Optimal Multiplication Coefficients in the Application of Avalanche Photodiodes in Optical Communication Systems

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Abstract

This paper deals with the engineering design of a multi-channel digital optical communication system with wavelength-division multiplexing (WDM) in most cases, in which this design is based on a given probability of an error in one one-bit interval (BER). There is a solution to the problem of deriving the given value of BER through a maximum economical energy balance of the system. There is an analytical derivation of the optimal multiplication coefficient in the photo-detector (avalanche photodiode) of the optical receiver, i.e. the multiplication coefficient under which the technical task is performed with a minimal signal radiant flux.

There are derivations of expressions for engineering calculation of the optimal avalanche multiplication coefficient as well as of the minimal signal optical flow. There are model project calculations with a set of incoming data adequate to the task. The results are discussed and conclusions relevant to them are made..

1. INTRODUCTION

The contemporary development of communication technologies is related to a great extent with the optical span of the electromagnetic spectrum [1–8]. This circumstance leads to an increasing interest in the parameter optimizing of the structural units of optical communication systems (OCS) with the aim of deriving the highest possible quality indicators of these systems. An essential position in the mentioned problematic is given to specifying the optimal avalanche multiplication coefficient in the photo detector of the optical receiver in the process of OCS design [1, 2, 4, 6, 8–10]. This problem has two aspects:

(1) Derivation of μ_{opt} with the goal to derive $(BER)_{\min}$, respectively $(SNR)_{\max}$

on the condition of a given signal optical flux Φ_{rs} at the input aperture of the photo detector.

(2) Derivation of μ_{opt} with the goal to derive $(\Phi_{rs})_{\min}$, respectively minimal output power for the laser (or LED) used in the OCS with a given value *BER*, or respectively a given *SNR*.

The first aspect is well-studied in references [9–10].

The second aspect, as far as we are aware, was studied only in our book [8]. But our opinion is that especially this second aspect has a greater impact on engineer practice. In fact during the design process *BER* is given most often, because this is the main quality characteristic of digital OCS.

For this reason the main goal of the present work is the analytical derivation of the optimal multiplication coefficient $\mu_{opt} = \mu_{opt,2}$ for the avalanche photodiode (APD) of the OCS, respectively of the corresponding to $\mu_{opt,2}$ minimal signal optical flux $(\Phi_{rs})_{\min}$ in APD input aperture, with a given value of *BER*, respectively value of *SNR*.

2. THEORETICAL ANALYSIS

The two problem aspects for μ_{opt} commented above have a common theoretical basis – a full analytical description of OCS. The latter was derived (with algorithms for engineering design and concrete numerical examples) in our book [7] – for fiber and for free space OCS.

In compliance with the relation

$$BER = \frac{1}{2} erfc\left(\frac{SNR}{2\sqrt{2}}\right), \; erfc(Q) = \frac{2}{\sqrt{\pi}} \int_Q^{\infty} e^{-x^2} dx, \tag{1}$$

the value of *BER* is directly connected with the value of *SNR* (when using the simple binary code).The ratio “signal to noise” was defined by the expression

$$SNR = \frac{S(\bar{\lambda})\Phi_{rs}\,\mu}{\sqrt{2\left[2k_B T\left(\frac{1}{R} + \frac{1}{R_{Fb}}\right)A + e\,S(\bar{\lambda})\left(\Phi_{rs}\mu^{2+x} + \Phi_{rb}\right)\right]\Delta f}} \tag{2}$$

[7–10], where: Φ_{rs} и Φ_{rb} are, respectively, signal and background optical fluxes in the aperture of the photodetector; $S(\bar{\lambda})$ is the spectral response of the photodetector; $\bar{\lambda}$ is the average wavelength in the chromatic span of the OCS with WDM;

$k_B = 1{,}38.10^{-23}\dfrac{J}{K}$ is the Boltzmann constant; T is the thermodynamic temperature of the optical receiver; R and R_{Fb} are, respectively, the input resistance and the resistance of the feedback of the transimpedance optical receiver amplifier ; $A > 1$ is a coefficient which normalizes the relative noise of the amplifier elements to the equivalent thermal noise [10]; $e = 1{,}6.10^{-19}C$ is the electrical charge of the electron; Δf is the width of the optical spectrum of the pulse-code signal in one given TDM - subsystem in OCS with WDM; x is the noise coefficient related to the avalanche multiplication.

In fiber optic OCS, the background flux is of course equal to zero. In free space optic OCS this flux is calculated with the relation $\Phi_{rb} = \pi^2 \tau_r l_b(\bar{\lambda}) R_r^2 \theta_r^2 (\Delta\lambda)_{IF}$ [8], where τ_r is the transparency of the group "receiver antenna – interference filter"; l_b is the chromatic density of background flux brightness; R_r and θ_r are, respectively, radius and half angle of observation of the receiver antenna; $(\Delta\lambda)_{IF}$ is the pass wave band of the interference filter.

Further on we mean a fiber optic OCS, i.e. we will assume $\Phi_{rb} = 0$. The taking into account of the background flux for a free space optic OCS, however, is not a serious problem.

For the sake of completeness we will present the value $\mu_{opt,1}$ of μ_{opt} for the first aspect of the optimal multiplication coefficient. This value was derived by the operation $\dfrac{\partial(SNR)}{\partial\mu} = 0$, based on (2) and is described by

$$\mu_{opt,1} = \left[\frac{4k_B T\left(\dfrac{1}{R} + \dfrac{1}{R_{Fb}}\right)A}{e\,S(\bar{\lambda})\Phi_{rs}x}\right]^{\frac{1}{2+x}} . \qquad (3)$$

Now we pass on to the problem of deriving the value $\mu_{opt,2}$ of μ_{opt} in terms of the second aspect for deriving the optimal avalanche multiplication coefficient – with a given *BER* (or *SNR*), i.e. we pass on to the problem of deriving $(\Phi_{rs})_{\min}$.

For this purpose we solve (2) in respect of Φ_{rs} (we assume $\Phi_{rb} = 0$) and present the result in the following form

$$\Phi_{rs} = \alpha\left[\beta\mu^{x} + \left(\beta^{2}\mu^{2x} + \gamma\mu^{-2}\right)^{1/2}\right], \tag{4}$$

where

$$\alpha = \frac{SNR}{S(\bar{\lambda})},$$

$$\beta = e(SNR)\Delta f\,,\ \gamma = 4k_B T\left(\frac{1}{R} + \frac{1}{R_{Fb}}\right)A\Delta f\,. \tag{5}$$

Calculating the derivative $\frac{\partial\Phi_{rs}}{\partial\mu}$ and equalizing it to zero gives us the equation

$$\left(1 - x^{2}\right)\mu^{4x} + K^{2}\mu^{-4} = \left(2K + x^{2}\right)\mu^{2(x-1)}, \tag{6}$$

which defines $\mu_{opt,2}$ on the condition

$$K = \frac{\gamma}{\beta^{2}}. \tag{7}$$

In the simple particular cases $x = 0{,}5$ and $x = 1$ equation (6) has explicit solutions. They are:

$$\begin{aligned} \mu_{opt,2}(x = 0{,}5) &= \left[\frac{4}{3}K + \frac{1}{6} + \frac{2}{3}\left(K^{2} + K + \frac{1}{16}\right)^{1/2}\right]^{1/3}, \\ \mu_{opt,2}(x = 1) &= \left[\frac{K}{(2K+1)^{1/2}}\right]^{1/2}. \end{aligned} \tag{8}$$

Of course if during the solution process of (6) for some values of x, β and γ, i.e. for x and K (in compliance with (7)), is derived $\mu_{opt,2} < 1$ or $\mu_{opt,2} \approx 1$, for photo detector, it is reasonable to use a photo diode.

The corresponding to $\mu_{opt,2}(x, K)$ minimal value of the signal optical flux in the aperture of APD was derived by calculations with the expression

$$\left(\Phi_{rs}\right)_{\min} = \alpha\beta\left[\mu_{opt,2}^{x} + \left(\mu_{opt,2}^{2x} + K\mu_{opt,2}^{-2}\right)^{1/2}\right], \tag{9}$$

which was derived by (4), (7) and (5).

3. NUMERICAL RESULTS

In order to define $\mu_{opt,2}$ by the given value of *BER*, we have to use a significant part of the algorithm for engineering design of a multi-channel digital OCS with WDM

which we have developed in chapter V. of [8]. We assume the following sample input data: $BER = 10^{-12}$; $(C_I)_{WDM} = 10\frac{\text{Gbit}}{\text{s}}$ ($(C_I)_{WDM}$ is bit rate of OCS with WDM); $N = 8$ (N is the number of subsystems with TDM, which comprise the infrasystem with WDM); $\bar{\lambda} = 1{,}55\,\mu\text{m}$; $\eta(GaInAs; \bar{\lambda} = 1{,}55\,\mu m) = 0{,}7$ (η is the quantum efficiency of photodetector); $R_{Fb} = 1k\Omega$; $R = 100\,k\Omega$; $A = 5$; $T = 300\,K$; $x(GaInAs) = 0{,}6$.

From (1) we derive

$Q = 4{,}97$; $SNR = 2\sqrt{2}\,.\,4{,}97 = 14{,}06$.

using the formula

$$S(\bar{\lambda}) = 0{,}805\,.\,\eta(\bar{\lambda})\bar{\lambda}[\mu m]$$

we calculate

$$S(\bar{\lambda}) = 0{,}805\,.\,0{,}7\,.\,1{,}55 = 0{,}873\frac{A}{W}.$$

By the relation $\Delta f = (C_I)_{WDM} / 2N$ we derive

$$\Delta f = (\Delta f)_{TDM} = \frac{10.10^9}{2.8} = 0{,}625\,GHz.$$

By the expressions (5) and (7) we calculate

$$\alpha = \frac{14{,}06}{0{,}873} = 16{,}1\frac{W}{A},$$

$$\beta = 1{,}6.10^{-19}.0{,}625.10^9.14{,}06 = 1{,}406.10^{-9}A,$$

$$\gamma = 4.1{,}38.10^{-23}.0{,}625.10^9.3.10^2 \times$$
$$\times\left(\frac{1}{10^3} + \frac{1}{10^2.10^3}\right)5 = 5{,}227.10^{-14}A^2,$$

$$K = \frac{5{,}227.10^{-14}}{(1{,}406.10^{-9})^2} = 2{,}644.10^4.$$

Now we will derive $\mu_{opt,2}$ by the graphical solution of equation (6) for $x = 0{,}6$. For the sake of convenience we multiply the two sides of (6) by 10^{-4} and present it in the form $L(\mu) = R(\mu)$, where

$L(\mu) = 0{,}64.10^{-4}\mu^{2,4} + 6{,}991.10^{4}\mu^{-4}$, $R(\mu) = 5{,}288\mu^{-0,8}$.

The functions $L(\mu)$ and $R(\mu)$ have been drawn graphically in fig.1. By the

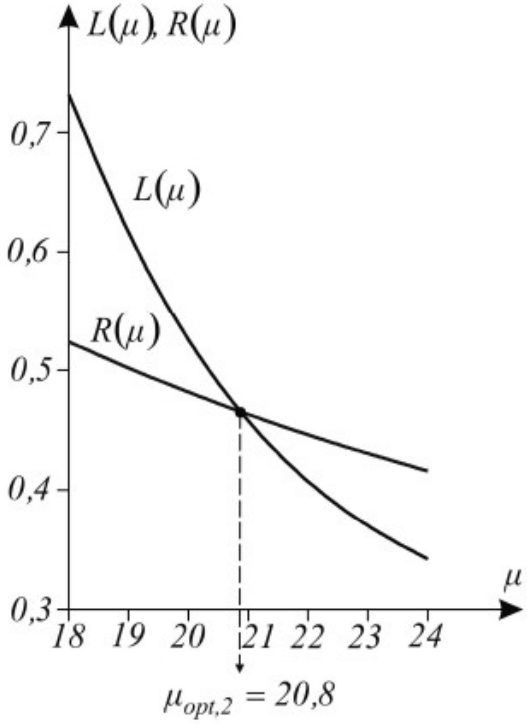

Figure 1. Functions $L(\mu)$ and $R(\mu)$

crossing-point of the two curves we derive $\mu_{opt,2}(x = 0{,}6) = 20{,}8$.

The well-expressed minimum of the function $\Phi_{rs}(\mu)$ has been shown in fig. 2. This expression proper gives sense to and justifies the search for $\mu_{opt,2}$.

To illustrate the essential dependence of $\mu_{opt,2}$ on x we will also make calculations with formulae (8). We derive

$$\mu_{opt,2}(x = 0{,}5) = 37{,}53$$

and

$$\mu_{opt,2}(x = 1) = 10{,}72 .$$

Our next step is the derivation of the minimal value of the signal optical flux in the

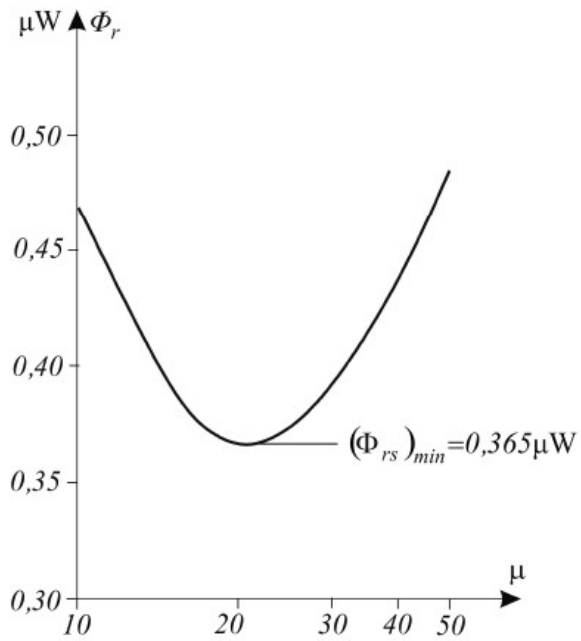

Figure 2. Function $\Phi_{rs}(\mu)$

APD, which corresponds to $\mu_{opt,2} = 20{,}8$ and guarantees the realization of $BER = 10^{-12}$. By the expression (9) we derive $(\Phi_{rs})_{\min} = 0{,}365\,\mu\text{W}$ (see also fig. 2).

With a view to a numerical comparison of the two aspects of the problem for the derivation of μ_{opt} we will assume that as a result of the solution of the forward problem we have derived $(\Phi_{rs})_{\min} = 0{,}2\,\mu\text{W}$.

Then, using (3) we find $\mu_{opt,1}(x = 0{,}6) = 36{,}34$.

By replacing (2), we calculate $(SNR)_{\max}(x = 0{,}6) = 13{,}3$.

Further on we have $Q = 4{,}702$, $erfc(Q) = 2{,}955.10^{-11}$ and relation (1) gives us $(BER)_{\min}(x = 0{,}6) = 1{,}498.10^{-11}$.

4. CONCLUSION

1. Defined and solved is the practically important problem of deriving the optimal coefficient $\mu_{opt,2}$ of multiplication in the photodetector (avalanche photo diode) of multi-channel digital optical communication system with WDM. In this case the sense of the optimum consists in the realization of the given value of BER with a minimal energy resource of the system.

2. We have derived expression (6), which (along with expressions (7) and (5)) defines the value of $\mu_{opt,2}$ with a random value of coefficient x of the noise of the avalanche multiplication.

3. We have derived expression (9) for the calculation of the minimal value $(\Phi_{rs})_{\min}$ of the signal optical flux in the input aperture of the avalanche photo diode, which guarantees the realization of the BER value given in advance.

4. We have made sample design calculations for the derivation of $\mu_{opt,2}$. By the graphical materials we can see that the minimum of the function $\Phi_{rs}(\mu)$ is clearly expressed and relatively "deep", which gives sense to task of deriving $\mu_{opt,2}$. We have shown an essential dependence of $\mu_{opt,2}$ on x. We have also calculated the value of $(\Phi_{rs})_{\min}$ corresponding to $\mu_{opt,2}$. We have made a numerical comparison of the two aspects of the problem opf deriving μ_{opt}. There is a confirmation of the expectation that the decreasing of Φ_{rs} leads to the decreasing of SNR (to the increasing of *BER*), irrespective of the greater value of μ – the reason, obviously, is in the different situations of the two experimental tasks.

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