IMAGE TRANSFER BY SPATIALLY MODULATED LASER RADIATION

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Abstract

A Laser communication system is investigated for transfer of pseudo-stochastic two-dimensional signals (images) by means of transverse spatial amplitude modulation of the carrier laser radiation. The optical channel system is structured to assure the wave-optical restoration of the initial image in the receiving aperture. The minimum aperture dimensions are analytically determined to guarantee adequate signal and spatial transformations in the optical channel. Analytical expressions for the statistical characteristics of optical intensity distribution in the receiving aperture have been derived. A particular analytical description (based on experimental data) of the intensity power spectrum has been found which is necessary for optimal filtration of the signal image from the additive mixture with the two-dimensional noise field. The theoretical conclusions are used for numerical examples. Analysis of the investigation results is performed.

1. INTRODUCTION

In cases of short communication distance the image transfer by means of scanning (transforming from two-dimensional spatial modulation to time modulation) can be replaced by the simpler implemented direct image transfer by means of a cross-spatial modulation of the laser beam [1]. The specific features of this method are connected with the aperture dimensions of the corresponding optical subsystems [2] and the image pseudo-stochastic structure [3]. The tasks of this investigation are: determination of the aperture dimensions that guarantee satisfactory realization of the signal and spectral transformations in the communication system optical channel; determining the statistical characteristics of optical intensity distribution in the

receiving aperture related to the signal separation from two dimensional noise fields of different physical origin [4].

2. BASIS FOR CONSIDERATION

The *optical channel* scheme (with equivalent lens transmitting aperture) is shown on Figure 1. The practical implementation of the considered systems involves application of mirrors because of the necessity of aperture of decimeter size.

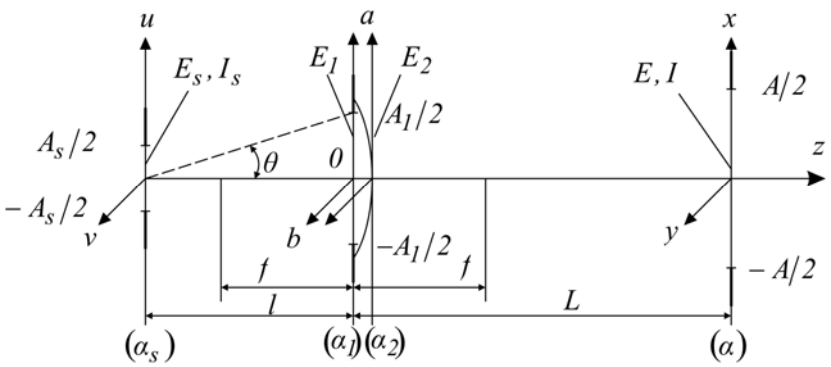

Figure 1. Optical channel scheme

We have cross-coherent wave optical signal

$$E_s(u,v) = E_m S(u,v) rect\left(\frac{u}{A_s}, \frac{v}{A_s}\right) \tag{1}$$

in the plane (α_s) as a result of spatial modulation of a plane laser wave by image $S(u,v)$ – a normalized two dimensional signal originated by means of a quadratic diaphragm $A_s \times A_s$. We have $S(u,v) \in [(1/2)-t,(1/2)+t]$ and

$$\langle S \rangle = \frac{1}{2}, \ \langle E_s \rangle = \frac{E_m}{2} \quad \left(u,v \in \left[-\frac{A_s}{2}, \frac{A_s}{2}\right]\right), \tag{2}$$

where $t > 0$ is the maximal deviation of S from $\langle S \rangle$.

As the intensity is

$$I_s(u,v) = E_m^2 |S(u,v)|^2 = E_m^2 S^2(u,v), \tag{3}$$

we have

$$\langle I_s \rangle = E_m^2 \left\langle \left[\frac{1}{2} + \overset{\circ}{S}(u,v) \right]^2 \right\rangle = E_m^2 \left(\frac{1}{4} + \sigma_s^2 \right), \tag{4}$$

where $\overset{\circ}{S}(u,v)$ and σ_s^2 are the mean value and the variance of $S(u,v)$. For normal distribution of the pseudo-stochastic two-dimensional signals $\overset{\circ}{S}(u,v)$ and the "3 σ" rule [5] we obtain the relation

$$\sigma_s^2 = \frac{t^2}{9}, \ \langle I_s \rangle = E_m^2 \left(\frac{1}{4} + \frac{t^2}{9} \right) E_m^2. \tag{5}$$

The thin lens between the plane (α_1) and (α_2) with focal length f is situated on distance $l > f$ from (α_s), while plane (α) of the receiving aperture is at distance L from (α_1), (α_2). The following well known relation is satisfied

$$\frac{1}{f} = \frac{1}{l} + \frac{1}{L}, \tag{6}$$

which ensures the image function of the lens in (α) [6]. In $(\alpha_1)/(\alpha_2)$ (for the fine lenses $(\alpha_1) \equiv (\alpha_2)$) and (α) there are diaphragms with dimensions $A_1 \times A_1$ and $A \times A$ respectively.

3. SIGNAL AND SPECTRAL TRANSFORMATIONS IN THE OPTICAL CHANNEL

For symbols used in (α), (α_1) and (α_s) see table 1.

Table 1

Plane	Coordinate	Coordinate increment	Spatial frequency
(α)	x,y	$\Delta x = \xi$, $\Delta y = \eta$	p,q
(α_1), (α_2)	a,b		
(α_s)	$u = \frac{1}{L}x$, $v = \frac{1}{L}y$	$\Delta u = \mu = \frac{1}{L}\xi$, $\Delta v = \upsilon = \frac{1}{L}\eta$	$\alpha = \frac{1}{L}p$, $\beta = \frac{1}{L}q$

Let us assume that the paraxial propagation in the (α_s) – (α_1) and (α_1) is situated in the Fraunhofer area of the aperture $A_s \times A_s$, which means that

$$l_{min} = \frac{A_s^2}{\lambda}, \; cos\,\theta = \frac{1}{\sqrt{l^2 + (A_1/2)^2}} \approx 1 . \tag{7}$$

Then for (α_1) we have

$$E_1(a,b) = \frac{kE_m}{2\pi l} e^{jk\frac{a^2+b^2}{2l}} \iint_{-\infty}^{\infty} S(u,v) rect\left(\frac{u}{A_s}, \frac{v}{A_s}\right) e^{-jk\frac{au+bv}{l}} dudv = \tag{8}$$

$$= \frac{kE_m}{2\pi l} e^{jk\frac{a^2+b^2}{2l}} G(\alpha,\beta),$$

[7], where

$$G(\alpha,\beta) = G_s(\alpha,\beta) \otimes G_{rect}(\alpha,\beta) \tag{9}$$

is the Fourier spectrum of the final signal field (1),

$$G_s(\alpha,\beta) = \iint_{-\infty}^{\infty} S(u,v)\, e^{-jau} e^{-j\beta v} dudv , \tag{10}$$

$$G_{rect}(\alpha,\beta) = A_s^2 \frac{sin\frac{\alpha A_s}{2}}{\frac{\alpha A_s}{2}} \frac{sin\frac{\beta A_s}{2}}{\frac{\beta A_s}{2}}, \tag{11}$$

$$\alpha = \frac{ka}{l}, \; \beta = \frac{kb}{l}, \; k = \frac{2\pi}{\lambda} . \tag{12}$$

The maximum values $\alpha_1 = \beta_1$ of $|\alpha|$ and $|\beta|$ in $G_s(\alpha,\beta)$ are determined by the dimensions Λ_{min} of the smallest image $E_s(u,v)$ details, which means

$$\alpha_1 = \frac{2\pi}{\Lambda_{min}} = \frac{2\pi m}{A_s}, \; m >> 1 . \tag{13}$$

From (11) follows that the maximum values $\alpha_2 = \beta_2$ of $|\alpha|$ and $|\beta|$ in $G_{rect}(\alpha,\beta)$ can be determined by $\alpha_2 A_s/2 = \pi$, i.e. by

$$\alpha_2 = \frac{2\pi}{A_s}, \tag{14}$$

It follows from (9) that the maximum value γ of $|\alpha|$ and $|\beta|$ in $G(\alpha,\beta)$ is the sum of $\alpha_1 = \beta_1$ and $\alpha_2 = \beta_2$ [7], $\gamma = \alpha_1 + \alpha_2$. According to (13) and (14) we get $\alpha_2 << \alpha_1$ ($\gamma \approx \alpha_1$) or

$$\gamma = \frac{2\pi m}{A_s}. \tag{15}$$

Then equations (8) and (9) yield:

$$E_1(a,b) = \frac{kE_m}{2\pi l} e^{jk\frac{a^2+b^2}{2l}} \iint_{-\infty}^{\infty} G_s(\alpha',\beta') G_{rect}(\alpha-\alpha',\beta-\beta') d\alpha' d\beta'. \tag{16}$$

In case $\alpha_2 >> \alpha_1$ the area of $G_{rect}(\alpha,\beta)$ is much smaller than the $G_s(\alpha,\beta)$ one, which allows us to present $G_{rect}(\alpha,\beta)$ in (16) by delta function $\delta(\alpha,\beta)$. Having in mind the filtrating property of $\delta(\alpha,\beta)$ [7] and (10) and (12), we get

$$E_1(a,b) = \frac{kE_m}{2\pi l} e^{jk\frac{a^2+b^2}{2l}} \iiiint_{-\infty}^{\infty} S(u,v) e^{-j\alpha' u} e^{-j\beta' v} \times \tag{17}$$

$$\times \delta(\alpha-\alpha',\beta-\beta') du dv d\alpha' d\beta' = \frac{kE_m}{2\pi l} e^{jk\frac{a^2+b^2}{2l}} \iint_{-\infty}^{\infty} S(u,v) e^{-jk\frac{au+bv}{l}} du dv$$

Alternatively, we can apply the more precise Frenel approximation [7]

$$E_1(a,b) = \frac{kE_m}{2\pi l} e^{jk\frac{a^2+b^2}{2l}} \iint_{-\infty}^{\infty} S(u,v) e^{-jk\frac{au+bv}{l}} e^{jk\frac{u^2+v^2}{2l}} du dv \tag{18}$$

which gives a description common with the $(\alpha_2) - (\alpha)$ area.

The transformation in the area $(\alpha_1) - (\alpha_2)$ of the fine lens is a radial square phase modulation [7, 8], namely

$$E_2(a,b) = e^{-jk\frac{a^2+b^2}{2f}} E_1(a,b), \tag{19}$$

and in the area $(\alpha_2) - (\alpha)$ the Frenel approximation is used:

$$E(x,y) = \frac{k}{2\pi L} e^{jk\frac{x^2+y^2}{2L}} \iint_{-\infty}^{\infty} E_2(a,b) e^{-jk\frac{xa+yb}{L}} e^{jk\frac{a^2+b^2}{2L}} da db. \tag{20}$$

We replace the integration $-A_2/2$ to $A_2/2$ $(A_2 = A_1)$ with $-\infty$ to ∞ having in mind that A_1 is bigger than $|a|_{max} = |b|_{max}$ of $E_1(a,b)$ in (α_1), i.e. of $E_2(a,b)$ in (α_2).

Substituting (18) into (19) and (19) into (20), having in mind (6) we get

$$E(x,y) = \frac{k^2 E_m}{(2\pi)^2 lL} e^{jk\frac{x^2+y^2}{2L}} \iiiint_{-\infty}^{\infty} S(u,v) e^{jk\frac{u^2+v^2}{2l}} \times \tag{21}$$

$$\times e^{-jk\frac{au+bv}{l}} e^{-jk\frac{xa+yb}{L}} dudvdadb.$$

The integration with respect to a and b in (21) leads to

$$E(x,y) = \frac{E_m l}{L} e^{jk\frac{x^2+y^2}{2L}} \iint_{-\infty}^{\infty} S(u,v) e^{jk\frac{u^2+v^2}{2l}} \delta\left(u + \frac{l}{L}x, v + \frac{l}{L}y\right) dudv = \tag{22}$$

$$= \frac{E_m l}{L} e^{jk\frac{x^2+y^2}{2Q}} S\left(-\frac{l}{L}x, -\frac{l}{L}y\right) = \frac{E_m l}{L} e^{jk\frac{x^2+y^2}{2Q}} S(-u,-v), \quad Q = \frac{L^2}{L+l}.$$

It can be seen from (22) that $|E(x,y)|$ in (α) is a copy of $S(u,v)$ in scale L/l.

4. MINIMUM APERTURE DIMENSIONS. RELATIONS AMONG σ_s^2, A_s, A_1, A, l, L, f, m, λ, t

As a result of the insertion of (15) into (12) and the equation $\gamma = \alpha_1 = |\alpha|_{max}$ we get

$$\frac{k|\alpha|_{max}}{l} = \frac{2\pi m}{A_s}, \quad |\alpha|_{max} = \frac{ml\lambda}{A_s}$$

and the obvious requirement $|\alpha|_{max} \le A_1$ is reduced to

$$A_1 = A_2 \ge \frac{2ml\lambda}{A_s} = A_{1,min}. \tag{23}$$

For the field (1) in (α_s) we have $|u|_{max} = A_s/2$. For the field (22) in (α) this expression becomes

$$\frac{l}{L}|x|_{max} = \frac{A_s}{2},$$

where considering $|x|_{max} \le A$ we get

$$A \ge \frac{LA_s}{l} = A_{min} . \tag{24}$$

It is clear that $A_{s,min}$ is to be connected with the resolving ability in the image (1). This ability is determined by $\Lambda_{min} \ge \lambda$, and it leads to

$$A_{s,min} = m\lambda . \tag{25}$$

So we get to the way of determining σ_s^2, A_s, A_1, l, f and A knowing L, m, λ and t: from (5) we calculate σ_s^2; from (25) we calculate $A_{s,min}$ and select A_s; from (7) we calculate l_{min} and select l; from (6) we calculate f; from (23) we calculate $A_{1,min}$ and select $A_1 = A_2$ (here is the place to check the paraxial approximation $\cos\theta \approx 1$ by means of (7)); from (24) we calculate A_{min} and select A.

5. STATISTICAL CHARACTERISTICS OF THE PSEUDOSTOCHASTIC OPTICAL FIELD INTENSITY IN (α)

On the basis of (22) and (3) we have

$$I(x,y) = |E(x,y)|^2 = \left(\frac{l}{L}\right)^2 I_s(u,v) = E_m^2 \left(\frac{l}{L}\right)^2 S^2(u,v). \tag{26}$$

And using (4) and (5) we get for the ***mean value of*** $I(x,y)$

$$\langle I \rangle = \left(\frac{l}{L}\right)^2 \langle I_s \rangle = E_m^2 \left(\frac{l}{L}\right)^2 \left(\frac{1}{4} + \sigma_s^2\right) = E_m^2 \left(\frac{l}{L}\right)^2 \left(\frac{1}{4} + \frac{t^2}{9}\right). \tag{27}$$

From (26) and (27) we find the ***variance of*** $I(x,y)$

$$\sigma_I^2 = \left\langle [I(x,y) - \langle I \rangle]^2 \right\rangle = \left(\frac{l}{L}\right)^4 \left\langle [I_s(u,v) - \langle I_s \rangle]^2 \right\rangle = \left(\frac{l}{L}\right)^4 \sigma_{I_s}^2 . \tag{28}$$

By (3), (4) and

$$\left\langle \overset{\circ}{S}{}^{2n+1} \right\rangle = 0 , \quad \left\langle \overset{\circ}{S}{}^{2n} \right\rangle = \sigma_s^{2n} \times 1 \times 3 \times \ldots \times (2n-1), \quad n = 0,1,2,\ldots \text{ [5]}$$

for normal distribution of $\overset{\circ}{S}(u,v)$ we get for the connection between $\sigma_{I_s}^2$ and σ_s^2

$$\sigma_{I_s}^2 = E_m^4 \sigma_s^2 \left(2\sigma_s^2 + 1\right). \tag{29}$$

Substituting (29) in (28) gives

$$\sigma_I^2 = E_m^4 \left(\frac{l}{L}\right)^4 \sigma_s^2 \left(2\sigma_s^2 + 1\right). \tag{30}$$

Expressions (27) and (30) allow calculating the contrast image $\sigma_I / \langle I \rangle$ in the receiving antenna aperture.

Autocorrelation function of $I(x,y)$**.**

From (26) we get

$$B_I(\xi,\eta) = \left\langle \left[I(x,y) - \langle I \rangle\right]\left[I(x+\xi, y+\eta) - \langle I \rangle\right]\right\rangle = \left(\frac{l}{L}\right)^4 B_{I_s}(\mu,\upsilon). \tag{31}$$

Power spectrum of $I(x,y)$.

The Viner-Hintchin theorem [4] can be presented in the form

$$\Gamma_I(p,q) = \iint_{-\infty}^{\infty} B_I(\xi,\eta) e^{-jp\xi} e^{-jq\eta} d\xi d\eta, \tag{32}$$

as $B_I(\xi,\eta)$ decreases strongly when ξ and η increase [3]. Inserting (31) into (32) leads to

$$\Gamma_I(p,q) = \left(\frac{l}{L}\right)^2 \iint_{-\infty}^{\infty} B_{I_s}(\mu,\upsilon) e^{-j\frac{L}{l}p\mu} e^{-j\frac{L}{l}q\upsilon} d\mu d\upsilon = \left(\frac{l}{L}\right)^2 \Gamma_{I_s}(\alpha,\beta). \tag{33}$$

6. PARTICULAR FORMULAS FOR $B_I(\xi,\eta)$ AND $\Gamma_I(p,q)$ ON THE BASIS OF EXPERIMENTAL DATA

In [3] it is shown that $B_{I_s}(\mu,\upsilon)$ can be approximated by

$$B_{I_s}(\mu,\upsilon) = \sigma_{I_s}^2 e^{-2\frac{|\mu|+|\upsilon|}{r}}, \tag{34}$$

where r is the radius of a correlation in (α_s).

Replacing (34) into (31) leads to

$$B_I(\xi,\eta)=\left(\frac{l}{L}\right)^4 \sigma_{I_s}^2 e^{-2\frac{|\mu|+|\upsilon|}{r}} = \sigma_I^2 e^{-2\frac{l}{L}\frac{|\xi|+|\eta|}{r}} . \tag{35}$$

When $|\xi|+|\eta|=rL/l$ we have $B_I=\sigma_I^2/e^2$, consequently $|\xi|+|\eta|\le rL/l$ is the area of significant correlation.

The approximation (34) turns the determination of the power spectrum of $I_s(u,v)$ by means of the Wiener-Hinchin theorem into simple integration. Using the Wiener-Hinchin theorem and (29) we get

$$\Gamma_{I_s}(\alpha,\beta)=\iint_{-\infty}^{\infty} B_{I_s}(\mu,\upsilon)e^{-j\alpha\mu}e^{-j\beta\upsilon}d\mu d\upsilon = \tag{36}$$

$$=4E_m^4\sigma_s^2\left(2\sigma_s^2+1\right)\frac{(2/r)^2}{\left[(2/r)^2+\alpha^2\right]\left[(2/r)^2+\beta^2\right]}.$$

To express r in terms of γ we assume $\Gamma_{I_s}(\alpha,\beta)=\Gamma_{I_s}(0,0)/10$ in the points $|\alpha|=\gamma$, $\beta=0$ and $\alpha=0$, $|\beta|=\gamma$ of the frequency field (α,β). Determining $\Gamma_{I_s}(0,0)$ and $\Gamma_{I_s}(\pm\gamma,0)=\Gamma_{I_s}(0,\pm\gamma)$ from (36) and inserting the results in the above accepted we get

$$r=\frac{6}{\gamma}. \tag{37}$$

Replacing (36) and (37) into (33) leads to the particular expression for the power spectrum of $I(x,y)$

$$\Gamma_I(p,q)=4E_m^4\left(\frac{l}{L}\right)^2\sigma_s^2\left(2\sigma_s^2+1\right)\frac{(\gamma/3)^2}{\left[(\gamma/3)^2+(L/l)^2p^2\right]\left[(\gamma/3)^2+(L/l)^2q^2\right]}, \tag{38}$$

where according to $|\alpha|_{max}=\gamma$ and table 1 (see above) we have

$$|p_I|_{max}=|q_I|_{max}=\frac{l}{L}\gamma. \tag{39}$$

Expression (38) can be presented as

$$\Gamma_I(p,q)=\Gamma_I(0,0)\frac{(\gamma/3)^4}{\left[(\gamma/3)^2+(L/l)^2p^2\right]\left[(\gamma/3)^2+(L/l)^2q^2\right]}, \tag{40}$$

where

$$\Gamma_I(0,0)=4E_m^4\left(\frac{l}{L}\right)^2\sigma_s^2\left(2\sigma_s^2+1\right)\left(\frac{3}{\gamma}\right)^2. \tag{41}$$

Obviously, we have

$$\Gamma_I\left(\pm\frac{l}{L}\gamma,0\right)=\Gamma_I\left(0,\pm\frac{l}{L}\gamma\right)=\frac{1}{10}\Gamma_I(0,0).$$

From (40) and (41) we can find the line $q=q(p)$ in (p,q), corresponding to $\Gamma_I(p,q)=\Gamma_I(0,0)/10$ and limiting the area of the significant part of $\Gamma_I(p,q)$. We obtain:

$$\frac{Lq}{l\gamma}=\sqrt{\frac{1-(Lp/l\gamma)^2}{1-(3Lp/l\gamma)^2}}. \tag{42}$$

We need the power spectrum to determine the transfer function of the optimal filter, which can be used for the selection of the signal image from its additive mixture with a two dimensional noise field [7].

7. NUMERICAL CALCULATION

Let us have: $L=1{,}5$ km, $m=100$, $\lambda=0{,}63$ μm, $t=1/4$. Then we get:

$\sigma_s^2=1/144$, $A_{s,min}=0{,}063$ mm (we choose $A_s=2$ mm), $l_{min}=6{,}35$ m (we choose $l=10$ m), $f=9{,}93$ m $(l-f=7\text{ cm})$, $A_{1,min}=0{,}63$ (we choose $A_1=0{,}7$ m) $\cos\theta=0{,}999\approx 1$, $A_{min}=0{,}3$ m (we choose $A=0{,}35$ m), $\sigma_I/\langle I\rangle=0{,}166$.

From (15) we get $\gamma=\pi\times 10^5$ m^{-1} and by (40) we calculate $\Gamma_I(p,0)/\Gamma_I(0,0)$. The result is shown on Figure 2. By (42) we determine the isoline $q=q(p)$, by limiting the area through the relation $\Gamma_I(p,q)\geq\Gamma_I(0,0)/10$ (Figure 3).

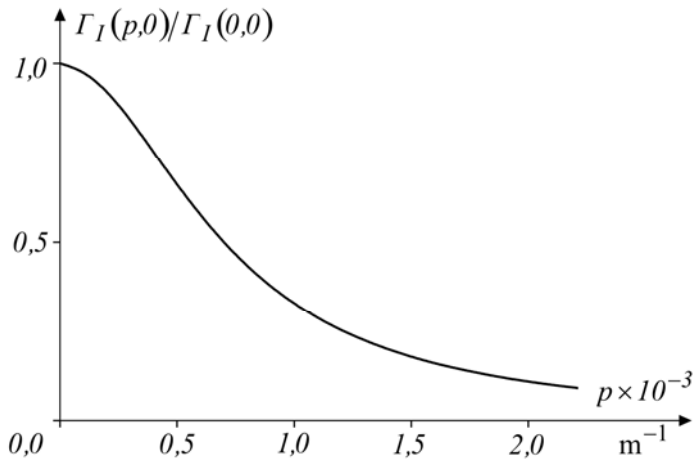

Figure 2. Normalized power spectrum of the optical intensity in the receiving aperture

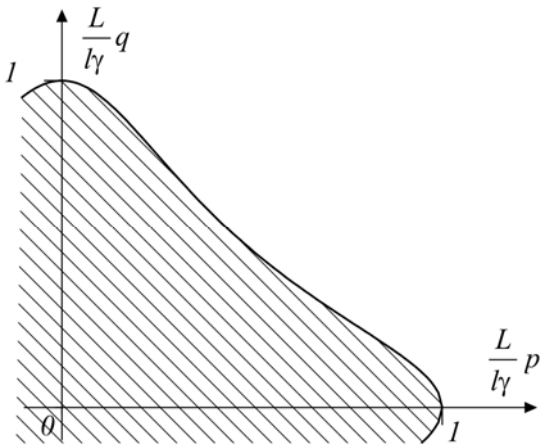

Figure 3. Area of the significant part of the optical intensity power spectrum in the spatial frequency domain of the receiving aperture

8. CONCLUSIONS

The expressions (23) and (24) show that A_{min} does not depend on the choice $A_1 \geq A_{1,min}$ of the transmitting aperture.

The value of $A_{1,min}$, is inversely proportional to A_s (see eqn. (23)) of the initial image due to the Fraunhofer diffraction in the area $(\alpha_s)-(\alpha_1)$, guaranteed by the condition (7). The value of A_{min} is proportional to A_s due to the optical channel structure adopted in this paper, which leads to the approximation of the wave image-forming to ray image-forming in the paraxial conditions.

From (23) and (24) it follows that the minimization of $A_{1,min} + A_{min}$ on A_s leads to $A_s = (2m\lambda/L)^{1/2} = A_s^{'}$. It is easy to check that $A_s = A_s^{'}$ means also $A_{1,min} = A_{min} = (2m\lambda L)^{1/2}$. But $A_s = A_s^{'}$ is possible when $A_s^{'}$ satisfies (25). For the example calculated we get $A_s^{'} = 0{,}29$ mm $> A_{s,min} = 0{,}063$ mm and $A_{1,min} = A_{min} = 0{,}43$ m, which is an acceptable result.

Knowing $\sigma_I / \langle I \rangle$ equs. (27) and (30), allows us to investigate the noise field influence on the image quality, and the knowledge of spatial power spectrum $\Gamma_I(p,q)$ - can be used to apply optimal filtration in order to reduce noise. From (27) and (30) it can be seen that when $\sigma_s << 1$ we have $\sigma_I / \langle I \rangle \approx \sigma_s / \langle S \rangle$, which means that the image contrast is practically not changed through the optical channel.

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