singular behavior of a solution at the ends of a screen. All this permits us to seek a solution in the Hilbert function space. To prove

the convergence of the GMoM solution in such a space, we use the regularization procedure for the traditional MoM equation in the space of sequences, which is analogous to that of used in function space for static case, [5,6]. Such procedure permits us to transform the traditional 1st kind MoM equation to a Fredholm 2nd kind one. Regularization procedure in the space of sequences is none other than adding and subtracting of a special matrix (regularizer) to the MoM matrix. Such a trivial procedure yields completely nontrivial result for the GMoM because the matrix elements in this case are Fourier integrals, decay rate of which with harmonic number growth depends drastically on the smoothing properties of integrand. Then, a difference between the traditional MoM matrix and the regularizer represents a matrix, which elements are Fourier integrals with smoother integrand and then they decay more rapidly than that for both these matrices separately. It is precisely this issue that permits us to transform the traditional GMoM equation to a Fredholm one and to prove the convergence of the GMoM in the Hilbert function space.

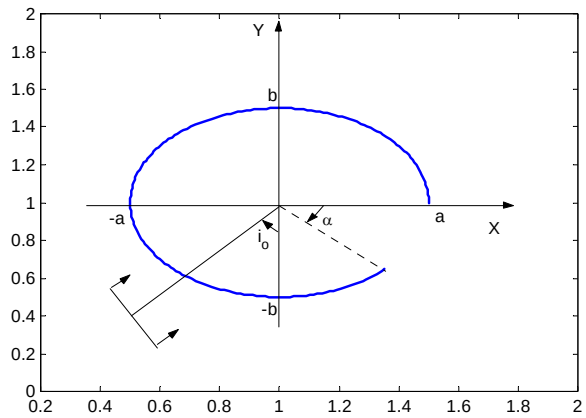

Figure 1. Geometry of α -degree-slot elliptical screen.

Besides the convergence problem, we also consider the actual error estimation with the use of known residual error. The completeness of basis function used permits us to obtain an analytic relationship between actual and residual errors in L_2 space. The obtained result permits one to avoid the using of more complicated Sobolev space when computing actual error as has been proposed in [7]. Besides that, practical merits of the new GMoM equation obtained are highlighted. The regularization procedure leads to smoother integrand in Fourier integrals, which represent matrix elements. This one increases significantly the accuracy of GMoM compared to the traditional method evaluation of matrix elements. Advantages of the new solver for the EFIE are

demonstrated by comparing results with that of traditional method. At the same time, having the rigorous solution we obtain an opportunity to verify the adequacy of traditional GMoM widely used in practice. This is done in the fifth section.

2. THE CONVERGENT GALERKIN MoM

Represent the electric field outside a screen as the sum of an incident field

$$E_0(x,y) = \exp(-jkx\sin i_0 - jky\cos i_0) \tag{1}$$

and a scattered field, which we will seek as a single-layer potential

$$E_{sc}(x,y) = \int_{\partial L} G(x,y,s')\varphi(s')ds' . \tag{2}$$

Here, the incidence angle i_0 is counted clockwise from the –Y direction (see Fig. 1), k is the free space wave number, and φ is an unknown function, which is proportional to a surface current. The function

$$G(x,y,s) = \frac{1}{4j} H_0^{(2)}(kR(x,y,s)) \tag{3}$$

stands for the free space Green's function with

$$R(x,y,s) = \sqrt{(x - x(s))^2 + (y - y(s))^2}$$

and

$$x = x(s), y = y(s), \;\; s \in [0, L] \tag{4}$$

being a C^∞-parameterization of a smooth boundary ∂L of the length L of a screen. Let us go to a new parametrization using a new variable

$$\sigma = -1 + 2s/L, \quad \sigma \in [-1,1]. \tag{5}$$

The scattered field (2) can be represented then as

$$E_{sc}(x,y) = \frac{L}{2}\int_{-1}^{1} G(k\overline{R}(x,y,\sigma'))\overline{\varphi}(\sigma')d\sigma' \tag{6}$$

with

$$\overline{R}(x,y,\sigma) = \sqrt{(x - \overline{x}(\sigma'))^2 + (y - \overline{y}(\sigma'))^2} , \tag{7}$$

new parametrization of a boundary ∂L

$$x = \overline{x}(\sigma) = x(0.5L + 0.5L\sigma), \; y = \overline{y}(\sigma) = y(0.5L + 0.5L\sigma) , \tag{8}$$

and new unknown function $\overline{\varphi}(\sigma) = \varphi(0.5L + 0.5L\sigma)$. The Dirichlet condition yields the EFIE

$$0.5L\int_{-1}^{1} G(k\overline{R}(\sigma,\sigma'))\overline{\varphi}(\sigma')d\sigma' = F(\sigma), \quad \sigma \in [-1,1] \tag{9}$$

when using (6). Here, $F(\sigma) = -E_0(\overline{x}(\sigma), \overline{y}(\sigma))$ and

$\overline{R}(\sigma,\sigma') = \sqrt{(\overline{x}(\sigma) - \overline{x}(\sigma'))^2 + (\overline{y}(\sigma) - \overline{y}(\sigma'))^2}$.

It is known that the solution of (9) has the singular behavior $\sim|1-\sigma^2|^{-1/2}$ at the end points $\sigma = \pm 1$. This issue prevents to seek a solution in familiar Hilbert space L_2. This obstacle can be circumvented with the aid of special change of variable in (9) proposed in [4]: $\sigma' = -\cos t'$, $t' \in [0,\pi]$. Using this procedure, the equation (9) can be written as

$$\int_0^{\pi} H_0^{(2)}(k\widetilde{R}(t,t'))\psi(t')dt' = f(t), \quad t \in [0,\pi] \tag{10}$$

with

$$\widetilde{R}(t,t') = \sqrt{(\widetilde{x}(t) - \widetilde{x}(t'))^2 + (\widetilde{y}(t) - \widetilde{y}(t'))^2}\ , \tag{11}$$

new unknown function

$$\psi(t) = \frac{L}{8j}\overline{\varphi}(\cos t)\sin t\,, \tag{12}$$

new parametrization

$$x = \widetilde{x}(t) = \overline{x}(\cos t), \quad y = \widetilde{y}(t) = \overline{y}(\cos t)\,, \tag{13}$$

and new right-hand side (RHS)

$$f(t) = F(\cos t)\,. \tag{14}$$

It is clear that the new unknown function (12) is nonsingular now at points $t = 0,\pi$ despite the mentioned singular behavior of the surface current function $\overline{\varphi}$. The scattered field can be written then as

$$E_{sc}(x,y) = \int_0^{\pi} H_0^{(2)}(k\widetilde{R}(x,y,t'))\psi(t')dt' \tag{15}$$

with

$$\widetilde{R}(x,y,t) = \sqrt{(x - \widetilde{x}(t'))^2 + (y - \widetilde{y}(t'))^2}\ .$$

Summarizing, we eliminate the singular behavior of a solution to the EFIE for screens and this procedure leads automatically to a new domain of integral equation, viz., $[0,\pi]$. Now we can seek a solution of (10) in L_2, using the Fourier representation of unknown function

$$\psi(t) = \sum_{n\geq 0} a_n \cos nt\,. \tag{16}$$

The known procedure yields the traditional GMoM equation in the space of sequences

$$Ka = f \tag{17}$$

with

$$K_{mn} = \int_0^{\pi} \cos mt dt \int_0^{\pi} H(t,t') \cos nt' dt', \tag{18}$$

$$f_m = \int_0^{\pi} f(t) \cos mt dt, \tag{19}$$

$$H(t,t') = H_0^{(2)}(k\tilde{R}(t,t')), \tag{20}$$

Prove that this equation can be transformed to a Fredholm one with the aid of equivalent transformations making use of special operator (regularizer) $\overset{\circ}{K}$, which has the following matrix elements:

$$\overset{\circ}{K}_{mn} = \int_0^{\pi} \cos mt dt \int_0^{\pi} \overset{\circ}{H}(t,t') \cos nt' dt',$$

with

$$\overset{\circ}{H}(t,t') = -\frac{2j}{\pi} \ln |2e^{-1}(\cos t - \cos t')|. \tag{21}$$

These matrix elements can be evaluated analytically making use of the known integral [4, 8]

$$-\frac{1}{\pi} \int_{-\pi}^{\pi} \ln |2e^{-1/2} \sin \frac{t-t'}{2}| e^{jnt'} dt' = \rho_n e^{jnt} \tag{22}$$

with $\rho_n = (\delta_{0n} + |n|)^{-1}$ and δ_{mn} being the Kroneker delta. With the aid of (22), we can obtain

$$\overset{\circ}{K}_{mn} = \pi j \frac{1+\delta_{0m}}{\delta_{0m}+m} \delta_{mn}. \tag{23}$$

Using the operator $\overset{\circ}{K}$, we can transform the traditional GMoM equation (17) as follows:

$$\overset{\circ}{K} a + Ta = f \tag{24}$$

with

$$T = K - \overset{\circ}{K} \tag{25}$$

having the matrix elements

$$T_{mn} = \int_0^{\pi} \cos mt dt \int_0^{\pi} P(t,t') \cos nt' dt' \tag{26}$$

Here, the function

$$P(t,t') = H(t,t') - \overset{\circ}{H}(t,t') \tag{27}$$

is nonsingular, and more, $P \in C^{\infty}(\Omega \times \Omega)$ with $\Omega = [0,\pi]$ for smooth boundary of the mentioned properties. This one can be easily seen from the following representation:

$$P(t,t) = 1 - \frac{2j}{\pi}\left(1 + C - 2\ln 2 + \ln ka\right) - \frac{2j}{\pi} \ln \frac{\Lambda(\cos t)}{a} \tag{28}$$

where $\Lambda(\sigma) = (\dot{\bar{x}}^2 + \dot{\bar{y}}^2)^{1/2}$ with pointed quantities being the derivatives of (8) with respect to σ, C = 0.577… the Euler constant, and a being a half of maximum size of a scatterer. For $\partial L \in C^{\infty}$ $\Lambda \geq \varepsilon > 0$, and then $P(t,t) \in C^{\infty}(\Omega)$ analogously to the static case [4]. Such behavior of the function P permits us to integrate by parts in (26) that, in turn, permits one to prove the decay rate of matrix elements T_{mn} as being $O(m^{-2}n^{-2})$ with harmonic numbers (m and n) increasing. At the same time, both the traditional GMoM matrix elements (K_{mn}) and auxiliary matrix ones ($\overset{\circ}{K}_{mn}$) decay separately only as $O(m^{-1}n^{-1})$. This is the point that permits us to transform the traditional GMoM equation (17) to a Fredholm one. To this end, let us apply the operator S^{-1} with matrix elements

$$S_{mn}^{-1} = (1+m)\delta_{mn} \tag{29}$$

to both sides of equation (24). Then, it becomes

$$S^{-1}\overset{\circ}{K}a + S^{-1}Ta = S^{-1}f \tag{30}$$

which is non other than a Fredholm equation. It is worthwhile to note that one can also use the inverse operator $\overset{\circ}{K}^{-1}$ instead of S^{-1}. It is just simpler from theoretical point of view but gives larger condition number when solving this equation numerically. Prove that the equation (30) is Fredholm. Using the decay rate of T_{mn} as being $O(m^{-2}n^{-2})$ and asymptotic behavior of S_{mn}^{-1} as (29), we can see that $(S^{-1}T)_{mn} = O(m^{-1}n^{-2})$ and then, the operator $S^{-1}T$ is compact in l_2. We can easily

see that the operator $S^{-1}\overset{\circ}{K}$ is Fredholm in l_2 as well because its matrix elements can be written as

$$(S^{-1}\overset{\circ}{K})_{mn} = \pi j(1+\delta_{0m})\delta_{mn} + \pi j(1+\delta_{0m})\frac{1-\delta_{0m}}{\delta_{0m}+m}\delta_{mn}.$$

Then, the first matrix corresponds to an invertable operator, and the second one corresponds to a compact one. Further, according to [9], for existence of a solution to the equation (30) the RHS vector $S^{-1}f$ have to be square summable. Prove that it is really so. Integrating by parts in (19), one can obtain that asymptotically $f_m = O(m^{-2})$ and then, using (29), $(S^{-1}f)_m = O(m^{-1})$. Because of that, $S^{-1}f \in l_2$. All this proves that the equation (30) is Fredholm and has a unique solution if the determinant of the system is nonzero. In this case, as has been proved in [10], $a \in l_2$ and the approximate solution

$$\psi_N(t) = \sum_{n=0}^{N} a_n \cos nt \tag{31}$$

converges to an exact solution of integral equation (10) in L_2. The uniqueness of a solution is guaranteed by the determinant of K_{mn}, and hence of $(S^{-1}\overset{\circ}{K}+S^{-1}T)_{mn}$, being nonzero. Prove this issue. As has been proved in [11], the homogeneous integral equation (10) has trivial solutions only. Because of that, the corresponding integral operator has no eigenvalue that equals to zero. Further, because of the spectrum of integral operator in (10) coincides with that of its discretized counterpart (17), and hence (30), if we use the Galerkin method [10], the determinant of the system in (17), and hence in (30), is nonzero. All this proves the convergence of the GMoM solution to the EFIE in L_2.

3. ACTUAL ERROR

One of the main problems of the EFIE solving procedure is the estimation of an actual error of an approximate solution with the aid of residual error computed in L_2. The latter stands for the boundary condition's error and can be easily computed in practice. This problem has been solved in [7] in the energy Sobolev space. We consider this problem in the familiar L_2 space. The completeness of basis functions used

permits us to obtain a rigorous relationship between actual and residual errors. To this end, let us write down an actual error as a function

$$\varepsilon(t) = \psi(t) - \psi_N(t)\,. \tag{32}$$

One can easily see that this function satisfies the integral equation

$$\int_0^{\pi} H(t,t')\varepsilon(t')dt' = d_N(t), \quad t \in \Omega \tag{33}$$

where

$$d_N(t) = f(t) - \int_0^{\pi} H(t,t')\psi_N(t')dt' \tag{34}$$

is a residual error function. Let us derive the actual error function from (33). This equation is exactly the same as (10) except the RHS. As is clear from the above, the developed GMoM procedure for solving the equation (33) converges if the RHS function (34) is continuous on Ω. It is really so. The first addend in (34) is obviously continuous. The second one is continuous as well because it is a single-layer potential. Hence, we can use the abovementioned technique to obtain a convergent GMoM solution for the error function

$$\varepsilon(t) = \sum_{n\geq 0} \alpha_n \cos nt \tag{35}$$

in L_2. As is clear from (30), the corresponding solution in l_2 can be written then as

$$\alpha = (S^{-1}\overset{\circ}{K} + S^{-1}T)^{-1} D \tag{36}$$

where vector D has the coordinates

$$D_m = (1+m)\int_0^{\pi} d_N(t)\cos mt dt\,. \tag{37}$$

Because $d_N(t)$ is continuous and its derivative is integrable, one can easily prove by integrating by parts that integral in (37) decays asymptotically as m^{-2} with m. Then, $D_m = O(m^{-1})$ and hence, $D \in l_2$. This is sufficient for the GMoM convergence in L_2, and this one permits us to use the Parseval equality

$$\| \varepsilon \|_{L_2} = \| \alpha \|_{l_2}\,,$$

which permits one to relate the actual and residual errors in the form

$$\| \psi - \psi_N \|_{L_2} = \| (S^{-1}\overset{\circ}{K} + S^{-1}T)^{-1} D \|_{l_2}\,, \tag{38}$$

if we account for (32), (36), and (37).

4. COMPUTATIONAL FEATURES

We use in this work an elliptic screen with α-degree slot (Fig. 1) for numerical calculations. For this boundary, the chosen parametrization (13) has the following form:

$$\tilde{x}(t) = a\cos 0.5\beta(1-\cos t); \quad \tilde{y}(t) = b\sin 0.5\beta(1-\cos t) \tag{39}$$

with $\beta = 2\pi - \alpha\pi/180$ and α being the angle measure of a slot (Fig. 1). Further, the relative boundary condition's error e_N is computed as the Euclidean norm of residual error for the (N+1)-term approximation (ψ_N) to an exact solution divided by $\| E_0 \|_2$ versus progressively greater value of the grid resolution $g_r = \lambda / h$ (a number of sub-intervals or points per wavelength) with h the grid spacing. The latter parameter, g_r, has been proposed in [12] instead of the number of subintervals (n) and has, to our opinion, an insightful physical meaning. When using the GMoM, a truncation number N is related with n as $N = [n/2]$, to support the needed accuracy when evaluating peripheral MoM matrix elements. The number (n) of subintervals, which is used for approximate evaluation of MoM matrix elements, is derived from the formula: $n = [Lkg_r / 2\pi]$ with $[\cdot]$ being the integer part of a number.

To compare different numerical algorithms in terms of total computational labor (TCL), we estimate this notion as numeric quantity. The TCL can be estimated as a ratio of the CPU time, $t_{CPU}(ka, n)$, of tested algorithm enacted on given meshing (n) to that of a standard one with the same meshing. As such a standard algorithm, we use the (2N+1)-terms approximation to the MFIE solution with the aid of GMoM on $\{e^{jnt}\}_{|n|\le N}$ basis for scattering of an E-polarized plane wave by a circular metallic cylinder of the same electric size (ka) and meshing (n). The CPU time of this standard algorithm we denote as $t_{ST}(ka, n)$. Then, the TCL can be estimated numerically as

$$tcl = t_{CPU}(ka, n) / t_{ST}(ka, n). \tag{40}$$

Abovementioned estimations of the main characteristics of a solution permit one to compare different numerical methods independently from their machinery realization. Besides these characteristics, we also use a monostatic RCS (MRCS) in unitless form

$$\sigma_b = \lim_{\rho\to\infty}(2\pi k\rho\,|E_{sc}|^2 / |E_0|^2) = 4\,|F(1.5\pi - i_0)|,$$

were $F(\varphi)$ is the scattering amplitude function (SAF). To verify the adequacy of the mentioned traditional GMoM, we solve in this work both the equation (17) and the transformed one (30). The matrix elements (18) of the traditional GMoM are evaluated in the Bogolyubov-Krylov approach [13] as

$$K_{mn} \approx h\sum_{i,k=1}^{n}\cos mt_i \cos mt_k \chi_{ik} \tag{41}$$

with

$$\chi_{ik} = \begin{cases} hH(t_i, t_k), & i \neq k \\ J_i, & i = k \end{cases}$$

and

$$J_i = \int_{t_i-d}^{t_i+d} H(t_i, t')dt' \tag{42}$$

with $d = h/2$. To evaluate the EFIE integral (42), we use the singularity extraction (SE) method [14] as the most accurate and fast. As the regularizer, we use here the function

$$\overset{\circ}{G}(t-t') = -\frac{2j}{\pi}\ln(ka\,|t-t'|)\,. \tag{43}$$

For this function, we need the following parameterization of the boundary ∂L:

$$\tilde{x} = a\cos\frac{\beta}{\pi}t; \quad \tilde{y} = a\sin\frac{\beta}{\pi}t, \quad t \in \Omega \tag{44}$$

instead of (39). The function H in (42) is defined then as (20) with corresponding replacements. Using (43) in the SE method, we obtain for the EFIE integral

$$J_i = \int_{t_i-d}^{t_i+d} \overset{\circ}{G}(t_i - t')dt' + \int_{t_i-d}^{t_i+d}\{H(t_i, t') - \overset{\circ}{G}(t_i - t')\}dt' \approx$$

$$-\frac{2j}{\pi}(\ln(kad) - 1 + \ln(L_i/2a))h + (1 - 2jC/\pi)h\,, \tag{45}$$

if we use the Bogolyubov-Krylov approach when evaluating the second integral. Here, $L_i = (\tilde{\dot{x}}^2(t_i) + \tilde{\dot{y}}^2(t_i))^{1/2}$, and a is a half of a maximum size of a scatterer. An improvement of this approach is possible, in principle, but leads to significant increasing of TCL and in fact is not effective. When solving the new GMoM equation (30), we use the same approach when evaluating the matrix elements, T_{mn} (26), as

$$T_{mn} \approx h^2 \sum_{i,k=1}^{n} \cos mt_i \cos nt_k P(t_i, t_k) . \tag{46}$$

Some improvement of this approach is not useful as well, as to our opinion. At last, the relative actual error is computed as

$$e_N^{(\psi)} = \| \psi - \psi_N \|_2 / \| \psi_N \|_2 \tag{47}$$

making use of (38).

5. NUMERICAL RESULTS

This section is devoted to research both the main characteristics of the new version of GMoM for EFIE and the adequacy problem for the mentioned traditional GMoM. As is known, the traditional GMoM procedure is coupled with two main problems: not proven convergence and condition number increasing with grid resolution (g_r). As has been shown in this work, the traditional GMoM equation (17) can be easily transformed to the Fredholm one (30). This issue ensures both the convergence of the used GMoM and the boundness of a condition number of GMoM system. On the other side, the regularization procedure used results in substantial increasing in accuracy of the new GMoM compared to the traditional one with the same TCL. General overlapping of the new and traditional solutions and some differences between them are demonstrated in Figure 2, where the scattering amplitude functions (SAF) are shown for the 90-degree-slot circular screen of *ka* = 20,

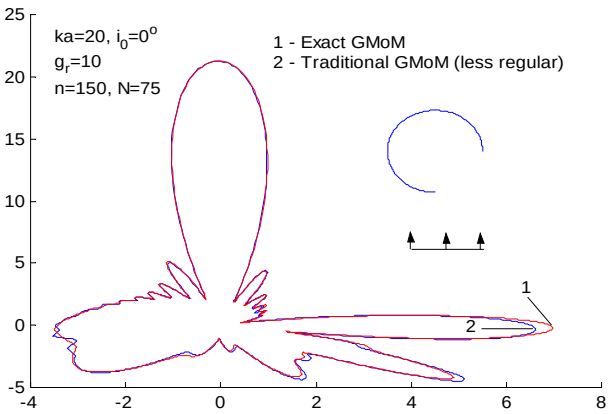

Figure 2. SAFs for 90°-slot circular screen of *ka* = 20, $i_0 = 0^\circ$, g_r =10 obtained bythe new (1) and traditional (2) methods.

$\alpha = 90^\circ, b/a = 1, i_0 = 0^\circ$, $2a \approx 6.4\lambda$, and $g_r = 10$. Small differences observed are

due to less accuracy of the traditional method, and they vanish with g_r growth. The scattering characteristics for this problem are listed more detailed in Table 1 for the new method and in Table 2 for the traditional one.

Table 1 New GMoM. 90°-slot circular screen, *ka*=20, $i_0 = 0^\circ$, *N*=[*n*/2].

g_r (n)	10 (150)	20 (300)	30 (450)	40 (600)	50 (750)
e_N	$6.2\cdot10^{-4}$	$3.9\cdot10^{-5}$	$7.9\cdot10^{-5}$	$2.5\cdot10^{-6}$	$1.0\cdot10^{-6}$
$e_N^{(\psi)}$	$3.4\cdot10^{-4}$	$1.5\cdot10^{-5}$	$2.5\cdot10^{-6}$	$6.9\cdot10^{-7}$	$2.5\cdot10^{-7}$
σ_b, dB	8.6594	8.6720	8.6733	8.6736	8.6737
cond	75.02	75.01	75.01	75.01	75.01
tcl	0.74	0.66	0.56	0.45	0.46

Table 2 Traditional GMoM. 90°-slot circular screen, *ka*=20, $i_0 = 0^\circ$, *N*=[*n*/2].

g_r (n)	10 (150)	20 (300)	30 (450)	40 (600)	50 (750)
e_N	0.0284	0.0137	0.0090	0.0067	0.0054
σ_b, dB	7.9804	8.3619	8.4743	8.5268	8.5578
cond	9.31	20.03	30.49	40.90	51.24
tcl	0.81	0.63	0.53	0.44	0.43

Table 3 New GMoM. Strip, *ka*=100, $i_0 = 45^\circ$, *N*=[*n*/2]..

g_r (n)	8 (254)	9 (286)	10 (318)	11 (350)	12 (380)	13 (412)	14 (444)
e_N	0.0086	0.0037	0.0011	$7.2\cdot10^{-4}$	$5.0\cdot10^{-4}$	$3.5\cdot10^{-4}$	$2.6\cdot10^{-4}$
$e_N^{(\psi)}$	$4.9\cdot10^{-4}$	$2.6\cdot10^{-4}$	$1.5\cdot10^{-4}$	$9.4\cdot10^{-5}$	$6.3\cdot10^{-5}$	$4.2\cdot10^{-5}$	$2.9\cdot10^{-5}$
σ_b, dB	$8.7\cdot10^{-4}$	0.0118	0.0113	0.0113	0.0113	0.0113	0.0113
cond	428.2	487.7	628.7	642.1	642.0	642.0	642.0
tcl	0.71	0.65	0.64	0.59	0.54	0.55	0.55

We can see herein substantially larger accuracy of the new method with practically the same TCL if we compare the boundary condition's errors (e_N). The actual error ($e_N^{(\psi)}$), which we can compute for the new method only, is less than the residual error (e_N) because the latter one is integrated along the boundary when computing $e_N^{(\psi)}$. As has been predicted, the condition number of the new GMoM equation is bounded in contrast to the traditional one. It is worthwhile, however, to note that for the traditional GMoM algorithm used a condition number is very small for screens, and its growing rate with g_r is small as well. This one does not give the sound for

incipience of inadequacy of a traditional solution due to the ill-posedness in sufficiently large range of electric size of a scatterer. To demonstrate this issue, we consider the scattering problem for scatterer of moderate size. As an example we use a strip of ka = 100 ($\alpha = 180^\circ, b/a = 0 i_0 = 45^\circ$, $2a \approx 32\lambda$).

Table 4 Traditional GMoM. Strip, ka=100, $i_0 = 45^\circ$, N=[n/2].

g_r (n)	8 (254)	9 (286)	10 (318)	11 (350)	12 (380)	13 (412)	14 (444)
e_N	0.0243	0.0225	0.0207	0.0192	0.0179	0.0167	0.0156
σ_b, dB	0.0858	0.0775	0.0709	0.0654	0.0612	0.0573	0.0540
cond	18.0	20.5	23.0	25.4	27.7	30.1	32.5
tcl	0.70	0.65	0.61	0.56	0.54	0.55	0.53

The scattering characteristics for this problem obtained by the new and traditional methods are gathered in Table 3 and 4 respectively. We can see herein that the residual error for both methods diminishes with g_r, but the new method gives much more rapid diminution. A condition number for the new method is bounded as well but its magnitude became much more large than for ka = 20. At the same time, a condition number for the traditional method remains not large despite the increasing with g_r.

6. CONCLUSION

In this paper, the convergence of the Galerkin MoM solution to the EFIE has been proved for smooth two-dimensional screens. To this end, a special technique for eliminating the singular behavior of a solution at the ends of a screen and a regularization procedure for eliminating the singular behavior of the EFIE kernel has been applied.

Completeness of the basis functions used has permitted one to obtain a rigorous relation between residual and actual errors. This relation permits one to compute an actual error simply by integration of a residual error along the boundary of a screen.

Adequacy problem of the traditional GMoM for the EFIE has also been considered. The closeness of solutions obtained by the traditional and new methods has been demonstrated for scatterers of moderate electric size. Despite the boundness of condition number for the new method, its magnitude increases substantially with dimensionality of a scatterer. On the other hand, that of traditional method remains very moderate despite general increasing. This state is very contrasted with that for closed

boundary and permits one to use in practice this version of traditional GMoM in wide range of scatterer dimensionalities without apprehension of ill-conditioning. At the same time, the new method gives much more accurate solution with theoretically proven convergence, which gives the opportunity to verify a traditional method if it is necessary.

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