DESIGN AND SIMULATION OF RADON-MULTIWAVELET BASED OFDM SYSTEM

O. Q. J. Al-Thahab, W. A. Mahmoad, M. S. Abdul-Wahab

Department of Electrical and Electronic Engineering, University of Technology, Baghdad , Iraq

e-mail: oalthahab@yahoo.com

Abstract

In this work, the Orthogonal Frequency Division Multiplexing (OFDM) system is designed based on Finite radon transform (FRAT) instead of 16QAM, and QPSK mapping schemes, while using Discrete Multiwavelet Transform (DMWT) as multi-carier modulation technique. The system is proposed to improve the Bit Error Rate (BER) and spectral efficiency of the traditional OFDM system over multi-path Rayleigh selective and flat fading channels followed by Additive White Gaussian Noise (AWGN) and taking into account the effect of varying the Doppler shift frequency, and the variation of channel parameters. It can be seen from the proposed system that a good performance was obtained over the traditional OFDM which uses the fast Fourier Transform (FFT) as multi-carrier modulation technique. This is because the advantages of DMWT of providing multi-filter banks which reduces the BER, while the use of FRAT provided the enhancement of spectrum of the transmitted symbols.

1. INTRODUCTION

Due to the enormous growth of wireless services (cellular telephones, wireless LAN's, etc.) during the last decade, the need of a modulation technique that can transmit reliably high data rates at a high bandwidth efficiency arises In a mobile radio channel, the signal is disturbed by multipath fading which generally exhibits both time-selectivity and frequency-selectivity. The signal power is carried by a large number of paths with different strengths and delays [1].

Orthogonal frequency-division multiplexing (OFDM) has recently received increased attention due to its capability of supporting high-data-rate communication in frequency selective fading environments which cause inter-symbol interference (ISI). Instead of using a complicated equalizer as in the conventional single carrier systems, the ISI in OFDM can be eliminated by adding a guard interval which significantly simplifies the receiver structure [2,3].

Many of the proposed modern wireless communication standards, such as WLAN 802.11a/g, and WMAN 802.16a employ different Orthogonal Frequency Division Multiplexing (OFDM) modulation schemes. In an OFDM system, the processes of modulation and demodulation are carried out by means of an Inverse Fast Fourier Transform (IFFT) and a Fast Fourier Transform (FFT) respectively [4].

In an OFDM communication system, the broadband is partitioned into many orthogonal sub-carriers, in which data is transmitted in a parallel fashion. Thus the data rate for each sub-carrier is lowered by a factor of N in a system (where N sub-carriers). By this method, the channel is divided into many narrowband flat fading sub-channels. This makes the OFDM system more resistant to the multi-path frequency selective fading than the single carrier communication system. The sub-carriers are totally independent and orthogonal to each other. The sub-carriers are placed exactly at the nulls in the modulation spectral of one another. At the peak point of one sub-carrier waveform, the symbol values of other sub-carriers at the nulls are zeros and thus contribute no ISI to the symbol sub-carrier. This is where the high spectral efficiency of OFDM comes from. It can be shown that keeping the orthogonality of the sub-carriers is very critical for an OFDM system to be free from inter carrier interference.

In a typically OFDM system, a cyclic prefix is used as a guard interval to avoid the ISI and ICI. The cyclic prefix is the insertion of the last Ng symbols to the original symbol sequence, where Ng is the length of the guard interval. This guard interval is required to be at least equal to or longer than the maximum channel delay spread of the system. Using a cyclic prefix as the guard interval is a simple way to reduce the ISI and ICI, however it also reduces the transmission efficiency of the system [5].

Another approach for guaranteeing symbol delectability over ISI channels is to modify the OFDM setup. Instead of inducing the CP, each IFFT processed block is

zero padded ZP, by at least as many zeros as the channel order. If the number of zero-padded equals the CP length, the ZP-OFDM and CP-OFDM transmission have the same spectral efficiency. For mobile communication, when transmitter and receiver are in relative motion. The Doppler effect also has to be considered, which may lead to a shifted of the various frequency components of the signal and inter channel interference (ICI), however, to keep the ICI small the inter-carrier spacing of the system has to be chosen large, compared to the maximal Doppler frequency of the fading channel [6].

2. FINITE RADON TRANSFORM

The Radon transform (RT) is a mapping technique from image domain to a parameter domain, where the parameters characterize the curves to be identified. In this way, the Radon Transform converts a difficult global detection problem in the image domain into a more easly solved local peak detection problem in the parameter domain, and the actual parameters can be recovered by, e.g., thresholding the Radon Transform [7].

The Finite Radon Transform (FRAT), is defined as summations of image pixels over a certain set of "lines". Those lines are defined in a finite geometry in a similar way as the lines for the continuous Radon transform in the Euclidean geometry. Denote Zp= {0,1,.., p-1}, where p is a prime number. Note that Zp is a finite field with modulo p operations. The FRAT of a real discrete function f on the finite grid z_p^2 is defined as:

$$FRAT_f(k,l) = \frac{1}{\sqrt{p}} \sum_{(i,j)\in L_{k,l}} f(i,j) \tag{1}$$

Here $L_{k,l}$, denotes the set of points that make up a line on the lattice z_p^2 ie

$$L_{k,l} = \begin{cases} \{(i,j) : j = (ki+l)(\mathrm{mod}p), i \in Z_p\} \; if \; 0 \le k < p \\ \{(l,j) : j \in Z_p\} \; if \; k = p \end{cases} \tag{2}$$

The inverse transform is obtained through the Finite Back-Projection operator (FBP) defined as a sum of Radon coefficients of all the lines that go through a given point. Here *f* is supposed to be a zero-mean image:

$$FBP_r(i,j) = \frac{1}{\sqrt{p}} \sum_{(k,l)\in P_{i,j}} FRAT_f(k,l) \tag{3}$$

From (2) it can be found that $P_{i,j}$, is:

$$P_{i,j} = \{(k,l) : l = (j - ki)(\text{mod } p), k \in Z_p\} \cup \{(p,i)\}$$

Substituting (3) into (1) we obtain that

$$FBP_r(i,j) = \frac{1}{p} \sum_{(k,l)\in P_{i,j}} \sum_{(i',j')\in L_{k,l}} f(i',j') =$$
$$= \frac{1}{p}\left(\sum_{(i',j')\in Z_p^2} f(i',j') + p \cdot f(i,j) \right) = f(i,j) \tag{4}$$

and so the perfect reconstruction is achieved [8].

To find the FRAT, the following steps can be done as shown in Figure (1).

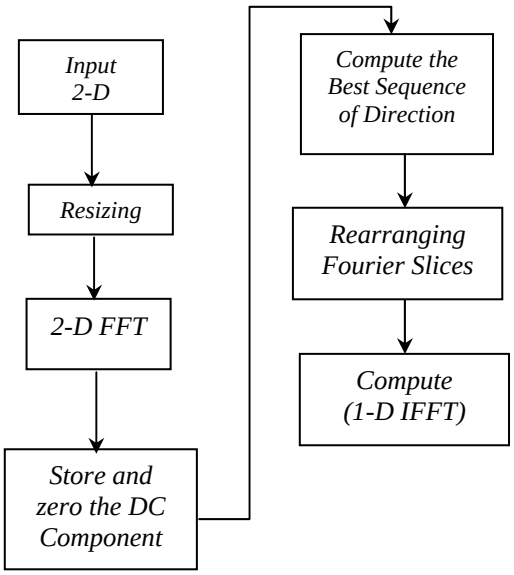


Figure 1. The construction of FRAT

3. DISCRETE MULTIWAVELET TRANSFORM (DMWT)

Wavelet transforms (including decomposition and reconstruction) with single-mother wavelet functions have been studied extensively. One of their main properties is the time-frequency localization property of wavelet functions. However, it is known that there is a limitation for the time-frequency localization of a single wavelet function.

Multiwavelets are extension from wavelets, and several mother wavelet functions are used to expand a function. Wavelets have an associated scaling function $\phi(t)$ and wavelet function $\psi(t)$, while Multiwavelets have two or more scaling and wavelet functions. Multiwavelets have attracted much attention due to their nice properties, and they are very similar to wavelets but have some important differences.

For notational convenience, the set of scaling functions can be written using the vector notation $\Phi(t)\equiv[\phi_1(t), \phi_2(t)\ldots \phi_r(t)]^T$,where $\Phi(t)$ is called the multiscaling function. Likewise, the Multiwavelet function is defined from the set of wavelet functions as $\Psi(t)\equiv[\psi_1(t), \psi_2(t)\ldots \psi_r(t)]^T$. When r=1, $\Psi(t)$ is called a scalar wavelet, or simply wavelet. While in principle r can be arbitrarily large. The Multiwavelets studied to date are primarily for r=2.

The Multiwavelet two-scale equations resemble those for scalar wavelet

$$\Phi(t) = \sqrt{2}\sum_{k=-\infty}^{\infty} H_k \Phi(2t-k) \tag{5}$$

$$\Psi(t) = \sqrt{2}\sum_{k=-\infty}^{\infty} G_k \Phi(2t-k) \tag{6}$$

Note, however, that $\{H_k\}$ and $\{G_k\}$ are matrix filters, i.e., H_k and G_k are r× r matrices for each integer k. The matrix elements in these filters provide more degrees of freedom than a traditional scalar wavelet. These extra degrees of freedom can be used to incorporate useful properties into the Multiwavelet filters, such as orthogonality, symmetry, and high order of approximation. However, the multi-channel nature of Multiwavelets also means that the sub-band structure resulting from passing a signal through a multifilter bank is different [9].

One famous Multiwavelet filter is the GHM filter proposed by Geronimo, Hardian, and Massopust [10]. The GHM basis offers a combination of orthogonality, symmetry, and compact support, which can not be achieved by any scalar wavelet basis. According to Eqs.(3-54) and (3-55) the GHM two scaling and wavelet functions satisfy the following two-scale dilation equations:

$$\begin{bmatrix} \phi_1(t) \\ \phi_2(t) \end{bmatrix} = \sqrt{2}\sum_k H_k \begin{bmatrix} \phi_1(2t-k) \\ \phi_2(2t-k) \end{bmatrix} \tag{7}$$

$$\begin{bmatrix} \psi_1(t) \\ \psi_2(t) \end{bmatrix} = \sqrt{2}\sum_k G_k \begin{bmatrix} \phi_1(2t-k) \\ \phi_2(2t-k) \end{bmatrix} \tag{8}$$

where H_k for GHM system are four scaling matrices H_0, H_1, H_2, and H_3, also, G_k for GHM system are four wavelet matrices G_0, G_1, G_2, and G_3.

$$H_0 = \begin{bmatrix} \frac{3}{5\sqrt{2}} & \frac{4}{5} \\ -\frac{1}{20} & -\frac{3}{10\sqrt{2}} \end{bmatrix}, \quad H_1 = \begin{bmatrix} \frac{3}{5\sqrt{2}} & 0 \\ \frac{9}{20} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$H_2 = \begin{bmatrix} 0 & 0 \\ \frac{9}{20} & -\frac{3}{10\sqrt{2}} \end{bmatrix}, \quad H_3 = \begin{bmatrix} 0 & 0 \\ -\frac{1}{20} & 0 \end{bmatrix} \tag{9}$$

$$G_0 = \begin{bmatrix} -\frac{1}{20} & -\frac{3}{10\sqrt{2}} \\ \frac{1}{10\sqrt{2}} & \frac{3}{10} \end{bmatrix}, \quad G_1 = \begin{bmatrix} \frac{9}{20} & -\frac{1}{\sqrt{2}} \\ -\frac{9}{10\sqrt{2}} & 0 \end{bmatrix}$$

$$G_2 = \begin{bmatrix} \frac{9}{20} & -\frac{3}{10\sqrt{2}} \\ \frac{9}{10\sqrt{2}} & -\frac{3}{10} \end{bmatrix}, \quad G_3 = \begin{bmatrix} -\frac{1}{20} & 0 \\ -\frac{1}{10\sqrt{2}} & 0 \end{bmatrix} \tag{10}$$

Here H and G consists of an r by r matrices, and during the convolution step they must multiply vectors (instead of scalars). This means that multifilter banks need r input rows. Here r = 2 and two data streams enter the multifilter. This means that the input must be pre-filtered first then taking the DMWT to it, and this process called the preprocessing.

The aim of preprocessing is to associate the given scalar input signal of length N to a sequence of length-2 vectors. Here N is assumed to be a power of 2, and so it is of even length. If the preprocessing produces N length-2 vectors it is said to be an oversampling scheme, while if it produces N/2 length-2 vectors the result is a critical sampling.

After the multiwavelet transform, inverse transform and post-filtering should recover the input signal exactly if nothing else has been done this process called the postprocessing.

In this work, the use of critically sampling is done to maintain the same data rate of the input symbols and to prevent the change in the Bandwidth. There is two types of critically sampling:

a) First order approximation.

b) Second order approximation.

It can be seen that, the 2nd order approximation give an approximation to the inputs so that it can be reconstructed exactly without any error, which differ from 1st order approximation.

4. PROPOSED SYSTEM FOR RADON-MULTIWAVELET MODULATOR BASED OFDM

In this section, the Fourier based OFDM use the complex exponential bases functions and it's replaced by DMWT in order to reduce the level of interference, and the FRAT is proposed as a mapping technique in the communication systems instead of QPSK and QAM technique in FFT based system. The block diagram of the proposed system for OFDM is depicted in Figure (2).

In the proposed system there was no need to use the cyclic prefix due to the excellent orthogonality of both Radon and Multiwavelet Transforms. Here the length of the frame of the input data must be (p* p) (i.e 25,49,…..), where p must be prime number. After the FRAT, a modification is made here on the output matrix (R) of FRAT coefficients, for a purpose of increasing the bit per Hertz of the mapping. Construct the complex matrix C (which is a vector) from the real matrix R such that

its dimension will be (p * (p+1))/2, This can be achieved by using the general equation (11).

$$C(d,1) = \sum_{k=1,3,..}^{p} \sum_{j=1}^{p} R(j,k) + iR(j,k+1) \tag{11}$$

$$where \quad d = 1,2,..,(p*(p+1))/2$$

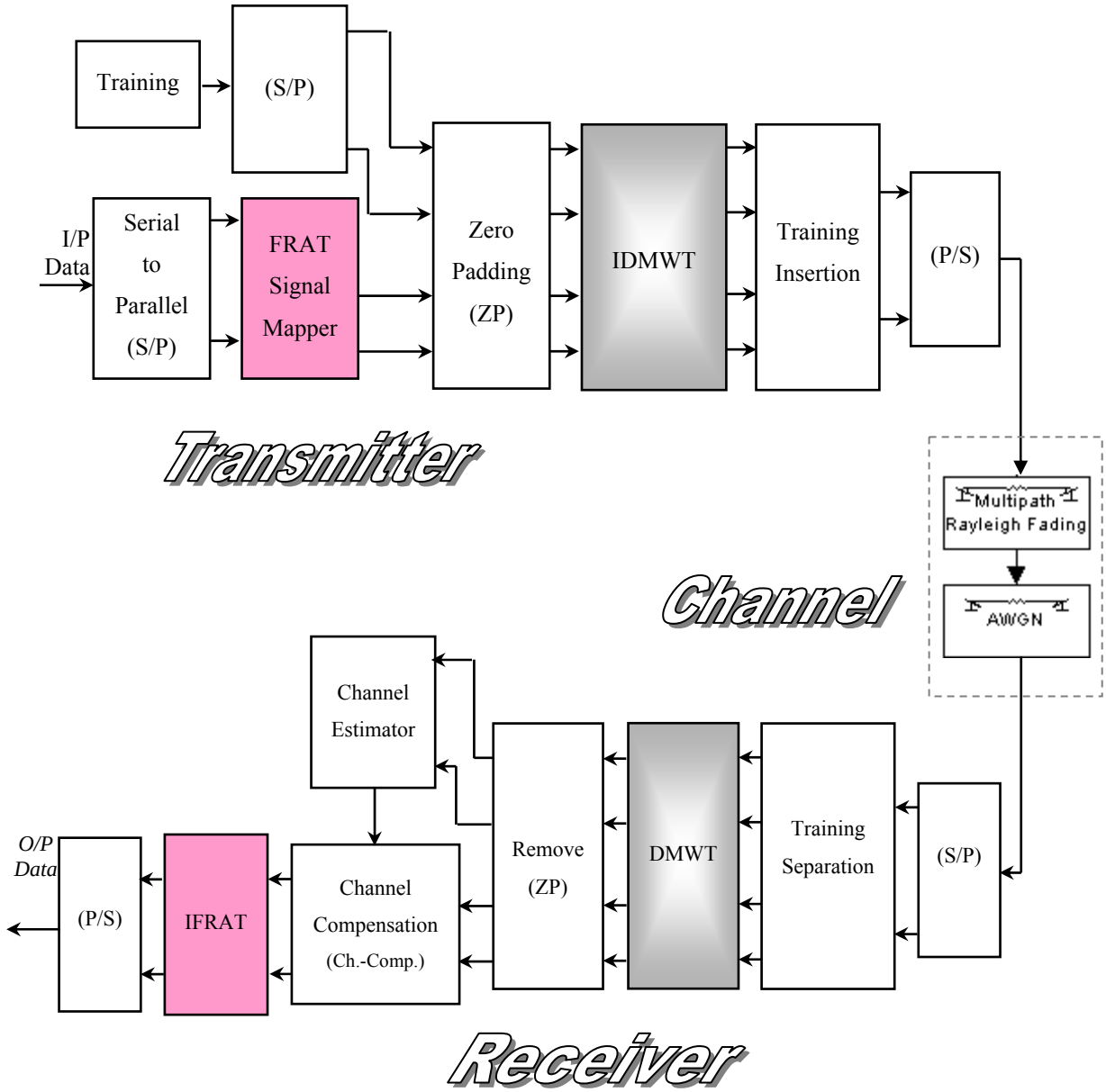


Figure 2. Proposed Radon-Multiwavelet based OFDM

Both the training and transmitted symbols are interred to IDMWT then transmitted through the multipath fading channel and AWGN.

In the receiver the operation done with reverse direction, the training sequence will be used to estimate the channel frequency response as follows:

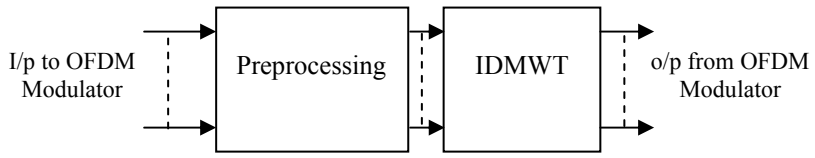
$$\mathrm{H(k)} = \frac{\text{Received Training Sample (k)}}{\text{Transmitted Training Sample (k)}} \tag{12}$$

The channel frequency response will be used to compensate the channel effects on the data, and the estimated data can be found using the following equation:

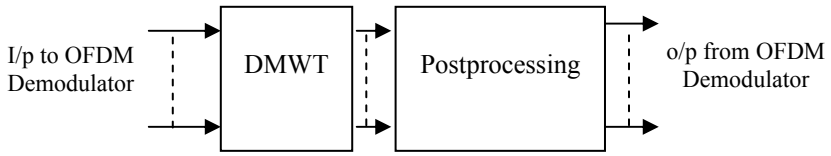
$$Estimate.data(k) = H^{-1}_{estimate}(k) * \mathrm{Re}ceived.data(k) \tag{13}$$

The output of channel compensator will be passed through the signal demapper which is IFRAT. The OFDM modulator and demodulator of DMWT-based OFDM are shown in figure (3). The system is simulated using MATLAB 7 Program.

The proposed system was tested via AWGN channel, Flat fading Rayleigh channel, and Selective fading Rayleigh channel.



(a) DMWT-OFDM modulator



(b) DMWT-OFDM modulator

Figure 3. DMWT-OFDM modem system.

5. THE RESULTS OF THE PROPOSED SYSTEM

The bandwidth used in the test was 10 MHz, and the proposed system was compared with the following systems: FFT based OFDM, Radon based OFDM, and Multiwavelet based OFDM. The performance of the BER over AWGN channel is shown in Figure (4).

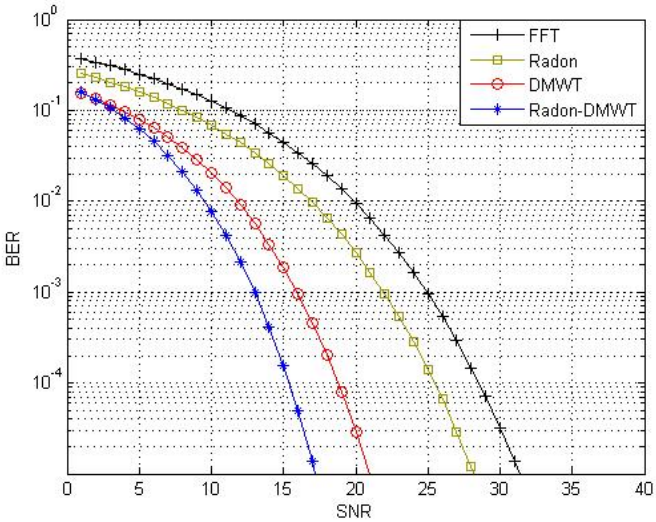
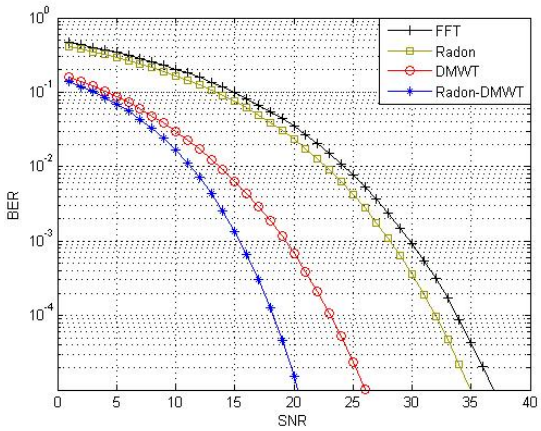


Figure 4. BER performance of Radon-DMWT-OFDM in AWGN channel

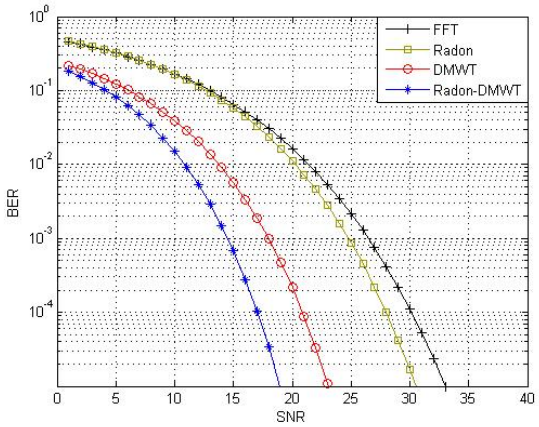
It is shown clearly that the Radon-DMWT-OFDM is much better than the three compared systems FFT-OFDM and DMWT-OFDM and Radon-OFDM. This is a reflection to the fact that the orthogonal bases of the Multiwavelets with Radon is much significant than the orthogonal bases used in FFT-OFDM and DMWT-OFDM and Radon-OFDM alone.

The resulting BER over Rayleigh Flat fading channel with Doppler frequency (4 Hz, 80Hz, 300Hz, and 500 Hz) are shown in Figures (5-a, 5-b, 5-c, and 5-d).

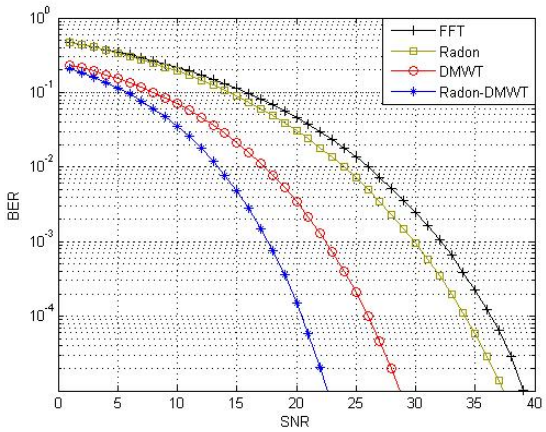
In this case all the frequency components in the signal will be effect by a constant attenuation and linear phase distortion of the channel, which has been chosen to have a Rayleigh's distribution.



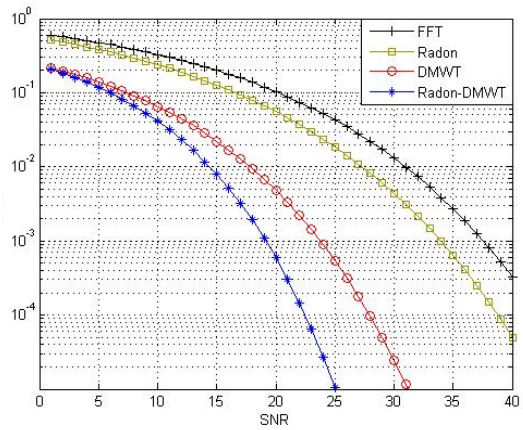
(a) BER performance of Radon-DMWT-OFDM in flat Fading Channel at Max Doppler Shift=4 Hz.



(b) BER performance of Radon-DMWT-OFDM in flat Fading Channel at Max Doppler Shift=80 Hz.



(c) BER performance of Radon-DMWT-OFDM in flat Fading Channel at Max Doppler Shift=300 Hz



(d) BER performance of Radon-DMWT-OFDM in flat Fading Channel at Max Doppler Shift=500 Hz.

Figure 5. BER performance of Radon- DMWT-OFDM in a Flat fading channel at different Doppler Shift frequency

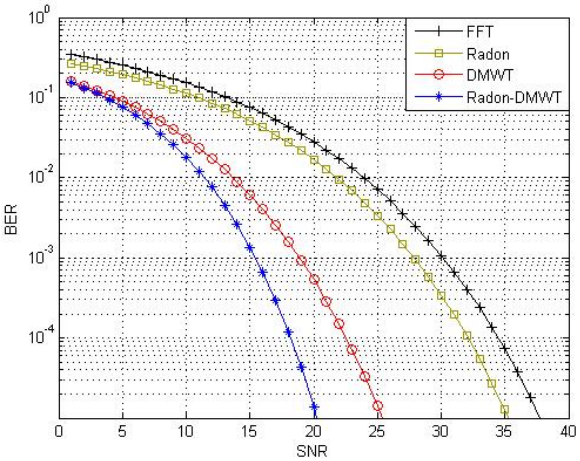
It can be shown From Figure 5, that the model of Radon-DMWT-OFDM again have the best performance over FFT-OFDM and DMWT-OFDM and Radon-OFDM. This is due to the perfect orthogonality of the Radon Transform and the excellent orthogonality of DMWT.

Finally The resulting BER over Rayleigh Selective fading channel with Doppler frequency (4 Hz, 80Hz, 300Hz, and 500 Hz) are shown in Figures (6-a, 6-b, 6-c, and 6-d).

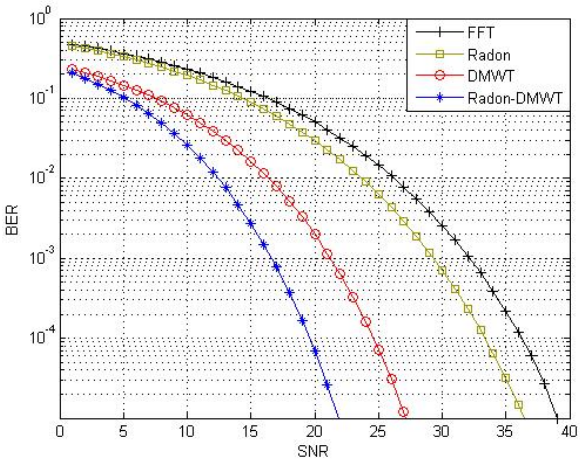
Here the channel model is assumed to be selective fading channel, where the parameters of the channel in this case corresponding to multipaths where two paths

are chosen the line of site (LOS) and second path. The LOS path have Average Path Gain equal 0dB and Path Delay 0, where the second path have Average Path Gain -8 dB and path Delay 0.1μsec.

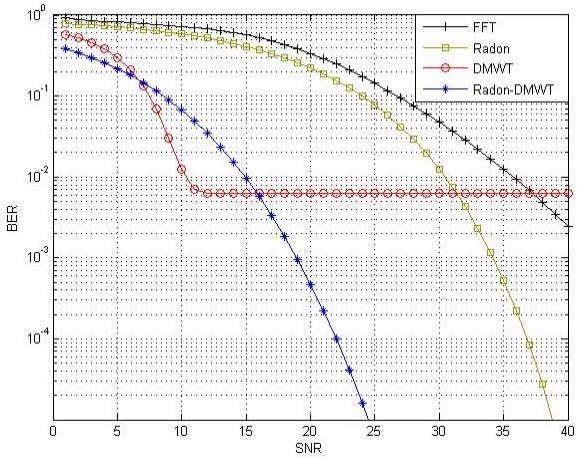
In figure (6), it is shown clearly that BER performance of Radon-DMWT-OFDM is better also than the three systems FFT-OFDM and DMWT-OFDM and Radon-OFDM.



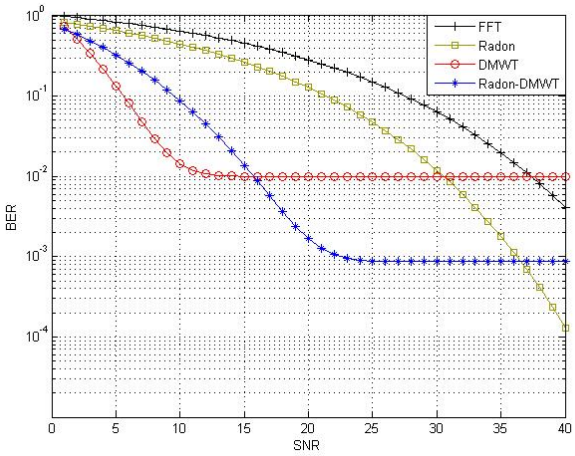
(a) BER performance of Radon-DMWT-OFDM in Selective Fading Channel at Max Doppler Shift=4Hz.



(b) BER performance of Radon-DMWT-OFDM in Selective Fading Channel at Max Doppler Shift=80Hz



(c) BER performance of Radon-DMWT-OFDM in Selective Fading Channel at Max Doppler Shift=300Hz



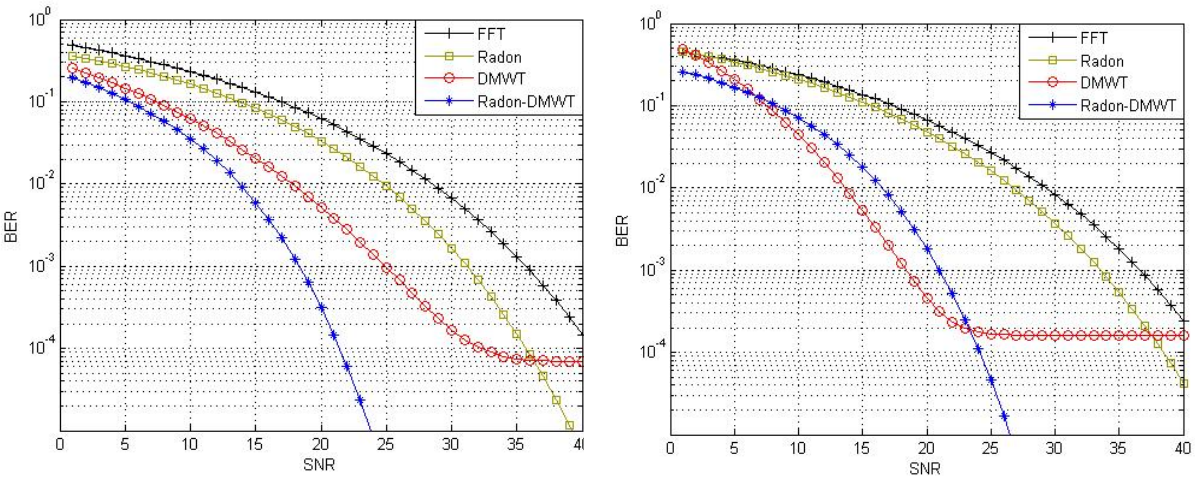
(d) BER performance of Radon-DMWT-OFDM in Selective Fading Channel at Max Doppler Shift=500Hz

Figure 6. BER performance of Radon- DMWT-OFDM in a selective fading channel at different Doppler Shift frequency

5.1 EFFECT OF THE VARIATION OF CHANNEL PARAMETERS ON THE PERFORMANCE OF THE PROPOSED RADON-MULTIWAVELET BASED OFDM

Now, the effect of variation of selective fading channel parameters on the performance of the proposed system is studied here. The first test gives the effect of changing the 2nd path gain on the performance of BER, the case study here is when the path gain of the second path becomes -4 dB.

The second test gives the effect of the 2nd path delay on the BER. The case study here is when the path delay of the second path become 1 μsec. The two cases were studied at Doppler frequency 80 Hz, and the results are shown in Figures (7-a) and (7-b) respectively.



(a) (b)

Figure 7. (a) BER performance of Radon-DMWT-OFDM in Selective Fading Channel at Doppler Shift frequency=80 Hz with path gain -4 dB. (b) BER performance of Radon-DMWT-OFDM in Selective Fading Channel at Doppler Shift frequency=80 Hz with path delay 1 μsec.

It can be seen from Figure (7-a) that the proposed system having the smallest sensitivity to the variation of the channel path gain against the other systems, and from Figure (7-b) the system is more sensitive to the variation of the path delay upon the variation of the path gain. This conclusion is applied to the other three systems, and it can be shown that the proposed system is still better than them.

6. DISCUSSIONS AND CONCLUSIONS

In this work a (FRAT) is used as a main building block into OFDM scheme. It was found that it is suitable for such implementation, and particularly, the FRAT was used as a data mapper instead of the conventional PSK and QAM techniques. This choice is easy to configure since it does not require a complex digital signal processing except some of 1D and 2D FFT algorithms which are available in all common DSP processors.

As a result of applying the FRAT, the BER performance was improved but it does not enough. DMWT offer the advantages of combining symmetry, orthogonality, and short support, which cannot be achieved by scalar two-channel wavelet systems at the same time. This advantages of DMWT over FFT pushing toward using it as a modulator instead of FFT in OFDM system. So the combining between the FRAT and DMWT will give an excellent performance and this was proved from the testing result of the proposed system.

These tests were carried out to verify its successful operation and its possibility of implementation. It can be concluded that this structure achieves much lower bit error rates assuming reasonable choice of the bases function and method of computation. It gave a SNR gain at 10^{-5} BER of about 4 dB against the DMWT based OFDM in AWGN channel and more than that gain against the other systems (Radon based OFDM, and FFT based OFDM).

In flat fading channel the SNR gain at 10^{-5} BER with Doppler frequency shift (4 Hz, 80Hz, 300Hz, and 500 Hz) were found to be (4 dB,5.8 dB,6.3 dB, and 6 dB respectively) against the DMWT based OFDM system. Wile in selective fading channel the SNR gain at 10^{-5} BER with Doppler frequency shift (4 Hz, and 80Hz) were found to be 4.4 dB, and 5 dB respectively.

It is shown from Figure (6-c) that the FFT-OFDM need SNR more than 40 dB to reach 10^{-4} BER, Radon-OFDM needs SNR around 39 dB to reach 10^{-5} which is so much SNR, while DMWT-OFDM doesn't exceed 0.002425 BER in spite of increasing of SNR so the system performance is not good, whereas the proposed system (Radon-Multiwavelet based OFDM) is more better than the three systems and reach 10^{-5} BER at 24.5 dB. It can be concluded from Figure (6-d) that when the Doppler frequency exceed 500 Hz then the proposed system faced the same problem

that the DMWT-OFDM system suffer from, so it can be seen that the proposed system will be sensitive to the variation of Doppler frequency when its equal or exceed 500 Hz.

In the proposed system there is no need of using CP as mentioned previously, and subsequently, reduce the system complexity, increase the transmission rate, and increase spectral efficiency.

Another conclusions can be obtained after the variation of channel parameters were done, which are the proposed system having the smallest sensitivity to the variation of the channel path gain against the other systems, and the system is more sensitive to the variation of the path delay upon the variation of the path gain. This conclusion is applied to the other three systems, and in spite of that the proposed system is more efficient.

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