The generalized solutions of Maxwell equations for the uniaxial crystal

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Abstract

The exact analytical solutions of Maxwell's equations are constructed by means of the method of generalized functions in vector form for unbounded uniaxial crystals. The fundamental solutions of a system of Maxwell's equations for uniaxial crystals are obtained. In particular, the solutions for elementary electric dipoles are deduced and the radiation patterns for Hertzian radiator dipole in such media are presented. Validity of the solutions have been checked up on balance of energy by integration of energy flow on sphere.

1. Introduction

Media with anisotropic properties are widely used in modern radio electronics, astrophysics and plasma physics. In recent years artificial anisotropic materials with set of chiral properties are especially useful for the design of microwave integrated circuits and optical devices [1]. By virtue of tensor character of dielectric permeability the analytical solution of the system of Maxwell equations becomes complicated. Therefore it is effectively solved by means of numerical methods, for example, the finite-element method and the Galerkin method are applied in Ref's [2], [3] and others. However numerical results do not provide a precise physical description of electromagnetic processes in anisotropic mediums.

Therefore the rigorous solution of the system of Maxwell equations in an anisotropic mediums is of great importance until now [4] - [6]. Analytical solutions for unlimited anisotropic mediums allow to construct the integral equations for the solution of the corresponding boundary value problems.

This paper is organized as follows: in the following section, we present the Maxwell equations in an unbounded crystal medium in the frequency domain. In

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section 3, we use the generalized method of Fourier transformation for solving the matrix form of Maxwell's equations [7], [8]. In section 4, we present radiation patterns in an unbounded crystal medium. Finally, in Section 5 conclusions and future related research are presented.

2. Maxwell's equations for a uniaxial anisotropic dielectric medium (one-axis crystal).

The electric and magnetic field strengths satisfy the following system of Maxwell's equations in frequency domain :

$$\begin{cases} \text{rot}\, \boldsymbol{H} + i\omega \cdot \boldsymbol{D} = \mathbf{J}, \\ \text{rot}\, \boldsymbol{E} - i\omega \cdot \boldsymbol{B} = 0, \end{cases} \tag{1}$$

where ω is the frequency of the electromagnetic (EM) field, E and H are the intensities of electric and magnetic field respectively, J is the current density, D is the electric displacement vector and B is the magnetic induction. The relations between (D,E) and (B,H) for an unbounded uniaxial medium are provided below.

Eq. (1) can be represented in matrix form as :

$$\mathbf{MU} = \mathbf{J}, \tag{2}$$

where

$$\mathbf{M} = \begin{pmatrix} -i\omega\varepsilon_0\hat{\varepsilon}\,\mathbf{I} & \mathbf{G}_0 \\ \mathbf{G}_0 & i\omega\mu_0\mu\,\mathbf{I} \end{pmatrix}, \qquad \mathbf{G}_0 = \begin{pmatrix} 0 & -\partial_z & \partial_y \\ \partial_z & 0 & -\partial_x \\ -\partial_y & \partial_x & 0 \end{pmatrix},$$

$$\mathbf{U} = \begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}, \qquad \boldsymbol{E} = \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}, \qquad \boldsymbol{H} = \begin{pmatrix} H_x \\ H_y \\ H_z \end{pmatrix}, \qquad \mathbf{J} = \begin{pmatrix} \boldsymbol{j} \\ \mathbf{0} \end{pmatrix},$$

$$\boldsymbol{j} = \begin{pmatrix} j_x \\ j_y \\ j_z \end{pmatrix}, \qquad \mathbf{0} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \qquad \hat{\varepsilon} = \begin{pmatrix} \varepsilon_1 & 0 & 0 \\ 0 & \varepsilon & 0 \\ 0 & 0 & \varepsilon \end{pmatrix},$$

where **M** is Maxwell's operator, **I** is a 3x3 identity matrix and μ represents the magnetic permeability.

The linear relation between the dielectric displacement D and the intensity of electric field E in uniaxial anisotropic dielectric mediums is:

$$\boldsymbol{D} = \hat{\varepsilon}\varepsilon_0 \boldsymbol{E}\,, \tag{3}$$

while the vector of magnetic induction is given by:

$$\boldsymbol{B} = \mu\mu_0 \boldsymbol{H} \tag{4}$$

If we choose a frame in main axes of dielectric tensor, the constitutive equation will be written as:

$$D_x = \varepsilon_1\varepsilon_0 E_x\,, \quad D_y = \varepsilon\varepsilon_0 E_y\,, \quad D_z = \varepsilon\varepsilon_0 E_z\,. \tag{5}$$

The elements of the dielectric permeability tensor $\hat{\varepsilon}$ correspond to a one-axis crystal, where the axis of the crystal is directed along axis *x*.

Moreover, it is required to define the intensities of the electromagnetic field $\boldsymbol{E}$, $\boldsymbol{H}$ in the space of generalized functions, as described in Section 3, below.

3. Problem Solution

A method based on the theory of the generalized functions of the Fourier transformation is used for solving the matrix equation (2) [9]:

$$\widetilde{\boldsymbol{E}}(\boldsymbol{k}) = \mathrm{F}[\boldsymbol{E}(\boldsymbol{r})] = \int_{\boldsymbol{R}^3} \boldsymbol{E}(\boldsymbol{r}) \cdot e^{-i\boldsymbol{kr}}\, dV, \tag{6}$$

where

$$\boldsymbol{E}(\boldsymbol{r}) = \mathrm{F}^{-1}[\widetilde{\boldsymbol{E}}(\boldsymbol{k})] = \int_{\boldsymbol{R}^3} \widetilde{\boldsymbol{E}}(\boldsymbol{k}) \cdot e^{i\boldsymbol{kr}} \frac{d^3k}{(2\pi)^3}, \tag{7}$$

and

$$\boldsymbol{k} = (k_x, k_y, k_z), \quad d^3k = dk_x dk_y dk_z\,, \quad dV = dxdydz\,, \quad \boldsymbol{r} = (x, y, z)\,.$$

By means of direct Fourier transformation we write down the system of equations (2) in matrix form:

$$\widetilde{\mathbf{M}}\widetilde{\mathbf{U}} = \widetilde{\mathbf{J}} \tag{8}$$

The solution of the problem is reduced to the determination of the system of the linear algebraic equations for the Fourier-components of the fields, where $\widetilde{\mathbf{U}}$ is defined by means of inverse matrix $\widetilde{\mathbf{M}}^{-1}$:

$$\widetilde{\mathbf{U}} = \widetilde{\mathbf{M}}^{-1}\widetilde{\mathbf{J}}\,. \tag{9}$$

By introducing new functions according to

$$\tilde{\Psi}_0 = \frac{1}{k_0^2 - k_x^2 - k_y^2 - k_z^2}, \qquad \tilde{\Psi}_1 = \frac{1}{k_n^2 - \frac{\varepsilon_1}{\varepsilon} k_x^2 - k_y^2 - k_z^2},$$

$$\tilde{\Psi}_2 = (\frac{\varepsilon_1}{\varepsilon} - 1)\tilde{\Psi}_1 \cdot \tilde{\Psi}_0, \tag{10}$$

the components of the electromagnetic field after transformations in Fourier space can be written as follows:

$$\begin{cases} \tilde{E}_x = -\frac{i}{\varepsilon\varepsilon_0\omega}\left(k_0^2 \tilde{j}_x - k_x(\boldsymbol{k}\tilde{\boldsymbol{j}})\right)\tilde{\Psi}_1, \\ \tilde{E}_y = -\frac{i}{\varepsilon_0\varepsilon\omega}\left(k_0^2 \tilde{\Psi}_0 \tilde{j}_y - k_y \tilde{\Psi}_1(\boldsymbol{k}\tilde{\boldsymbol{j}}) - k_0^2 k_y \tilde{\Psi}_2(\boldsymbol{k}\tilde{\boldsymbol{j}}_\perp)\right), \\ \tilde{E}_z = -\frac{i}{\varepsilon\varepsilon_0\omega}\left(k_0^2 \tilde{\Psi}_0 \tilde{j}_z - k_z \tilde{\Psi}_1(\boldsymbol{k}\tilde{\boldsymbol{j}}) - k_0^2 k_z \tilde{\Psi}_2(\boldsymbol{k}\tilde{\boldsymbol{j}}_\perp)\right), \end{cases} \tag{11}$$

$$\begin{cases} \tilde{H}_x = -i\tilde{\Psi}_0[\boldsymbol{k} \times \tilde{\boldsymbol{j}}]_x, \\ \tilde{H}_y = -i\left(k_z \tilde{j}_x \tilde{\Psi}_1 - k_x k_z \tilde{\Psi}_2(\boldsymbol{k}\tilde{\boldsymbol{j}}_\perp) + k_x \tilde{j}_z \tilde{\Psi}_0\right), \\ \tilde{H}_z = -i\left(k_x \tilde{j}_y \tilde{\Psi}_0 + k_y \tilde{j}_x \tilde{\Psi}_1 + k_x k_y \tilde{\Psi}_2(\boldsymbol{k}\tilde{\boldsymbol{j}}_\perp)\right), \end{cases} \tag{12}$$

where,

$$\tilde{\boldsymbol{j}} = \tilde{\boldsymbol{j}}_\perp + \tilde{\boldsymbol{j}}_0, \quad \tilde{\boldsymbol{j}}_0 = (\tilde{j}_x, 0, 0), \quad \tilde{\boldsymbol{j}}_\perp = \left(0, \tilde{j}_y \tilde{j}_z\right),$$

$$k_0^2 = \omega^2 \varepsilon_0 \varepsilon \mu \mu_0, \quad k_n^2 = k_0^2 \varepsilon_1 / \varepsilon \tag{13}$$

that is k_n is the propagation constant along the axis of the crystal (x-axis), and k_0 is the propagation constant along the y and z axis.

It is possible to represent the electromagnetic fields in vector form :

$$\tilde{\boldsymbol{E}} = -\frac{i}{\varepsilon_0\varepsilon\omega}\left(k_0^2\left\{\tilde{\boldsymbol{j}}_\perp \tilde{\Psi}_0 + \tilde{\boldsymbol{j}}_0 \tilde{\Psi}_1 + \left(\boldsymbol{k}_x(\boldsymbol{k}\tilde{\boldsymbol{j}}_\perp) - \boldsymbol{k}(\boldsymbol{k}\tilde{\boldsymbol{j}}_\perp)\right)\tilde{\Psi}_2\right\} - \boldsymbol{k}(\boldsymbol{k}\tilde{\boldsymbol{j}})\tilde{\Psi}_1\right), \tag{14}$$

$$\tilde{\boldsymbol{H}} = -i[\boldsymbol{k}, (\tilde{\boldsymbol{j}}_\perp \tilde{\Psi}_0 + \tilde{\boldsymbol{j}}_0 \tilde{\Psi}_1 + \boldsymbol{k}_x(\boldsymbol{k}\tilde{\boldsymbol{j}}_\perp)\tilde{\Psi}_2)], \quad \boldsymbol{k}_x = \mathbf{i}(\mathbf{i}\boldsymbol{k}) \tag{15}$$

where $\mathbf{i}$ is the unit vector along x-axis.

Using the property of convolution:

$$\mathrm{F}^{-1}[\widetilde{\mathbf{M}}^{-1}\cdot\widetilde{\mathbf{J}}]=\mathbf{M}^{-1}*\mathbf{J} \tag{16}$$

and by considering the inverse Fourier transformation

$$\mathbf{M}^{-1}=\mathrm{F}^{-1}[\widetilde{\mathbf{M}}^{-1}],\qquad \mathbf{J}=\mathrm{F}^{-1}[\widetilde{\mathbf{J}}] \tag{17}$$

it is possible to get the solution of the Maxwell equations (2) as:

$$\mathbf{U}=\mathbf{M}^{-1}*\mathbf{J} \tag{18}$$

where symbol "$*$" denotes the convolution on coordinates x, y, z.

The given solution is written in the form of the sum of two independent solutions:

$$\boldsymbol{E}=\boldsymbol{E}_1+\boldsymbol{E}_2,\quad \boldsymbol{H}=\boldsymbol{H}_1+\boldsymbol{H}_2 \tag{19}$$

The first of them, fields $\mathbf{E_1}$ and $\mathbf{H}_1$, is defined by Green's function Ψ_1 and the density of the current $\widetilde{\boldsymbol{j}}_0$ along the axis of the crystal:

$$\begin{cases}\boldsymbol{E}_1=-\dfrac{i}{\varepsilon_0\varepsilon\omega}(\mathrm{graddiv}+k_0^2)(\Psi_1*\boldsymbol{j}_0),\\ \boldsymbol{H}_1=-\mathrm{rot}(\Psi_1*\boldsymbol{j}_0),\end{cases} \tag{20}$$

where

$$\Psi_1\equiv-\frac{1}{4\pi}\sqrt{\frac{\varepsilon}{\varepsilon_1}}\frac{\exp(ik_n r')}{r'}, \tag{21}$$

and

$$r'=\sqrt{(\varepsilon/\varepsilon_1)\cdot x^2+y^2+z^2}\,.$$

The second solution, fields $\mathbf{E_2}$ and $\mathbf{H}_2$, can be written by using the component of the density of the current $\widetilde{\boldsymbol{j}}_\perp$ perpendicular to axis x and Green's functions Ψ_0, Ψ_1 and Ψ_2:

$$\begin{cases}\boldsymbol{E}_2=-\dfrac{i}{\varepsilon\varepsilon_0\omega}\left(k_0^2(\boldsymbol{j}_\perp*\Psi_0+\mathrm{grad}_\perp\mathrm{div}\boldsymbol{j}_\perp*\Psi_2)+\mathrm{graddiv}\boldsymbol{j}_\perp*\Psi_1\right),\\ \boldsymbol{H}_2=-\mathrm{rot}\left(\boldsymbol{j}_\perp*\Psi_0-\mathbf{i}\dfrac{\partial}{\partial x}\mathrm{div}\boldsymbol{j}_\perp*\Psi_2\right),\end{cases} \tag{22}$$

$$\Psi_0\equiv-\frac{1}{4\pi}\frac{\exp(ik_0 r)}{r},\quad r=\sqrt{x^2+y^2+z^2}\,,\quad \mathrm{grad}_\perp=\mathrm{grad}-\mathbf{i}\frac{\partial}{\partial x}, \tag{23}$$

$$\Psi_2\equiv(\varepsilon_1/\varepsilon-1)\Psi_0*\Psi_1, \tag{24}$$

We note here that the function Ψ_0, Eq. (23), is a fundamental solution of the Helmholtz operator for isotropic medium, while Ψ_1, Eq. (21), corresponds to the function Ψ_0 for the space deformed along the axis of the crystal.

Furthermore, the following useful identities are valid :

$$\mathrm{F}^{-1}\left[\tilde{\Psi}_0 \cdot \frac{1}{k_0^2 - k_x^2}\right] = \Psi_0 * \mathrm{F}^{-1}\left[\frac{1}{k_0^2 - k_x^2}\right] = -\frac{i}{8\pi \mathrm{k}_0}\Big[\exp(ik_0x)\cdot\big(\mathrm{Ci}(k_0(r-x)) + i\cdot Si(k_0(r-x))\big) + \exp(-ik_0x)\cdot\big(\mathrm{Ci}(k_0(R+x)) + i\cdot Si(k_0(R+x))\big)\Big] \tag{25}$$

$$\mathrm{F}^{-1}\left[\tilde{\Psi}_1 \frac{1}{k_0^2 - k_x^2}\right] = \Psi_1 * \mathrm{F}^{-1}\left[\frac{1}{k_0^2 - k_x^2}\right] = -\frac{i}{8\pi \mathrm{k}_0}\Big[\exp(ik_0x)\cdot\big(\mathrm{Ci}(k_n r' - k_0 x) + i\cdot Si(k_n r' - k_0 x)\big) + \exp(-ik_0x)\cdot\big(\mathrm{Ci}(k_n r' + k_0 x) + i\cdot Si(k_n r' + k_0 x)\big)\Big] \tag{26}$$

$$\mathrm{F}^{-1}[\tilde{\Psi}_2] = \mathrm{F}^{-1}[\tilde{\Psi}_0] * \mathrm{F}^{-1}\left[\frac{1}{k_0^2 - k_{\tilde{\sigma}}^2}\right] - \mathrm{F}^{-1}[\tilde{\Psi}_1] * \mathrm{F}^{-1}\left[\frac{1}{k_0^2 - k_{\tilde{\sigma}}^2}\right] \tag{27}$$

Therefore, by also using Eqs. (25), (26) above we find that the function Ψ_2 is given by:

$$\begin{aligned}\Psi_2 = -\frac{i}{8\pi k_0}\Big[&\exp(ik_0x)\big(\mathrm{Ci}(k_0(r-x)) + i\cdot Si(k_0(r-x))\big) + \exp(-ik_0x)\cdot \\ &\cdot\big(\mathrm{Ci}(k_0(r+x)) + i\cdot Si(k_0(r+x))\big) - \exp(ik_0x)\cdot \\ &\cdot\big(\mathrm{Ci}(k_n r' - k_0 x) + i\cdot Si(k_n r' - k_0 x)\big) - \exp(-ik_0x)\cdot \\ &\cdot\big(\mathrm{Ci}(k_n r' + k_0 x) + i\cdot Si(k_n r' + k_0 x)\big)\Big]\end{aligned} \tag{28}$$

where integral cosine and integral sine functions are defined by the following formulae:

$$\mathrm{Ci}(z) = \gamma + \ln(z) + \int_0^z \frac{\cos t - 1}{t}\,dt, \qquad Si(z) = \int_0^z \frac{\sin t}{t}\,dt - \frac{\pi}{2},$$

and Euler constant $\gamma = 0{,}5772$.

The solutions (19), (20) and (22) can be also represented with the help of vector potentials $\mathbf{A}_0$, $\mathbf{A}_1$ and $\mathbf{A}_2$ as follows :

$$\begin{cases} \boldsymbol{E} = i\omega\left(\mathbf{A}_0 + \mathbf{i}(\mathbf{i}\mathbf{A}_1) + \mathrm{grad}_\perp \mathrm{div}\mathbf{A}_2 + \dfrac{1}{k_0^2}\mathrm{graddiv}\mathbf{A}_1\right) \\ \boldsymbol{H} = \dfrac{1}{\mu\mu_0}\mathrm{rot}\left(\mathbf{A}_0 + \mathbf{i}(\mathbf{i}\mathbf{A}_1) - \mathbf{i}\dfrac{\partial}{\partial x}\mathrm{div}\mathbf{A}_2\right) \end{cases} \tag{29}$$

where $\mathbf{i}$ is the unit vector along x-axis. The vector potentials $\mathbf{A}_0$, $\mathbf{A}_1$ and $\mathbf{A}_2$ satisfy the following equations:

$$(\Delta + k_0^2)\mathbf{A}_0 = -\mu\mu_0 \cdot \boldsymbol{j}_\perp, \qquad (\Delta' + k_n^2)\mathbf{A}_1 = -\mu\mu_0 \cdot \boldsymbol{j} \tag{30}$$

$$(\Delta + k_0^2)(\Delta' + k_n^2)\mathbf{A}_2 = -\mu\mu_0(\varepsilon_1/\varepsilon - 1)\boldsymbol{j}_\perp \tag{31}$$

where Δ is the Laplacian operator. Finally, the solutions of Eqs. (30), (31) can be written as follows:

$$\mathbf{A}_0 = -\mu\mu_0 \cdot \boldsymbol{j}_\perp * \Psi_0, \qquad \mathbf{A}_1 = -\mu\mu_0 \cdot \boldsymbol{j} * \Psi_1, \qquad \mathbf{A}_2 = -\mu\mu_0 \boldsymbol{j}_\perp * \Psi_2 \tag{32}$$

4. Radiation patterns of a Hertzian dipole in uniaxial anisotropic media.

On the basis of the results obtained above, we shall consider the radiation of the electric Hertzian dipole in unbounded one-axis crystals. The point dipole moment is given by

$$\boldsymbol{p} = \mathbf{n}p_e \exp(-i\omega \cdot t), \tag{33}$$

where p_e is a constant, $\mathbf{n}$ is a unit vector parallel to the direction of the dipole moment, and the current density is defined by means of Dirak delta-functions:

$$\boldsymbol{j} = -i\omega \cdot \boldsymbol{p}\delta(\boldsymbol{r}), \quad \boldsymbol{p} = \boldsymbol{p}_0 + \boldsymbol{p}_\perp \tag{34}$$

The last formula of current density follows from the expression of charge density for the point dipole, as given by :

$$\rho = -(\boldsymbol{p}\nabla)\delta(\boldsymbol{r}) \tag{35}$$

and also from the charge conservation law, which can be written as :

$$\mathrm{div}\boldsymbol{j} - i\omega\rho = 0. \tag{36}$$

Furthermore, the expression of the radiated electromagnetic field for electric Hertzian dipole will take the following form, when the direction of the dipole moment $\boldsymbol{p}_0$ is parallel to the axis x of the crystal (Fig.1):

$$\begin{cases} \boldsymbol{E}_1 = -(\varepsilon\varepsilon_0)^{-1}(\text{graddiv} + k_0^2)(\Psi_1 \boldsymbol{p}_0), \\ \boldsymbol{H}_1 = i\omega \cdot \text{rot}(\Psi_1 \boldsymbol{p}_0). \end{cases} \tag{37}$$

Also, when the direction of the dipole moment is perpendicular to the axis x, we obtain (Fig. 2.):

$$\begin{cases} \boldsymbol{E}_2 = -\dfrac{1}{\varepsilon\varepsilon_0}\left(k_0^2(\boldsymbol{p}_\perp \Psi_0 + \text{grad}_\perp \text{div}(\boldsymbol{p}_\perp \Psi_2)) + \text{graddiv}(\boldsymbol{p}_\perp \Psi_1)\right), \\ \boldsymbol{H}_2 = i\omega \cdot \text{rot}\left(\boldsymbol{p}_\perp \Psi_0 - \mathbf{i}\dfrac{\partial}{\partial x}\text{div}(\boldsymbol{p}_\perp \Psi_2)\right) \qquad (\boldsymbol{p}_0 \perp \boldsymbol{p}_\perp). \end{cases} \tag{38}$$

Furthermore, we note here that the numerical calculation of the above solution of Maxwell equations satisfies the *energy conservation law*.

Moreover, we note that the independent solutions (20) and (22) define the corresponding polarization of electromagnetic waves. In addition, when ε_1 tends to ε, from Eq. (10) it follows that the potential Ψ_2 tends to zero and the well-known expressions of electromagnetic field follow from formula (20):

$$\mathbf{E} = i\omega\left(\frac{1}{k_0^2}\text{graddiv} + 1\right)\mathbf{A}, \tag{39}$$

$$\mathbf{H} = (\mu\mu_0)^{-1}\text{rot}\mathbf{A}, \tag{40}$$

where the well - known vector potential of the electromagnetic field for isotropic media is calculated from (32) as:

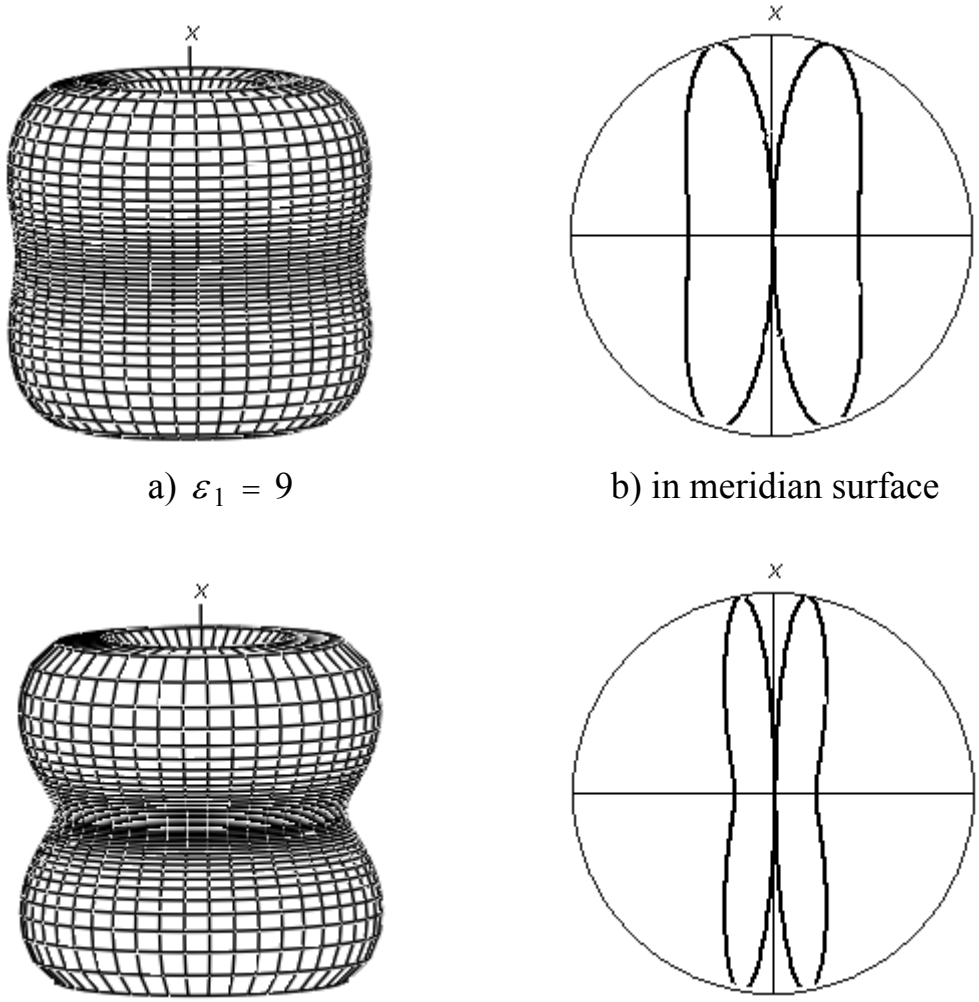
$$\mathbf{A} = \frac{\mu\mu_0}{4\pi}\int_V \mathbf{j}\frac{\exp(ik|\mathbf{r}-\mathbf{r}'|)}{|\mathbf{r}-\mathbf{r}'|}dV\,. \tag{41}$$

The obtained generalized solutions of the Maxwell equations are valid for any values of ε_1 and ε, as well as for sources of the electromagnetic waves, described by discontinuous or singular functions. The radiation pattern of the Hertzian dipole in isotropic medium is shown in Fig. 3, which of course possesses rotation symmetry around x-axis.

5. Conclusions

In this paper we have investigated the problem of propagation of electromagnetic (EM) waves in unbounded uniaxial anisotropic media. The solution of the problem was analyzed in Fourier space and closed form analytical solutions were

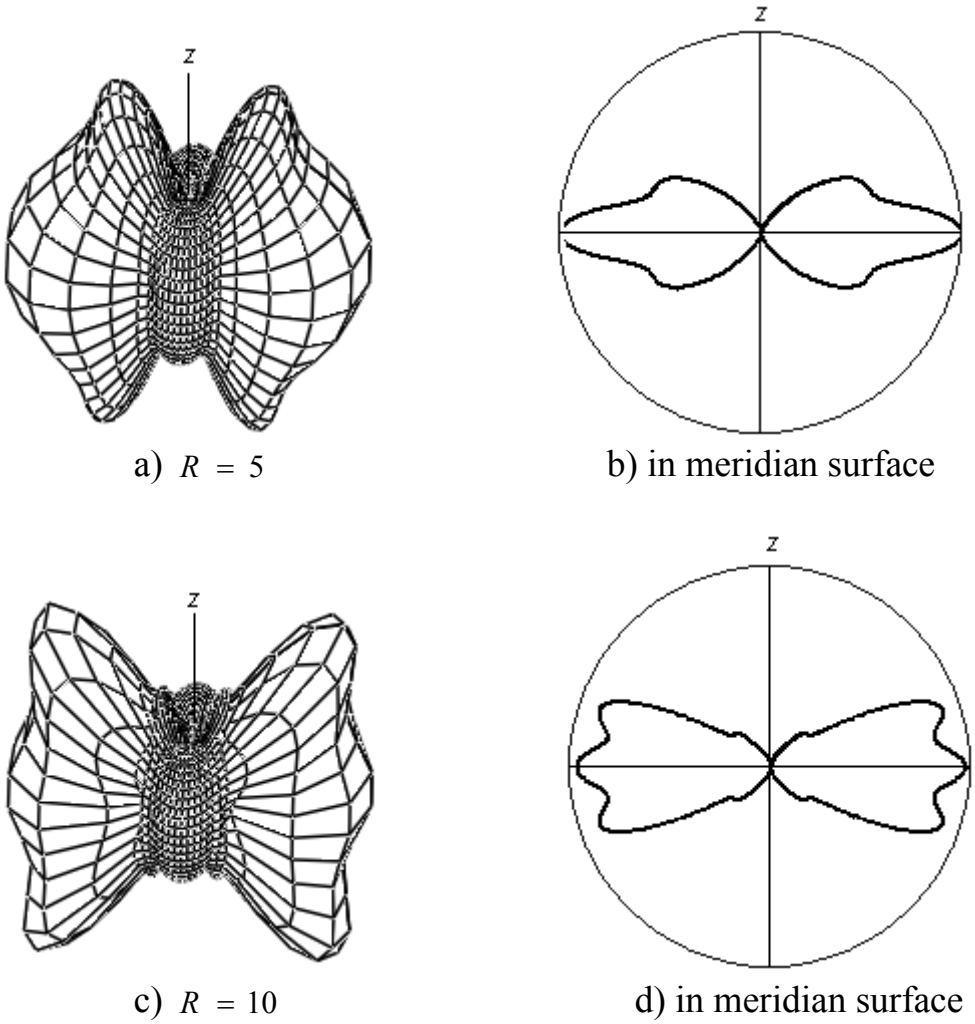
derived in Section 3, above. Then, when the current (source) distribution is defined in such a medium, the corresponding radiated electric and magnetic fields can be calculated anywhere in space. As a specific application, radiation from a Hertzian dipole in such a medium was examined in Section 4, and the corresponding radiation patterns were presented (figs. 1,2) and were verified through the energy conservation criterion. Finally, the governing equations and radiation pattern in the case of an unbounded isotropic medium were obtained as a special case.



a) $\varepsilon_1 = 9$ b) in meridian surface

c) $\varepsilon_1 = 25$ d) in meridian surface

Fig. 1. Radiation patterns of a Hertzian dipole inside an unbounded uniaxial anisotropic crystal. The axis of the dipole (x-axis) is parallel to the axis of the crystal.



a) $R = 5$ b) in meridian surface

c) $R = 10$ d) in meridian surface

Fig. 2. Radiation patterns of a Hertzian dipole inside an unbounded uniaxial anisotropic crystal. Here the axis of dipole (z) is perpendicular to axis of the crystal (x-axis). The value $\varepsilon_1 = 7$ is chosen here.

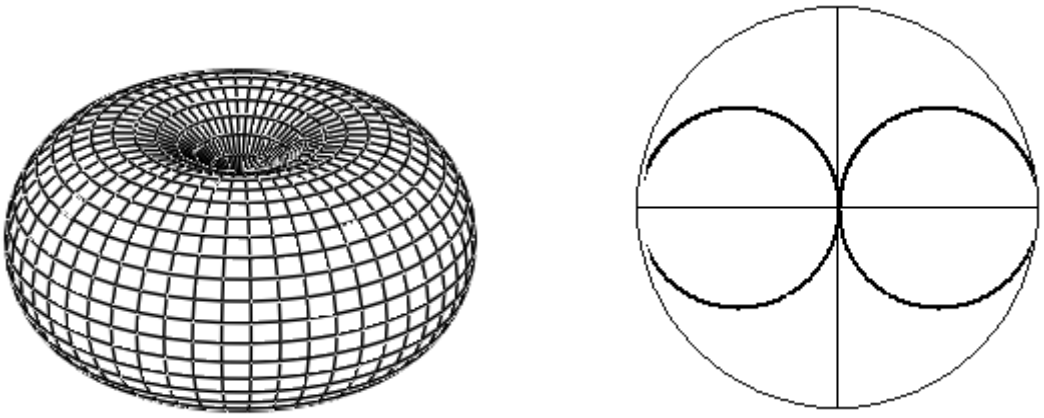


Fig. 3. Radiation pattern of a Hertzian dipole in the limiting case of an isotropic medium, $\varepsilon_1 = 1$. On the left, the three – dimensional radiation pattern is shown, while on the right the radiation pattern at the meridian surface is depicted.

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