HYBRID GALERKIN-RAYLEIGH METHOD: A UNIQUE SOLUTION FOR E-SCATTERING FROM 2-D ROUGHNESS ON A PEC GROUND PLANE

A.G. Tyzhnenko

Department of Mathematics
Kharkiv National Economic University
Prospect Lenina 9A
61059, Kharkiv, Ukraine
tyzhnenko@ksue.edu.ua

Abstract

A unique solution for scattering from a roughness (boss or indentation) on a PEC ground plane is obtained making use of a new method that combines the Galerkin MoM on entire-domain basis and the Rayleigh's method expansion of a solution in terms of the eigensolutions of Helmholtz equation in cylindrical coordinates. This method uses the dividing of upper space into two parts with the aid of semi-circumference that encloses the roughness. Numerical calculations are enacted on the roughness surface only and hence give a solution in a very economical way for any grazing. Uniqueness of a solution is proved and demonstrated. The two-dimensional case is considered. However, this method is applicable for the tree-dimensional scalar case as well.

1. INTRODUCTION

Scattering by a roughness of finite extent on a perfectly conducting (PEC) half-space is one of the main problems in electromagnetics since Lord Rayleigh’s time, when he has proposed the original method of images for constructing an equivalent scattering problem in free space and used it for semicircular bump. For this configuration, one can obtain a unique solution to the mentioned scattering problem making use of Rayleigh's method of images (MoI). For other configurations, one can use the combined field integral equation (CFIE) in free space to obtain a unique solution with the MoI. However, it is well known that the CFIE is not applicable for thin structures. Because of that, one cannot obtain a correct solution for scattering for example from a sea-wave on a ground plane in the PEC approximation of sea water if we use the MoI. More than that, this method is not applicable, in principle, for troughs in a PEC ground plane. The latter problem can be solved with the aid of widely used technique, the generalized network formulation proposed first in [1]. However, such a solution suffers from the problem of spurious resonances at the eigenfrequences of indentation. In order to obtain a unique solution to this problem, Asvestas and Kleinman [2] developed a set of coupled singular integral equations with the aid of

very complicated manipulations. Wood and Wood [3] applied these equations to the two-dimensional (2-D) cavity. It was claimed that these integral equations are free of cavity resonance. However, this affirmation was not proved. Numerical investigations show that it is really so, but mathematical complexity prevents to apply this method for arbitrary scatterers [4].

Theoretically, to obtain a unique solution to the scattering problem for any roughness on a PEC plane, we can use either the electric field integral equation (EFIE) or the magnetic field integral equation (MFIE) on the whole infinite interface. Recently, it has been developed a powerful method, the forward-backward (F/B, in authors' notation) iterative method solution to the MFIE on the infinite interface [5]. Unfortunately, this iterative method converges not for any roughness. In the case of convergence, one can obtain a very accurate approximate solution for not very low grazing angles. The last restriction is due to the fact of truncating the integral equation's domain to a finite one. This issue leads to theoretically incorrect statement when one uses the MFIE in fact for open screen. To diminish the inadequacy error, which arises from such a truncation, one has to add very large pieces of plane interface to both sides of a scatterer. However, it is clear that the less the grazing the larger pieces must be added to prevent the influence of the ends of a screen. Because of that, one cannot diminish grazing angle close to zero if an integral equation is defined on the whole infinite interval and then it is truncated to a finite one. Additionally, the computational labor in such cases increases dramatically with grazing angle diminishing if we want to weaken the inadequacy. This refers not only to [5] but also to analogous works [6, 7] and to all other ones used the truncation principle without any version of resistive tapering. The last method is also considered intensively in the literature with the goal of reducing the edge diffraction (e.g., see [8]). Other than, it has to be noted that at low grazing this method does not improve a truncated solution in an acceptable manner as well. However, scattering features at close-to-zero grazing are of paramount importance for studying radar returns from sea surface in the PEC approximation of sea water, for example.

A unique solution to the mentioned problem for a specific geometry, viz., a semicircular trough in a ground plane, can be obtained with the aid of Rayleigh's method expansion of a solution in terms of the eigensolutions of Helmholtz equation in cylindrical coordinates. Many authors solved this problem in such a manner (e.g.,

see [9-11]). However, all cited works dealt with ill-conditioned equations for solving for modal coefficients. Because of that, these solutions are valid for the low-frequency case only. In [12], the most general case of scattering from cavity-backed semicircular troughs was considered. In this work, authors obtained very complicated system of integral-integrodifferential equations of the first kind but solved them in the low-frequency case only without unicity and convergence analysis and error estimation. Recently, this canonical problem has been rigorously solved by the author in [13] for the E-case, and in [14, 15] for the H-case. To this end, the very same method that was used in [9-11] was modified so the equation for modal coefficients became Fredholm. However, for arbitrary roughness shape the mentioned uniqueness problem is not solved to date, as to author's knowledge, nether in the 2-D nor in 3-D case.

To solve the uniqueness problem in the 2-D case, we consider in this paper a novel hybrid Galerkin-Rayleigh method (HGRM) solution for the mentioned scattering problem valid for any grazing and any roughness. The method combines the Rayleigh's method expansion of a solution in terms of eigensolutions of the Helmholtz equation in the cylindrical coordinates (Rayleigh's series) and the method of moments (MoM). Worth noting that the HGRM solves the above problem in a very economical way due to the fact of enacting of a solution on the rough contour only. The HGRM yields a simple matrix equation for modal coefficients, which is analogous to a common MoM equation for screens and therefore has a unique solution [16].

In this paper, we apply the HGRM for solving the E-scattering from sea wave modeled as the “exponential wedge” with the purpose of comparing our results to that obtained in [5] for 5° grazing angle with the aid of mentioned F/B method. Besides, we also consider backscattering at low-grazing regime, viz., for 1° and 0.1° grazing angles, to demonstrate the advantages of the HGRM in solving low-grazing-angle scattering problems. In addition, we consider the scattering from semicircular trough in a PEC ground plane with the goal of testing our numerical code and demonstrating the applicability of the HGRM for troughs. To this end, we compare this method to that of [13], in which the rigorous solution to semicircular trough has been obtained. At last, we note that presented method can be applied for solving appropriate three-dimensional scalar scattering problems as well if we use specifically oriented spherical coordinate system, in which θ varies in the range of $0<\theta<\pi$ for free semi-space. Finally, note that we have considered here the E-case only leaving the H-

case for the next paper. This is caused by the use of completely other technique for solving the hypersingular integral equation of the 1st kind, which arises on the rough metal boundary in the H-case.

2. HYBRID GALERKIN-RAYLEIGH METHOD

Suppose a plane wave impinging upon a metallic ground plane with two-dimensional roughness of a finite extent that occupies the interval [-a, a] on the x-axes (Fig. 1). Let this wave has the z-component only, viz.,

$$E_0(\rho,\varphi) = e^{-jk\rho\sin(i_0-\varphi)+j\omega t} \tag{1}$$

where $k = \omega / c$ is the free space wavenumber and i_0 is an incidence angle counted counterclockwise from the y-axes. We omit henceforth the time dependence and

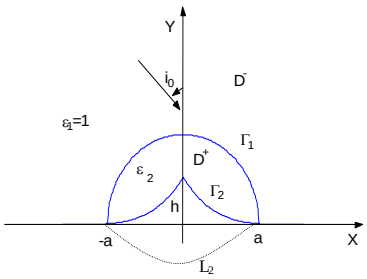

Figure 1. The geometry of scattering problem.

decompose the total electric field in D^- domain into the incident, reflected, and scattered field as follows:

$$E(\rho,\varphi) = e^{-jk\rho\sin(i_0-\varphi)} - e^{-jk\rho\sin(i_0+\varphi)} + \sum_{n>0} a_n H_n^{(2)}(k\rho)\sin n\varphi \tag{2}$$

In D^+, the total electric field is sought as a sum of the single-layer potential and the Rayleigh's expansion

$$E(\rho,\varphi) = \int_0^{\pi} H_0^{(2)}(k_2 R(x,y,t'))\psi(t')dt' + \sum_{n>0} b_n J_n(k_2\rho)\sin n\varphi\,, \tag{3}$$

where both (x,y) and (ρ,φ) represent the same observation point. We suppose here that D^- and D^+ are separated by a semi-circumference (Γ_1) of radius a (see Figure

1), and the dielectric constant ε_2 in the inward (D^+) domain may be complex number with $\mathrm{Jm}\varepsilon_2 \le 0$. Here, in the integral term we denote

$$R(x,y,t') = ((x - x_2(t'))^2 + (y - y_2(t'))^2)^{1/2} \tag{4}$$

with

$$x = x_2(t),\ y = y_2(t),\ t \in [0,\pi] \tag{5}$$

being a parametric equation of the boundary Γ_2 (or L_2) of the scatterer and $k_2 = k\sqrt{\varepsilon_2}$. Consider first the boundary conditions on Γ_1. In this case, it is convenient to represent all fields in the polar coordinates and use of

$$x = x_1(\varphi) = a\cos\varphi,\ y = y_1(\varphi) = a\sin\varphi \tag{6}$$

on Γ_1. Then, one can obtain the following function equations on $[0,\pi]$:

$$\sum_{n>0} a_n H_n \sin n\varphi - \sum_{n>0} b_n J_n \sin n\varphi - \int_0^{\pi} H_1(\varphi,t')\psi(t')dt' = E_0(\varphi)\,, \tag{7a}$$

$$k\sum_{n>0} a_n H_n^{'} \sin n\varphi - k_2 \sum_{n>0} b_n J_n^{'} \sin n\varphi - \int_0^{\pi} \tilde{H}_1(\varphi,t')\psi(t')dt' = \tilde{E}_0(\varphi) \tag{7b}$$

where

$$H_1(\varphi,t) = H_0^{(2)}(k_2 R_{21}(\varphi,t));\ \tilde{H}_1(\varphi,t) = \frac{\partial}{\partial a} H_0^{(2)}(k_2 R_{21}(\varphi,t))\,;$$

$$H_n = H_n^{(2)}(ka)\,,\ H_n^{'} = H_n^{(2)'}(ka)\,,\ J_n = J_n(k_2 a)\,,\ J_n^{'} = J_n^{'}(k_2 a)\,;$$

$$E_0(\varphi) = -e^{-jka\sin(i_0-\varphi)} + e^{-jka\sin(i_0+\varphi)}\,;$$

$$\tilde{E}_0(\varphi) = jk\sin(i_0-\varphi)e^{-jka\sin(i_0-\varphi)} - jk\sin(i_0+\varphi)e^{-jka\sin(i_0+\varphi)}$$

$$R_{21}(\varphi,t') = ((x_1(\varphi) - x_2(t'))^2 + (y_1(\varphi) - y_2(t'))^2)^{1/2}\,.$$

Here, the primes denote derivatives of the Bessel and Hankel functions with respect to the whole argument. Consider now the boundary condition on Γ_2 (L_2). Here, we use the rectangular coordinates and known relationship between polar and rectangular reference systems [17],

$$\rho = (x^2 + y^2)^{1/2}, \quad \varphi = \begin{cases} a\tan\dfrac{y}{x}, x \ge 0 \\ \pi + a\tan\dfrac{y}{x},\ x \le 0 \end{cases}. \tag{8}$$

We denote further

$$\rho_2(t) = (x_2^2(t) + y_2^2(t))^{1/2}, \quad \varphi_2(t) = \begin{cases} a\tan\dfrac{y_2(t)}{x_2(t)}, \ t \in [0, \dfrac{\pi}{2}] \\ \pi + a\tan\dfrac{y_2(t)}{x_2(t)}, \ t \in [\dfrac{\pi}{2}, \pi] \end{cases},$$

$$R_{22}(t,t') = ((x_2(t) - x_2(t'))^2 + (y_2(t) - y_2(t'))^2)^{1/2}$$

and write down the equation on Γ_2 (L_2) as

$$\int_0^{\pi} H_0^{(2)}(k_2 R_{22}(t,t'))\psi(t')dt' + \sum_{n>0} b_n J_n(k_2\rho_2(t))\sin n\varphi_2(t) = 0, \quad t \in [0,\pi]. \tag{9}$$

What is important in this equation is the use of relation (8) that yields a uniform discretization of the boundary if we use a uniform meshing of parameter *t*. Besides that, we do not need to find the polar equation of the boundary of a scatterer that is not an easy problem itself. Instead, we can use a parametric equation (5) of the boundary Γ_2 (L_2), deriving of which is not a problem. The first issue permits one to increase substantially the accuracy of a solution, and the second one to consider scatterers with complex boundaries. To obtain the solution of functional equations (7, 9), we represent the unknown function $\psi(t)$ as a Fourier series

$$\psi(t) = \sum_{n>0} c_n \sin nt \tag{10}$$

according to the Galerkin procedure. Then, multiplying the equations (7) by $\sin m\varphi$ and integrating from 0 to π, we obtain the following system for modal coefficients

$$\tilde{a}_m - \tilde{b}_m - \sum_{n>0} T_{mn}^{(1)} c_n = F_m^{(1)} \tag{11a}$$

$$\tilde{a}_m \frac{H_m'}{H_m} - \frac{k_2}{k}\frac{J_m'}{J_m}\tilde{b}_m - \sum_{n>0} S_{mn}^{(1)} c_n = F_m^{(2)} \tag{11b}$$

Additionally, multiplying the equation (9) by $\sin mt$ and integrating from 0 to π as well, we obtain the third matrix equation

$$\sum_{n>0} T_{mn}^{(2)} c_n + \sum_{n>0} E_{mn}\tilde{b}_n = 0. \tag{11c}$$

Here,

$$F_m^{(1)} = \frac{2}{\pi}\int_0^{\pi} E_0(\varphi)\sin m\varphi d\varphi, \ F_m^{(2)} = \frac{2}{\pi k}\int_0^{\pi} \tilde{E}_0(\varphi)\sin m\varphi d\varphi, \tag{12}$$

$$T_{mn}^{(1)} = \frac{2}{\pi}\int_0^{\pi} \sin m\varphi d\varphi \int_0^{\pi} H_1(\varphi,t')\sin nt' dt', \tag{13a}$$

$$S_{mn}^{(1)} = \frac{2}{\pi k}\int_0^{\pi} \sin m\varphi d\varphi \int_0^{\pi} \tilde{H}_1(\varphi, t')\sin nt' dt' , \tag{13b}$$

$$T_{mn}^{(2)} = \int_0^{\pi} \sin mt dt \int_0^{\pi} H_2(t,t')\sin nt' dt' , \tag{13c}$$

$$E_{mn} = \int_0^{\pi} \sin mt \sin n\varphi_2(t) \frac{J_n(k_2\rho_2(t))}{J_n(ka)} dt , \tag{13d}$$

$$H_2(t,t') = H_0^{(2)}(k_2 R_{22}(t,t')) ,$$

with

$$\tilde{a}_m = a_m H_m , \tilde{b}_m = b_m J_m . \tag{14}$$

The system obtained, (11), can be solved analytically with respect to modal coefficients $\tilde{a}_m$ and $\tilde{b}_m$. To this end, denote

$$\Delta_m = -1 + \frac{k_2}{k}\frac{H_m J_m^{'}}{H_m^{'} J_m} , \tag{15}$$

$$\Phi_{mn} = \frac{1}{\Delta_m}(T_{mn}^{(1)} - \frac{H_m}{H_m^{'}} S_{mn}^{(1)}) , \tag{16}$$

and

$$F_m^{(3)} = \frac{1}{\Delta_m}(F_m^{(1)} - \frac{H_m}{H_m^{'}} F_m^{(2)}) . \tag{17}$$

Then, these coefficients are as follows:

$$\tilde{b}_m = \sum_{n>0} \Phi_{mn} c_n + F_m^{(3)} , \tag{18a}$$

$$\tilde{a}_m = \tilde{b}_m + \sum_{n>0} T_{mn}^{(1)} c_n + F_m^{(1)} . \tag{18b}$$

Substituting (18a) into (11c) and denoting

$$D_{mn} = \sum_{k>0} E_{mk} \Phi_{kn} , \tag{19}$$

$$F_m^{(4)} = -\sum_{k>0} E_{mk} F_k^{(3)} , \tag{20}$$

we obtain the main matrix equation for $\{c_m\}$, viz.,

$$T^{(2)} c + Dc = F^{(4)} . \tag{21}$$

In this equation, $T^{(2)}$ is a common Galerkin MoM matrix for the EFIE on a non-closed contour (screen), D is a square summable matrix under certain condition we

consider a bit later. It is known that $T^{(2)}$ has highly expressed diagonal predominance and, then, permits one to obtain an adequate solution to the equation (21). Really, the equation (21) follows from the MoM procedure applied to functional equation (9), which has the same integral operator as the EFIE for a screen. The existence and uniqueness of a MoM solution to such an equation was proved in [17]. This one proves the uniqueness of a solution to considered scattering problem. Following further the work [18], one can transform this equation to a Fredholm one making use of both regularization and preconditioning procedures. Such an issue leads to substantial increasing in accuracy of an approximate solution and permits one to calculate an actual error of a solution for specific problem. We do not consider this method in presented work for brevity having in mind that it easily can be done making use of the mentioned work. Lastly, consider the square summability of matrix *D*. Because the matrix Φ (16) is compact, the compactness of *D* depends exclusively on compactness of the matrix *E*. The latter one is compact only for those scatterers for which $\rho(\varphi) \le a$. However, this restriction is not very hard because we always can increase the radius *a* of auxiliary semi-circumference to the needed value. These considerations prove completely the uniqueness of HGRM solutions. At last, we note that this method can be applicable to the tree-dimensional scalar case as well, if we use the spherical coordinate system so that metallic semi-space will be $y \le 0$ and hence $0 < \theta < \pi$ in the free semi-space. In this case, we can use the orthogonality properties of spherical functions and obtain modal coefficients on the semi-sphere analytically. The uniqueness of a HGRM solution can be proved in this case in very same manner as for 2-D case, if we account for the fact that the considered roughness inside the semi-sphere provides the same integral operator as the corresponding screen, for which the unique solvability has been proved in [19], for instance. Regardless dimensionality of the problem, we obtain in the HGRM an integral equation analogous to that for a screen of finite extent. Therefore, this integral equation one can solve making use of all of powerful methods developed to date.

3. COMPUTATIONAL FEATURES

The relative boundary condition's error e_N is computed as the Euclidean norm of residual error for the *N*-term approximation (ψ_N) to an exact solution to the equation

(9) versus progressively greater value of the grid resolution $g_r = \lambda / h$ (a number of subintervals or points per wavelength) with *h* the grid spacing. The latter parameter, g_r, has been proposed in [20] instead of the number of subintervals (n) and is very useful, as to our opinion. When using the GMoM, a truncation number *N* has been related with *n* as $N = [n/2]$, to support the needed accuracy when evaluating the peripheral MoM matrix elements. The number (*n*) of subintervals, which has been used for approximate evaluation of the MoM matrix elements, is derived then from the formula: $n = [Lkg_r / 2\pi]$ with $[\cdot]$ being the integer part of a number. The used relation between *N* and *n* corresponds approximately to the wide used relation: $h = 1/N$. In our work, this relationship is as follows: $h = \pi / 2N$. Besides these characteristics, we also use a monostatic RCS (MRCS) in unitless form

$$\sigma_b = \lim_{\rho \to \infty} (2\pi k\rho \,|\, E_{sc} \,|^2 \,/\, |\, E_0 \,|^2) = 4 \,|\, F(\varphi_b) \,|^2 \tag{22}$$

with $F(\varphi)$ being the amplitude scattering function and $\varphi = \varphi_b$ corresponds to backscattering. For comparison with [5], we also calculate the backscatter coefficient $B_H = 4 \,|\, F(\varphi_b) \,|$.

The matrix elements (13c) are evaluated in the Bogolyubov-Krylov approach [21] as

$$T_{mn}^{(2)} \approx h \sum_{i,k=1}^{n} \sin mt_i \sin nt_k \chi_{ik} \tag{23}$$

with

$$\chi_{ik} = \begin{cases} hH_2(t_i, t_k), & i \neq k \\ J_i, & i = k \end{cases}$$

and

$$J_i = \int_{t_i - d}^{t_i + d} H_2(t_i, t')dt', \tag{24}$$

where $d = h/2$. To evaluate the EFIE integral (24), we use the singularity extraction (SE) method [22] as the most accurate and fast. As the regularizer, we use here the function

$$\overset{\circ}{G}(t - t') = -\frac{2j}{\pi} \ln(ka \,|\, t - t' \,|) .$$

Using this function in the SE method, we transform the EFIE integral as

$$J_i = \int_{t_i-d}^{t_i+d} \overset{\circ}{G}(t_i - t')dt' + \int_{t_i-d}^{t_i+d} \{H_2(t_i, t') - \overset{\circ}{G}(t_i - t')\}dt' \approx$$

$$-\frac{2j}{\pi}(\ln(kad) - 1 + \ln(\sqrt{\varepsilon_2} L_i / 2a))h + (1 - 2jC/\pi)h\,. \qquad (25)$$

Here, $L_i = (\dot{x}_2{}^2(t_i) + \dot{y}_2{}^2(t_i))^{1/2}$, and C = 0.577… is the Euler constant. Above, we have used the Bogolyubov-Krylov approach when evaluating the second integral.

4. NUMERICAL RESULTS

What is the main problem of deriving a unique solution for scattering from rough metal surface for any grazing is the elimination of edge diffraction if we use a truncated integral equation. With grazing angle diminishing, it is necessary to increase substantially the leading and trailing flat sections that in turn increases dramatically the computational burden. To avoid this, one can use any version of resistive tapering across these sections as in [8]. However, such model methods need to be verified by some rigorous method. As such a method, one can use the HGRM proposed in this paper. This method permits one to obtain a unique solution to the mentioned problem for any grazing in a very economical way. Computational burden of this method is the same as for Galerkin MoM solution to the MFIE for the circular cylinder of the same electric size.

To demonstrate the uniqueness of HGRM solutions, we give in Figure 2 the behavior of matrix condition number via electric size, *ka*, for scattering from semi-elliptic boss with axes ratio *b/a* = 0.1 derived by both the Rayleigh's method of images making use of the MFIE (curve 1) and this method (curve 2). Both graphics are calculated with *ka* spacing 0.1. We can see herein practically unchanged matrix condition number for the HGRM in the vicinity of resonance frequencies in contrast to that for the MFIE.

To demonstrate the effectiveness of the new method, we consider the same scattering problem as in [5], where sea wave model called "exponential wedge" (*ew*) has been used. The profile of this wave is outlined by the function

$$f(x) = \begin{cases} \frac{h}{2}(1 + \cos\frac{\pi x}{a})\exp(-s\,|x|/h), & |x| \le a \\ 0, & |x| > a \end{cases} \qquad (26)$$

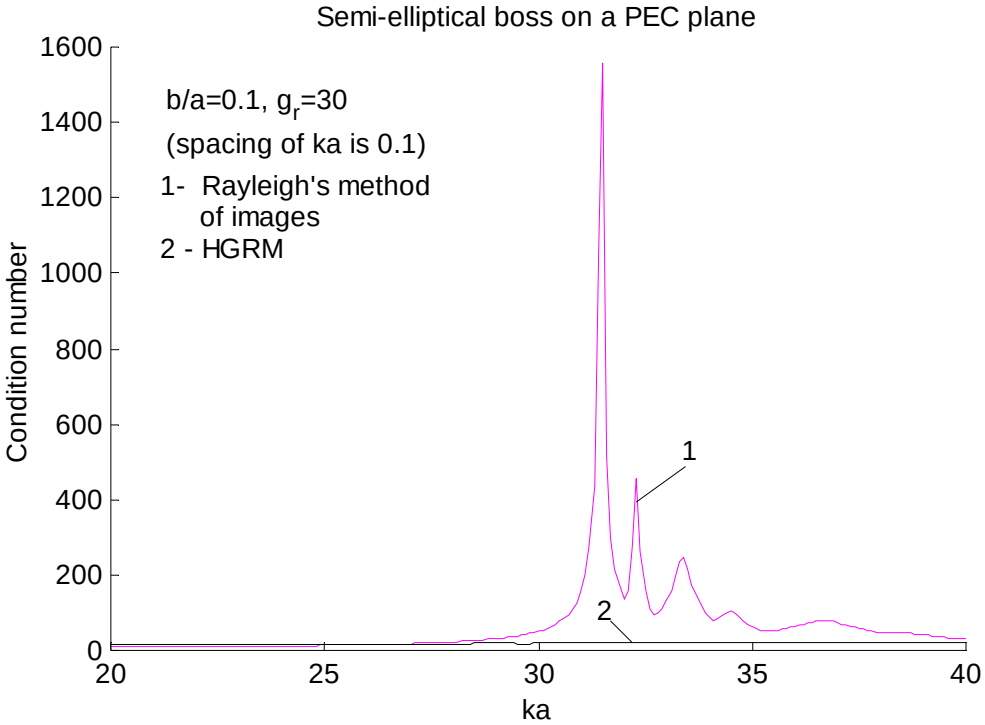

Figure 2. The semi-elliptic bump (*b/a* = 0.1) under normal incidence ($i_0 = 0^\circ$). The condition number of matrix system both for the Rayleigh's method of images with the MFIE (curve 1) and the HGRM (curve 2).

where *h* is the wave's height, *2a* is its length (Figure 1), and parameter *s* has been chosen equal to $\tan 22^\circ$ as in [5]. Scattering of the plane wave with wavelength 3cm impinging on a half space with this scatterer of the length 2 m and height from 0.1 cm to 12 cm at grazing angle 5° (incidence angle 85°) has been considered in mentioned paper. To obtain an adequate solution for such a low grazing, the authors needed to add 3 m flat pieces to both sides of scatterer so the total number of meshing points was 10001. For this meshing, approximately 0.5% relative error in boundary conditions has been achieved using a workstation of modest cost. Using new method presented in this paper, we obtain the same accuracy with 1462 points meshing used for integrals evaluation and 366 modal harmonics. The calculations have been provided with the aid of Pentium Pro 200 MHz computer. The comparative results are listed in Table 1 for two wave heights, 10 cm and 1 cm. It has to be noted that the HGRM solution is sufficiently accurate almost for 1048 points and 262 harmonics (error is less than 1%). What is of principle importance in the mentioned scattering problem is obtaining an accurate solution in the low grazing regime (grazing angle

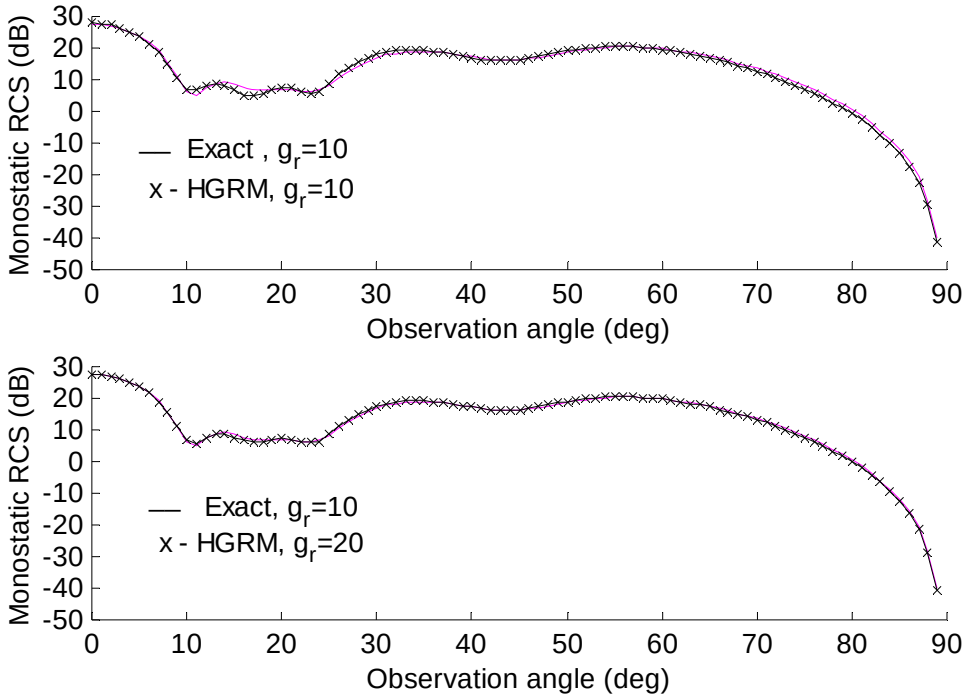

Figure 3. The monostatic RCS for scattering from the trough of *ka* = 10 obtained by the HGRM compared with the exact method [13] versus observation angle for various meshing (g_r).

less than 5°). This range is practically unachievable for methods that use the truncated integral equations. However, namely this range of grazing angles is of paramount importance for radar remote sensing and targets detection. The HGRM presented here is applicable for any grazing. This is demonstrated in Table 2, where the scattering features for the same *ew*-configuration of scatterer are listed for 1° and 0.1° grazing angles.

Because any roughness on sea surface is caused by gravitation, we cannot use a boss as a model of sea wave. Gravity wave is constituted from both promotions and indentations. To demonstrate the validity of HGRM for indentations as well, we consider the scattering from a semi-circular trough in a PEC plane with the goal of comparison our results with rigorous solution obtained in [13]. The latter solution will be regarded further as "exact" solution. These two solutions are used for computing the monostatic RCS (MRCS). The obtained results are shown in Figure 3. The MRCS (in dB) behaviors via incidence angle are presented herein for both the "exact" method (solid curve) with $g_r = 10$ and the HGRM ("x"-marked curve) with $g_r = 10$ (Figure 3a) and with $g_r = 20$ (Fig. 3b). As we can see, the corresponding MRCS values for these two methods are close enough even for $g_r = 10$ and those ones

obtained with the HGRM approach to corresponding "exact" values with g_r increasing. The boundary condition relative errors for these two methods are shown in Figure 4. The "exact" method has been enacted on meshing with $g_r = 10$ whereas the HGRM has been used with progressively increasing g_r, viz., $g_r = 10$ (Figure 4a), $g_r = 20$ (Figure 4b), $g_r = 30$ (Figure 4c), and $g_r = 40$ (Figure 4d) with the goal of demonstrating the convergence and validity of presented method. It is worthwhile to point out that the boundary condition's error is decreasing with grazing angle diminishing as we can see from these figures.

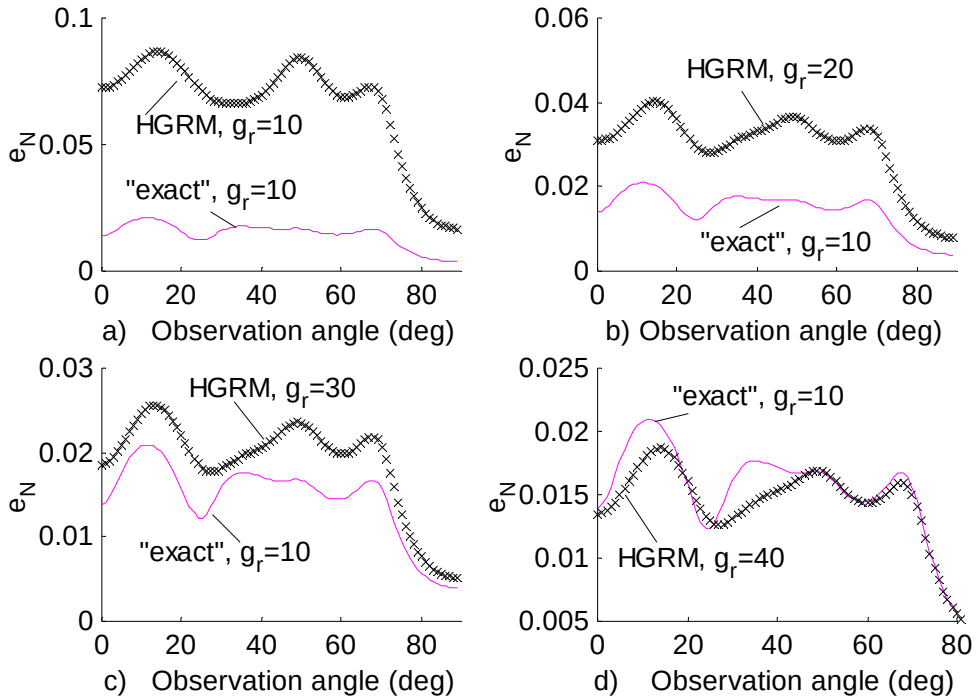

Figure 4. The boundary condition's relative error (e_N) for both the exact solution with g_r = 10 and the HGRM solution with progressively increasing grid resolution.

Table 1. Comparison of the HGRM and F/B for $i_0 85^\circ$ (5° grazing)

		HGRM					F/B
n		632	840	1048	1256	1462	10001
N		158	210	262	314	366	
h=10 cm	e_N	0.0545	0.0133	0.0069	0.0045	0.0031	~ 0.003
	$\lvert B_H \rvert$	0.2174	0.1837	0.1951	0.2015	0.2141	0.215
	σ_b, dB	-19.276	-20.736	-20.215	-19.933	-19.408	-19.371
h=1 cm	e_N	0.3098	0.0904	0.0461	0.0290	0.0280	~ 0.003
	$\lvert B_H \rvert$	0.0141	0.0160	0.0171	0.0180	0.0190	0.0191
	σ_b, dB	-43.057	-41.924	-41.337	-41.013	-40.445	-40.399

Table 2. The HGRM for low grazing.

h		10 cm	1 cm
$i_0 = 89^\circ$ 1° grazing	e_N	0.0059	0.0133
	$\lvert B_H \rvert$	0.0252	$7.23 \cdot 10^{-4}$
	σ_b, dB	-38.0088	-68.8290
$i_0 = 89.9^\circ$ 0.1° grazing	e_N	0.0060	0.0134
	$\lvert B_H \rvert$	$2.62 \cdot 10^{-4}$	$7.42 \cdot 10^{-6}$
	σ_b, dB	-77.6402	-108.8233

5. CONCLUSION

A rigorous method solution for scattering from any roughness of finite extent on a PEC ground plane has been presented. This method, the HGRM, is enacted on finite boundary of a scatterer only. In contrast to methods solution of the EFIE or MFIE on the whole infinite interface, the proposed method yields an accurate and fast convergent unique solution in a very economical way. Comparison of scattering from 67λ-"exponential wedge" obtained by well known "Forward-Backward" (F/B) method solution to the MFIE on the whole infinite interface to that obtained with this method exhibits approximately ten times reduction in complexity for our algorithm at the same level of accuracy. More than that, the presented method yields stable solution for any low-grazing angle in contrast to the F/B method, for instance. Therefore, the method presented gives a possibility to investigate effectively the backscattering from gravity sea waves at low grazing and many other related problems in two or three dimensions.

Summarizing we note that the method proposed in this paper solves in unique manner the scattering problem for any trough or boss on a ground plane in a rigorous and very economical way and finishes in a sense the long-time efforts to overcome this problem.

REFERENCES

[1] R. E. Harrington and J. R. Mautz, "A generalized network formulation for aperture problems", *IEEE Trans. Antennas Propag.*, vol. 24, pp. 870-873, Nov. 1976.

[2] J. S. Asvestas and R.E. Kleinman, "Electromagnetic scattering by intended screens," *IEEE Trans. Antennas Propag.*, vol. 42, pp. 22-30, Jan. 1994.

[3] W. D. Wood Jr and A. W. Wood, "Development and numerical solution of integral equations for electromagnetic scattering from a trough in a ground plane," *IEEE Trans. Antennas Propag.*, vol. 47, pp. 1318-1322, Aug. 1999.

[4] T. Van and A. W. Wood, "Finite element analysis of electromagnetic scattering from a cavity," *IEEE Trans. Antennas Propag.*, vol. 51, pp. 130-137, Jan. 2003.

[5] D. Holliday, L. L. DeRaad Jr, and G. J. St-Cyr, "Forward-Backward: a new method for computing low-grazing angle scattering," *IEEE Trans. Antennas Propag.*, vol. 44, pp. 722-729, May 1996.

[6] D. A. Kapp and G. S. Brown, "A new numerical method for rough-surface scattering calculations," *IEEE Trans. Antennas Propag.*, vol. 44, pp. 711-721, May 1996.

[7] M. R. Pino, L. Landesa, J. L. Rodriguez, F. Obeleiro, and R. J. Burkholder, "The generalized forward-backward method for analyzing the scattering from targets on ocean-like rough surfaces," *IEEE Trans. Antennas Propag.*, vol. 47, pp. 961-969, June 1999.

[8] J. C. West , "On the control of the edge diffraction in numerical rough surface scattering using resistive tapering," *IEEE Trans. Antennas Propag.*, vol. 51, pp. 3180-3183, Nov. 2003.

[9] M. K. Hinders and A. D. Yaghjian, "Dual series solution to scattering from a semicircular channel in a ground plane," *IEEE Microwave Guided Lett.*, vol. 1, pp. 239-242, Sept. 1991.

[10] T. J. Park, H. J. Eom, W.-M. Boerner, and Y. Yamaguchi, “TM scattering from a dielectric-loaded semi-sircular trough in a conducting plane,” Trans. IEICE Commun., vol. E75-B, no. 2, pp. 87-91, 1992

[11] T. J. Park, H. J. Eom, Y. Yamaguchi, W.-M. Boerner, and S. Kozaki, “TE plane wave scattering from a dielectric-loaded semi-circular trough in a conducting plane,” *J. Electromagn. Waves Applicat.*, vol. 7, no. 2, pp. 235-245, 1993.

[12] I. O. Vardiambasis, J. L. Tsalamengas, and J. G. Fikioris, “Plane wave scattering by slots on a ground plane loaded with semicircular dielectric cylinders in case of oblique incidence and arbitrary polarization,” *IEEE Trans. Antennas Propag.*, vol. 46, pp. 1571-1579, Oct. 1998.

[13] A. G. Tyzhnenko, “Two-dimensional TE-plane wave scattering by a dielectric-loaded semicircular trough in a ground plane,” *Electromagnetics*, vol. 24, pp. 357-368, July 2004.

[14] A. G. Tyzhnenko, “Two-dimensional scattering of H-wave from a dielectric loaded semicircular trough in a PEC ground plane," *Radiophysics and Electronics*, vol. 7, pp. 246-252, special issue, 2002 (in Russian).

[15] A. G. Tyzhnenko, "A unique solution to the 2-D H-scattering problem for a semicircular trough in a PEC ground plane," *J. Electromagn. Waves Applicat.*, 2005 (in press).

[16] E. Stephan and W. L. Wendland, "An augmented Galerkin procedure for the boundary integral method applied to two-dimensional screen and crack problems," Appl. Analysis, vol. 18, pp. 105-128, 1984.

[17] Courant R. *Differential and integral calculus*. Part II. Moscow: Nauka, 1970 (in Russian).

[18] A. G. Tyzhnenko, "Convergent Galerkin MoM solver for the EFIE in two dimensions," *Electromagnetics*, vol. 25, No. 3, 2005 (in press).

[19] E. P. Stephan, "Boundary integral equations for screen problems in R^3," *Integ. Equations Operator Theory*, vol. 10, pp.236-257, 1987.

[20] B. T. Nguyen, "On comparison of time-domain scattering schemes," *IEEE Antennas and Propag. Magazine*, vol. 39. pp. 99-102, Jan. 1997.

[21] L. V. Kantorovich and V. I. Krylov. *Approximate Methods of Higher Analyses*. 4th edition, New York: Wiley,1964.

[22] V. Sladek and J. Sladek, Eds. *Singular Integrals in Boundary Element Method*, Southampton, U.K.: Computational Mechanics Publications, 1998.