THE INFLUENCE OF ATMOSPHERIC TURBULENCE ON IMAGE TRANSFER BY SPATIALLY MODULATED LASER RADIATION

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Abstract

The influence of atmospheric turbulence (AT) on the functioning of a laser communication system for image transfer by transverse spatial modulation of the radiation is considered. The results from [1] concerning minimum aperture dimensions and the statistical characteristics of the optical signal intensity in the receiving aperture are used. Analytical expressions for the statistical characteristics of the intensity of noise spatial fluctuations caused by AT are found. The transfer function of analog optical processor for optimal filtration of the signal image in the receiving aperture from the additive mixture with the noise field is determined. Expressions have been found for the optimal filtration efficiency and the signal-to-noise rations at the filter input and output for different AT degrees. The theoretical conclusions have been used in numerical examples. Analysis of the investigation results is performed.

1. INTRODUCTION

Paper [1] shows a laser communication system for image transfer by transverse spatial modulation of the carrier wave. The optical channel of the system (Figure 1) is characterized with the well known from geometrical optics equation $\frac{1}{f}=\frac{1}{l}+\frac{1}{L}$, which assures wave-optic scale recovery of the pseudo-stochastic starting field

$E_s(u,v)$ in the aperture (α) of the receiving aperture. The minimum aperture dimensions (of the transmitting and receiving aperture) are analytically determined, and this assures adequate signal and spectral transformations in the optical channel of the system. Analytical expressions for the statistical characteristics of the intensity $I(x,y)$ of the signal optical field $E(x,y)$ in the receiving aperture have been found and a few relative calculations have been given.

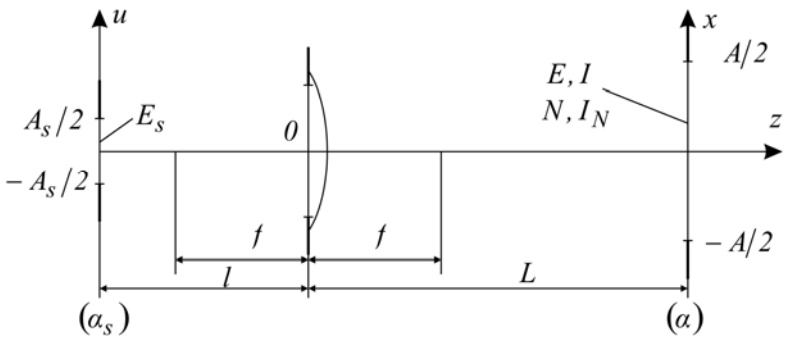

Figure 1. Equivalent scheme of the optical channel of a laser communication system

The cross-coherent signal field in the plane (α_s) is written in the following way:

$$E_s(u,v) = E_m S(u,v) rect\left(\frac{u}{A_s}, \frac{v}{A_s}\right), \tag{1}$$

where E_m is the amplitude of a plane laser wave, $S(u,v) \in [1/2 - t,\ 1/2 + t]$ is the normalized two-dimensional pseudo-stochastic signal, $t > 0$ is the maximal deviation of S from $\langle S \rangle$, A_s is the dimension of the opening of a square diaphragm in the plane (α_s). For the average value of the signal we have $\langle S \rangle = 1/2$ for $u,v \in [-A_s/2, A_s/2]$, and for its spatial variance – $\sigma_s^2 = t^2/9$ (according to the "3 σ" rule when the probability distribution of $S(u,v)$ is Gaussian [1, 2]). In this paper we will use the expressions from [1, 3]:

$$\gamma = \frac{2\pi m}{A_s}, \tag{2}$$

where γ is the module of the maximum spatial frequency in the Fourier spectrum of (1), $1 << m = A_s / \Lambda_{min}$, Λ_{min} is the dimension of the smallest details of the image (1),

$$\sigma_I^2 = E_m^4 \left(\frac{l}{L}\right)^4 \sigma_s^2 \left(2\sigma_s^2 + 1\right), \tag{3}$$

where σ_I^2 is the spatial variance of $I(x,y)$,

$$\Gamma_I(p,q) = \Gamma_I(0,0) \frac{(\gamma/3)^4}{\left[(\gamma/3)^2 + (L/l)^2 p^2\right]\left[(\gamma/3)^2 + (L/l)^2 q^2\right]}, \tag{4}$$

where p and q are the spatial frequencies in the connected to (α) frequency plane, $\Gamma_I(p,q)$ is the power spectrum of $I(x,y)$ and

$$\Gamma_I(0,0) = 4E_m^4 \left(\frac{l}{L}\right)^2 \sigma_s^2 \left(2\sigma_s^2 + 1\right)\left(\frac{3}{\gamma}\right)^2. \tag{5}$$

It is clear that with the distribution of the spatially modulated laser beam along a way of total length L there are parasitical cross-spatial amplitude fluctuations of that beam, which are due to the atmospheric turbulence (AT). We can consider that a two-dimensional noise field $N(x,y)$ is added (as a result from the AT) to the signal optical field $E(x,y)$ in the plane (α) of the receiving aperture, which reduces the reception quality. The tasks of this investigation are the following: to determine the statistical characteristics of the AT determined noise field $N(x,y)$ and its intensity $I_N(x,y)$ in the receiving aperture for the optical channel, shown on Figure 1; to determine the transfer function of an analog optical processor for optimal filtration of the signal intensity $I(x,y)$ from its additive sum with the intensity $I_N(x,y)$ of the turbulent spatial fluctuations; to estimate the effectiveness of the optimal filtration and the transformation of the signal-noise ratio in the filter according to the power of the AT.

2. STATISTICAL CHARACTERISTICS OF THE TURBULENT NOISE FLUCTUATIONS $N(x,y)$ AND THEIR INTENSITY IN THE RECEIVING APERTURE

We will consider that the stochastic field $N(x,y)$ of the turbulent spatial fluctuations is statistically homogeneous and has a gaussian probability distribution. We will analyze the influence of AT on the cross-spatial structure of the laser beam, using the so-called “heuristic theory” [4]. Alongside, we will have in mind the specification in [5] for the physical model of the optical properties of AT. This theory

has been chosen because the strict investigation of the spreading of the laser beam in a turbulent atmosphere by means of the theory of high-ordered statistical moments of the optical field [6] still cannot lead to practical useful analytical results [3, 7, 8, 9, 10].

The amplitude level [4] of $N(x,y)$ in the plane (α) will be defined with the relation

$$X(x,y)=\frac{N(x,y)}{E_0}, \tag{6}$$

where $E_0=\langle|E(x,y)|\rangle$ is the average module of the optical field in the aperture of the receiving aperture.

According to (1) and to $\langle S\rangle=1/2$ we have $\langle E_s(u,v)\rangle=E_m/2$; having in mind the expression for $E(x,y)$ in [1], we write $|E(x,y)|=(l/L)E_s(-u,-v)$; and this lead us to

$$E_0=\frac{E_m l}{2L}. \tag{7}$$

The heuristic theory gives us

$$\langle X\rangle=0 \tag{8}$$

and

$$\sigma_X^2=\frac{1}{3.\,2^{1/4}}C_n^2k^{7/6}L^{11/6}=0{,}28C_n^2k^{7/6}L^{11/6}, \tag{9}$$

where σ_X^2 is the variance of X, C_n^2 is structural constant of the turbulent fluctuations of the refraction coefficient of the atmosphere and $k=2\pi/\lambda$. The condition for use of (9) [4, 6] (according to the experimental results) is

$$C_n^2k^{7/6}L^{11/6}\le 1. \tag{10}$$

On the basis of (6) and (8) we can write

$$\langle N\rangle=0,\ \sigma_N^2=E_0^2\sigma_X^2, \tag{11}$$

where σ_N^2 is the variance of $N(x,y)$, as well as

$$I_N(x,y)=N^2(x,y)=E_0^2X^2(x,y). \tag{12}$$

(in (12) the multiplier $1/(2Z_w)$ has been dropped out, because it is not significant for the following analysis, where Z_w is the wave impedance of the environment).

From (12), having in mind (8) and (11), we find

$$\langle I_N \rangle = E_0^2 \left\langle X^2(x,y) \right\rangle = E_0^2 \sigma_X^2 = \sigma_N^2 \tag{13}$$

and

$$\sigma_{I_N}^2 = \left\langle \left[I_N(x,y) - \langle I_N \rangle \right]^2 \right\rangle = \left\langle N^4(x,y) \right\rangle - E_0^4 \sigma_X^4 . \tag{14}$$

From the Gaussian probability distribution of $N(x,y)$ and from (11) it follows

$$\left\langle N^4(x,y) \right\rangle = 3\sigma_N^2 = 3E_0^2 \sigma_X^2 . \tag{15}$$

With the help of (15) and (7) we present (14) as

$$\sigma_{I_N}^2 = 2E_0^4 \sigma_X^2 = \frac{1}{8} E_m^4 \left(\frac{l}{L} \right)^4 \sigma_X^4 \tag{16}$$

In order to express the autocorrelation function of $I_N(x,y)$ using that of $X(x,y)$ we use (12), (13) and write

$$B_{I_N}(\xi,\eta) = \left\langle \left[I_N(x,y) - \langle I_N \rangle \right] \left[I_N(x+\xi, y+\eta) - \langle I_N \rangle \right] \right\rangle = \tag{17}$$

$$= E_0^4 \left[\left\langle X^2(x,y) X^2(x+\xi, y+\eta) \right\rangle - \sigma_X^4 \right],$$

where ξ and η are the increments of x and y $(\xi = \Delta x, \eta = \Delta y)$.

From (6) we can conclude, that the Gaussian probability distribution of $N(x,y)$ is true for $X(x,y)$. Then we have

$$\left\langle X^2(x,y) X^2(x+\xi, y+\eta) \right\rangle = 2B_X^2(\xi,\eta) + \sigma_X^4$$

[2] and from (17) we find

$$B_{I_N}(\xi,\eta) = 2E_0^4 B_X^2(\xi,\eta) . \tag{18}$$

3. APPROXIMATION OF THE EXPRESSION FOR $B_X(\xi,\eta)$. POWER SPECTRUM OF $I_N(x,y)$

Heuristic theory leads to the relation

$$\rho_c = \sqrt{L\lambda} \tag{19}$$

for the correlation radius of the turbulent spatial fluctuations of the amplitude level [4]. The autocorrelation

$$B_X(\rho) = \langle X(r)X(r+\rho)\rangle \quad (\rho = |\vec{\rho}|)$$

can be approximated with the simple function

$$B_X(\xi,\eta) = \sigma_X^2\, e^{-2\frac{|\xi|+|\eta|}{\rho_c}} \tag{20}$$

Replacing of (7), (19) and (20) in (18) yields

$$B_{I_N}(\xi,\eta) = 0{,}125 E_m^4 \left(\frac{l}{L}\right)^4 \sigma_X^4\, e^{-4\frac{|\xi|+|\eta|}{\sqrt{L\lambda}}}. \tag{21}$$

In order to determine the power spectrum of $I_N(x,y)$ we will use the Wiener-Hinchin theorem [11] and the expressions (9), (21). We write

$$\Gamma_{I_N}(p,q) = \iint_{-\infty}^{\infty} B_{I_N}(\xi,\eta) e^{-jp\xi} e^{-jq\eta} d\xi d\eta = 0{,}98 \times \tag{22}$$

$$\times 10^{-2} E_m^4 \left(\frac{l}{L}\right)^4 C_n^4 k^{7/3} L^{11/3} \iint_{-\infty}^{\infty} e^{-4\frac{|\xi|+|\eta|}{\sqrt{L\lambda}}} e^{-jp\xi} e^{-jq\eta} d\xi d\eta,$$

where p and q are the spatial frequencies on the axes x and y. We make the integration in (22) and after a suitable transformation we reach to

$$\Gamma_{I_N}(p,q) = \Gamma_{I_N}(0,0) \frac{(16/L\lambda)^2}{\left(16/L\lambda + p^2\right)\left(16/L\lambda + q^2\right)} \tag{23}$$

where

$$\Gamma_{I_N}(0,0) = 0{,}178 E_m^4 l^4 C_n^4 L^{2/3} \lambda^{-4/3}. \tag{24}$$

4. ANALOG OPTICAL PROCESSOR FOR OPTIMAL FILTRATION OF THE SUM OF $I(x,y)$ AND $I_N(x,y)$

Let us suppose, that the signal optical image $I(x,y)$ and the turbulent noise optical intensity $I_N(x,y)$ are transferred in the plane (β_1) on Figure 2. We will drop out the scale multiplier $M<1$ and will name the optical intensities in (β_1) as those on plane (α) on Figure 1. Also, from now on in this paper when we speak of p and q we mean p/M and q/M. Figure 2 shows the scheme of an analog optical processor for optimal filtration of the signal from its additive sum with the noise [11]. The processor consists of two consecutive and identical basic optical systems $((\beta_1)-(\beta'))$ and $((\beta')-(\beta_2))$ including lens with focal distances f_F, and between them a transparency with a transmission function $T(x,y)$ is situated.

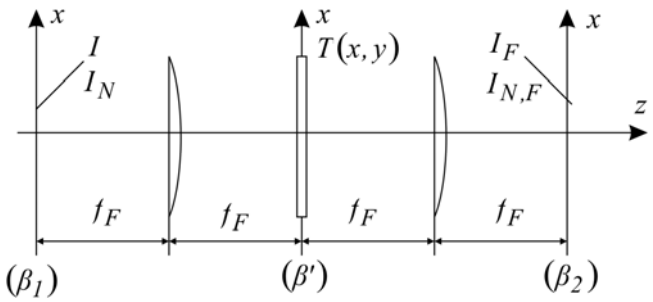

Figure 2. Analog optical processor for optimal filtration of $\Sigma(x,y)=I(x,y)+I_N(x,y)$

In the plane (β_1) there exists the total optical intensity $\Sigma(x,y)=I(x,y)+I_N(x,y)$, which is the only one possible estimation $\hat{I}(x,y)$ of the signal image $I(x,y)$ if no filter is used and we will have $\hat{I}(x,y)=\Sigma(x,y)$. The error due to the AT will be $\varepsilon(x,y)=\hat{I}(x,y)-I(x,y)=I_N(x,y)$, and it is obvious that for this mistake we have

$$\sigma_s^2=\sigma_{I_N}^2 . \tag{25}$$

When we use a filter, in the plane (β_2) the total optical intensity $\Sigma_F(x,y)=I_F(x,y)+I_{N,F}(x,y)$ will exist, for which there will be the estimation

$\left(\hat{I}_F(x,y)\right)$ of the signal image $I(x,y)$, and we will have $\hat{I}_F(x,y)=\Sigma_F(x,y)$. The error due to the AT will be $\varepsilon_F(x,y)=\hat{I}_F(x,y)-I(x,y)$, and its variance – $\sigma^2_{\varepsilon_F}=\left\langle\varepsilon^2_F(x,y)\right\rangle$.

The optimal filtration assures the minimization of $\sigma^2_{\varepsilon_F}$:

$$\sigma^2_{\varepsilon_F}=\left(\sigma^2_{\varepsilon_F}\right)_{min}. \tag{26}$$

It is known [11], that the minimization (26) can be performed, if the transfer function $H(p,q)$ of the filter satisfies the equation

$$|H(p,q)|=\frac{\Gamma_I(p,q)}{\Gamma_I(p,q)+\Gamma_{I_N}(p,q)}, \tag{27}$$

where the power spectrum $\Gamma_I(p,q)$ is represented by (4) and (5), and the power spectrum $\Gamma_{I_N}(p,q)$ – by (23) and (24).

According to the characteristics of the scheme on Figure 2 the transmission function of the transparency in (β') should be [11]

$$T(x,y)=H\left(\frac{kx}{f_F},\frac{ky}{f_F}\right)=H(p,q).$$

It is obvious that the participation of C_n^2 in (24) means that $|H(p,q)|$ will be different for the different values of AT.

5. EFFECTIVENESS OF THE OPTIMAL FILTRATION AND THE TRANSFORMATION OF THE SIGNAL–NOISE RATIO IN THE FILTER

The operation of the optimal filter can be estimated by means of the effectiveness factor $F>1$ of the optimal filtration, written in [11] as a ratio of (25) and (26), i.e. as a ratio of the variances of the errors before and after the optimal filtration. For the calculation of this factor we can use:

$$F=\frac{\sigma^2_{I_N}}{\sigma^2_{\varepsilon_F}}=\frac{\iint\limits_0^\infty \Gamma_{I_N}(p,q)dpdq}{\iint\limits_0^\infty |H(p,q)|\Gamma_{I_N}(p,q)dpdq}. \tag{28}$$

From (28) it is seen, that using F as an estimation of the effectiveness of the optimal filtration is equal to comparing the SNR in the input and output of the filter (SNR and $(SNR)_F$, accordingly). These ratios are determined by (3), (9), (16), (25), (26) and (28) with the obvious relations

$$SNR = \frac{\sigma_I^2}{\sigma_\varepsilon^2} = \frac{\sigma_I^2}{\sigma_{I_N}^2}, \quad (SNR)_F = \frac{\sigma_I^2}{\sigma_{\varepsilon_F}^2} = F \times SNR . \tag{29}$$

It is obvious that the values of F, SNR and $(SNR)_F$ will depend on the power of AT, i.e. on the value of C_n^2.

6. CALCULATIONS

The example calculations are made for the input data from [1] and for the values of the results, obtained there, namely: $L = 1{,}5$ km, $m = 100$, $\lambda = 0{,}63$ μm (He-Ne laser), $t = 1/4$, $\sigma_s^2 = 1/144$, $A_s = 2$ mm, $\gamma = \pi \times 10^5$ m^{-1}, $l = 10$ m.

The normalized power spectrum of $I(x,y)$ for $q = 0$ (on the frequency axis p), i.e. $\Gamma_I(p,0)/\Gamma_I(0,0)$, is calculated on the basis of (4). The calculation of the power spectrum of $I_N(x,y)$ for $q = 0$, i.e. $\Gamma_{I_N}(p,0)/\Gamma_{I_N}(0,0)$, is done by means of (23), and it is obvious, that this spectrum does not depend on the power of AT. The results are shown in Figure 3.

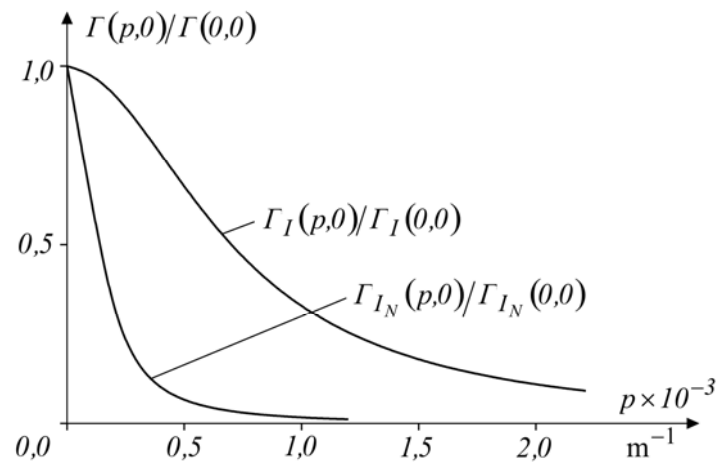

Figure 3. Normalized power spectrum of the signal and noise (of AT) optical intensities on the input of the optimal filter

The modulus of the transfer function of the optical filter for $q = 0$ and for different values of AT, i.e. $|H(p,0)|$ for different values of C_n^2 are calculated on the basis of (4), (5), (23) and (24), replaced in (27). The dependence of $\Gamma_{I_N}(0,0)/\Gamma_I(0,0)$ on C_n^2 has a great role, and it has been illustrated in Figure 4.

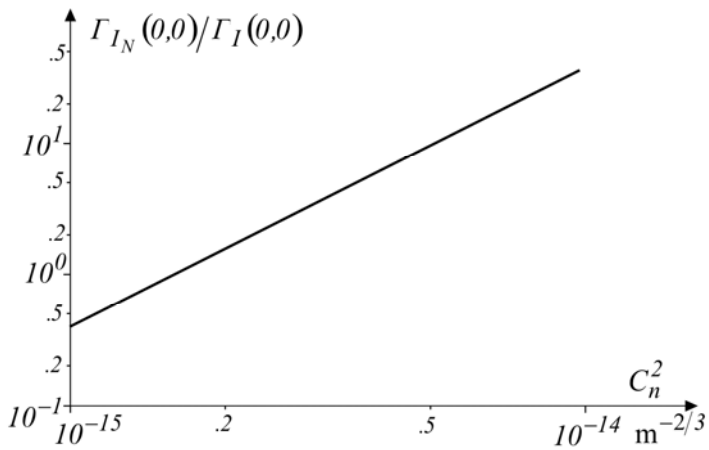

Figure 4. Relation of the noise (from AT) and the signal power spectrum for the zero spatial frequencies

Apart from this it meets the requirements of (10), and it means that for this example (for $L = 1{,}5$ km, $\lambda = 0{,}63$ μm) we have $\left(C_n^2\right)_{max} = 10^{-14}$ $m^{-2/3}$. The results for $|H(p,0)|$ are shown in Figure 5.

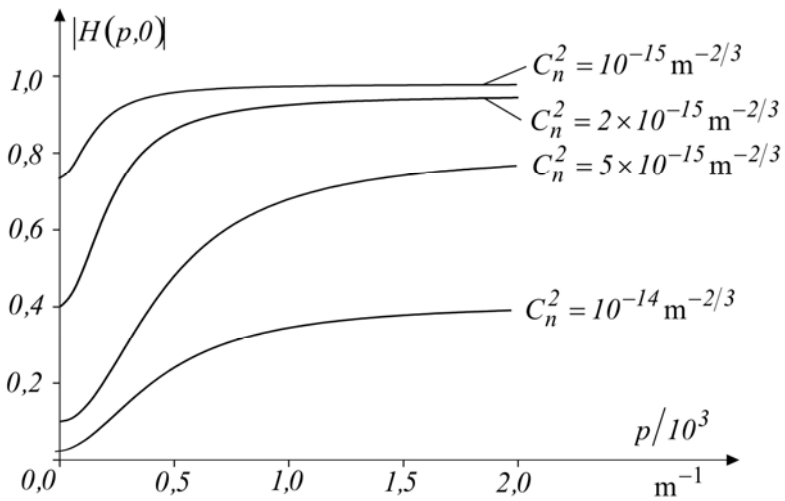

Figure 5. Transfer functions of the optimal filter

The effectiveness factor F of the optimal filtration is calculated with the help of (28) and is shown in Figure 6 as a function of C_n^2. Finally $SNR\left(C_n^2\right)$ and $(SNR)_F\left(C_n^2\right)$ are calculated, and their graphs are also in Figure 6.

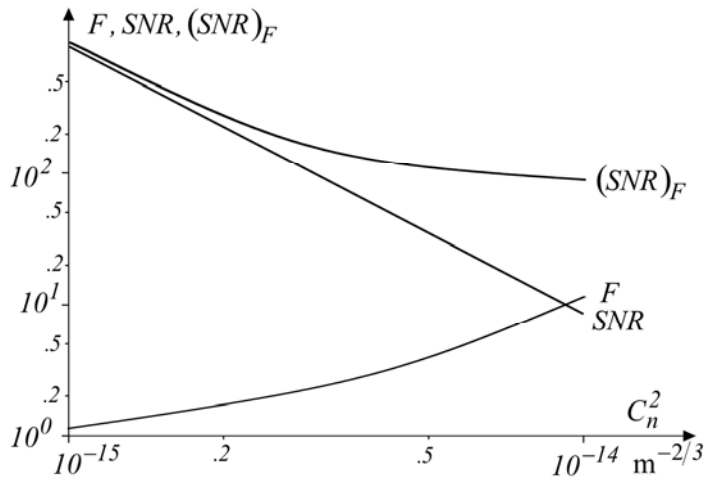

Figure 6. Effectiveness of the optimal filtration

7. CONCLUSIONS

Let us analyze Figure 3 – we can see that the power spectrum $\Gamma_{I_N}(p,q)$ of the dependent on AT noise optical intensity is much narrower than the power spectrum $\Gamma_I(p,q)$ of the signal optical intensity. This fact influences the dependence of the transfer function $|H(p,q)|$ of the optimal filter from the spatial frequencies p and q (Figure 5, for $q=0$) – this function has smaller values for lower spatial frequencies, for which the normalized/standard spectrum $\Gamma_{I_N}(p,0)/\Gamma_{I_N}(0,0)$ is closer to one. Naturally for $p=q=0$, $|H(p,q)|$ crosses the minimum. Moreover, Figure 5 illustrates the clear fact, from a physical point of view, that the greater the value of AT, i.e. the greater the value $\Gamma_{I_N}(0,0)$ of the noise power spectrum for $p=q=0$, relative to $\Gamma_I(0,0)$ (Figure 4), the more $|H(p,q)|$ differs from unity. The effectiveness factor F of the optimal filtration grows with the increasing value of AT, i.e. with the increase of C_n^2 (Figure 6). This circumstance is also physically clear – under weak AT we have $I_N(x,y) << I(x,y)$, $\Gamma_{I_N}(p,q) << \Gamma_I(p,q)$, $|H(p,q)| \le 1$ and the optimal filter

has an insignificant role. With the increase of C_n^2, the first two inequalities above do not hold anymore (Figure 4) and the modulus of $H(p,q)$ becomes less than one, which means that the optimal filtration is significant. The dependences of SNR before the filtration on the power of AT can be also interpreted according to these laws (Figure 6). The values of SNR and $(SNR)_F$ decrease with the increment of C_n^2, because the turbulent noise increases and the optimal filter only minimizes the error $\varepsilon_F(x,y)$ (its variance – see (26)), but it does not eliminate it. Besides, we should not forget that the aim of the determined by (27) optimal filtration is to ensure a minimal diversion of $\Sigma_F(x,y)$ from $I_F(x,y)$, and not the maximum value of $(SNR)_F$. The last one can be ensured by another optimal filtration, which is determined by a transfer function of the type $H(p,q) = const \times G_I^*(p,q) / \Gamma_{I_N}(p,q)$, where $G_I^*(p,q)$ is complex conjugated spectrum of the intensity I and is used when looking for a signal against the background of a noise field [11]. In this sense, $(SNR)_F$ here is of secondary importance.

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