ON THE APPLICATION OF MULTIFRACTAL METHODS FOR THE ANALYSIS OF SEA SURFACE IMAGES RELATED TO SEA STATE DETERMINATION

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Abstract

Digital images may register states of a process, and image analysis help us to study this process and its peculiarities. In spite of the fact that there are a lot of textbooks on techniques of image analysis, any new problem may requires a new method. Last decades fractal and multifractal methods are often used to analyze high resolution images having complex textures. In many cases fractal characteristics may be considered as classifying signs for further clustering. In this paper we present two multifractal methods. The first method allows us to obtain multifractal spectra and decompose an initial image into the union of nonintersecting sets. The second one is based on the transformation of an initial measure distribution under so called "direct multifractal transform" and calculation of information dimensions of measure supports. The examples of application of these techniques are given for various kinds of sea surface images.

1. INTRODUCTION

Now the methods of fractal and multifractal analysis are widely used to analyze images with complex structure. Such images are often fractals or multifractals. Fractal sets have a self-similarity property and may be described one numerical characteristic — fractal dimension. Multifractal sets are unions of several fractal subsets, which of them has its own fractal dimension, being these subsets are arranged in a complex intertwined manner. Hence a common characteristic for multifractals is multifractal spectrum — the set of fractal dimensions of its subsets. Our experience in analyzing biomedical preparation images (in [1] we calculated Rényi spectra for images of pharmacological solutions of Ag, and in [2,3] we applied such methods for sensitive crystallization images) testifies that the good separation of spectra results in successful classification of similar images.

In this work we consider two methods for calculation of multifractal characteristics for digital images: a method based on calculation so called density function, which is calculated for each pixel and characterizes intensity changes in its neighbourhood, and calculation of the set of information dimensions both for a given spectrum and its modified variants which are obtained by a renormalization of a given measure. Additionally, in this method we calculate the spectrum of averaged exponents of singularity. Thus to characterize an image we have 2 spectra. The combining of two methods increases the possibility for classification of images.

2. MULTIFRACTAL SPECTRUM BY USING DENSITY FUNCTION

We consider a special density function [4] to calculate the singularity power for every pixel. Then we combine all the pixels with close values of density function, which results in partition of the image on the subsets — so called level sets. For each level set we calculate its fractal dimension.

Let μ be a measure defined through pixel intensities for a given digital image. For $x \in R^2$ we denote $B(x,r)$ a square of length r with center x. Let $\mu\big(B(x,r)\big) = kr^{d(x)}(x)$, where $d(x)$ is so called the local density function of x, and k some constant. Taking several values for r we have

$$d(x) = \lim_{r \to 0} \frac{\log \mu(B(x,r))}{\log r}$$

The density function measures the non-uniformity of the intensity distribution in the square $\boldsymbol{B(x,r)}$. The set of all points $\boldsymbol{x}$ with local density $\boldsymbol{\alpha}$ is a level set $\boldsymbol{E_\alpha} = \{\boldsymbol{x} \in \boldsymbol{R^2}: \boldsymbol{d(x)} = \boldsymbol{\alpha}\}$. In practice, not to increase the number of level sets, one really consider the sets $\boldsymbol{E(\alpha,\varepsilon)} = \{\boldsymbol{x} \in \boldsymbol{R^2}: \boldsymbol{d(x)} \in [\boldsymbol{\alpha},\boldsymbol{\alpha}+\boldsymbol{\varepsilon}]$.

The set of fractal dimensions of s $\boldsymbol{E_\alpha}$ is the multifractal spectrum $\boldsymbol{f(\alpha)}$.

3. USING STATISTICAL SUM

It is usually assumed that an image is partitioned by cells with size $\boldsymbol{l}$, the number of cells is N(l) and the measure of $\boldsymbol{i}$-th cell is $\boldsymbol{p_i(l)} \sim \boldsymbol{l^{\alpha_i}}$. Consider the statistical sum $(\boldsymbol{q,l}) = \sum_{\boldsymbol{i=1}}^{\boldsymbol{N(l)}} \boldsymbol{p_i^q(l)}$) and the sequence of measures $\boldsymbol{\mu(q,l)} = \{\boldsymbol{\mu_i(q,l)}\}$ generated from the initial measure by the direct fractal transform $\mu_i(q,l) = \frac{p_i^q(l)}{\sum_{i=1}^N p_i^q(l)}$. The method was proposed in [5] and is based on the calculation of information dimension of a measure support M by the formula

$$\dim M = -\lim_{N\to\infty} \frac{\sum_{i=1}^N p_i \ln p_i}{\ln N}$$

The direct multifractal transform results in a transformation of the initial measure by using statistical sum, and hence it depends on q as well. For any measure from the generated sequence one may calculate the singularity power averaged over the measure and the fractal dimension of the support of the measure corresponding to this singularity power. Hence we obtain the averaged singularity spectrum $\alpha(q)$, and the fractal dimension of the support of the measure $f(q)$ as functions of the parameter q. Eliminating q one can obtain the relation between singularity values and fractal dimensions of corresponding subset.

For each measure $\mu(q,l)$ one can calculate information dimension of its support. As q changes, we have a set $f(q)$ of information dimensions of $\mu(q,l)$ supports, where

$$f(q) = \lim_{l\to 0} \frac{\sum_{i=1}^N \mu_i(q,l) \ln \mu_i(q,l)}{\ln l} = \lim_{l\to 0} \frac{f(q,l)}{\ln l}.$$

We also calculate averaging exponents over the measure $\mu(q,l)$, i.e.

$$\sum_{i=1}^N \alpha_i \mu_i(q,l) = \frac{\sum_{i=1}^N ln\, p_i(l) \mu_i(q,l)}{ln\, l} = \frac{\alpha(q,l)}{ln\, l},$$

and then the limit $\alpha(q)$ of these averagings when $l \to 0$. Hence, we obtain

$$\alpha(q) = \lim_{l\to 0} \frac{\alpha(\mathrm{q},l)}{\ln l}.$$

Such a method allows us to obtain the set of dimensions $f(q)$ and the set of averaging exponents $\alpha(q)$ as functions of the parameter q.

In practice, to obtain the above values we should do the following. For every q we take several values of variable l, calculate sets of points $(\ln l, f(q,l))$ and $(\ln l, \alpha(q,l))$ respectively. Then, by using the least square method, we determine the approximate values of $f(q)$ and $\alpha(q)$. Thus, we have the set of information dimensions of the supports of the measures that are obtained from the initial measure by the direct multifractal transform. We applied this method in [6], to analyze biomedical preparation images and in [7] to study crystallization images.

4. NUMERICAL EXPERIMENTS

Now we consider images of calm (1) and disturbed (2) sea surfaces.

(1) calm sea surface

(2) disturbed sea surface

Figure 1. Optical images of sea surface related to sea state determination

Values of density function lie in diapason [1.73, 2.23] for 1) and [1.52, 2.42] for 2). We note that wider interval corresponds to more complex structure of the image. Level sets were constructed with step 0.1. The graphs of multifractal spectra are given below.

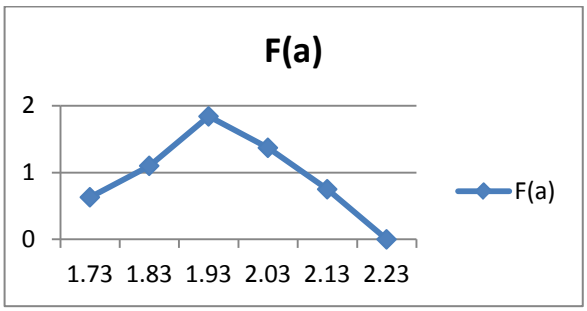


Figure 2 (a). Multifractal spectrum for image 1)

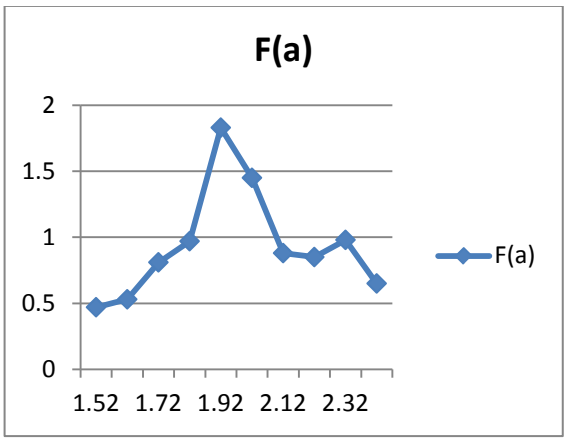
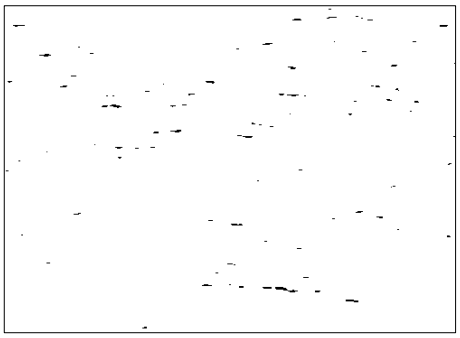
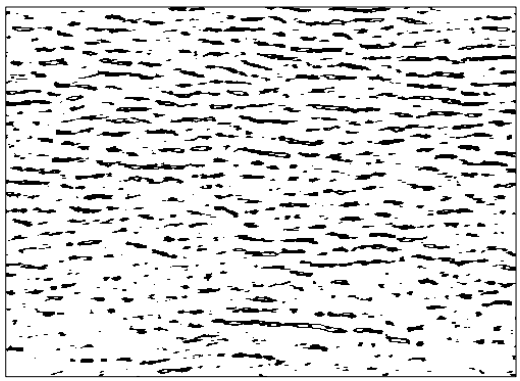


Figure 2 (b). Multifractal spectrum for image 2)

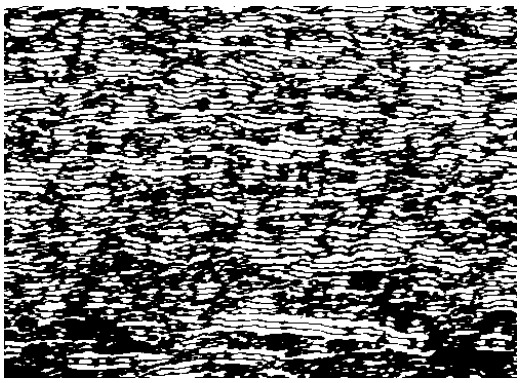
Level sets for image 1) are shown below. These sets illustrate a decomposition of an image on non-intersecting subsets. Each subset contains pixels having density function value in an interval [a,a+0.1), where a=1.73,1.83,193,2.03,2.13. Thus the number of subsets shown equals the number of intervals between values on OX axis. We see that image c) has the most intensive density, and on the graph we see that this subset has the maximal fractal dimension.



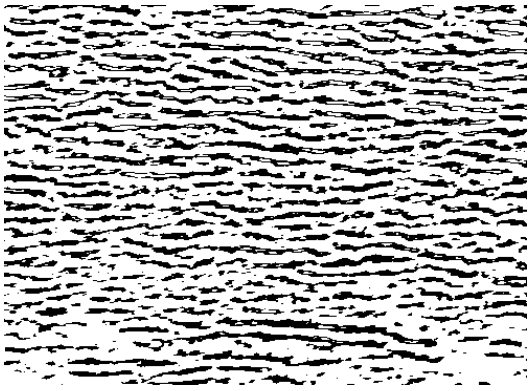
a)



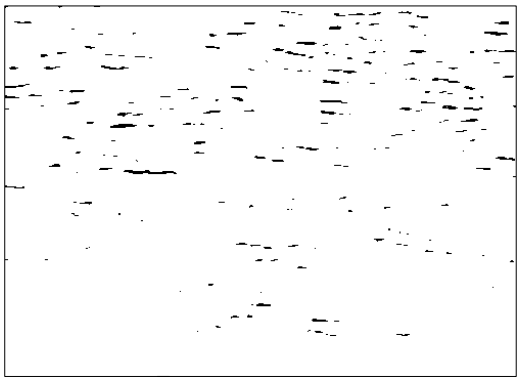
b)



c)



d)

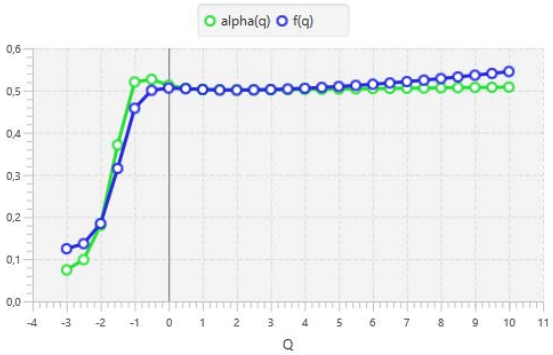


e)

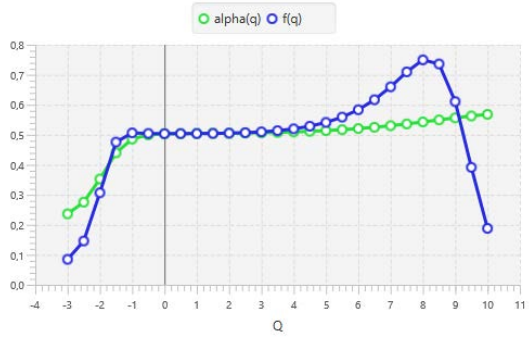
Figure 3. Level sets for image 1): the decomposition of the image on nonintersecting subsets

The corresponding graphic show values of fractal dimensions of these sets.

The application of the second method results in the graphics shown on Fig.4



f)



g)

Figure 4. Graphs of singularity and information dimensions for calm sea (f) and disturbed sea (g)

5. CONCLUSION

The results of numerical experiments show that the methods applied may be useful for analysis of sea surface images and sea state determination, since graphs of multifractal spectra look quite different for different types of sea surfaces. In the near future the authors intend to investigate more precise criteria based on the methods described to determine sea states.

6. ACKNOWLEDGMENTS

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