MODELLING THE DYNAMICS OF CHARGED PARTICLES USING UNITY ENGINE

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Abstract

The problem of computer modelling and visualization of low frequency electromagnetic fields is in the area of interest of such directions as education, research activity, medicine and industry. Visualization helps better understanding the influence of magnetic fields on various physical environments, and a clarifying the physical and mathematical models that are used. In industry visualization is often applied in the tools for non-destructive testing and in magnetic inspection systems. In medicine these fields are traditionally used in magnetotherapy, and at the moment computer modelling and visualization are often the only practical approach to the studying the influence magnetic fields on a living organism. In this work we demonstrate a computer tool both for modelling a magnetic field distribution and the motion of a charged particle in this field. The algorithms are implemented in a special environment Unity. That allows the combining calculations and visualization and results in considerable reducing run-time and the development of a flexible application that may be easily used by physicians. The results of experiments are given.

1. INTRODUCTION

Computer modelling and visualization of low frequency electromagnetic fields are of interest to education, research activities, medical practice and industry. Visualization furnishes insights into the nature of the influence of magnetic fields on various physical environments, reveals peculiarities of emerging phenomena and in many cases helps to clarify the physical and mathematical models that are used. In industry such a visualization is often applied when elaborating tools for non-destructive testing [6]. Low frequency magnetic fields are also used in magnetic inspection systems.

Of special interest is the application of the methods of computer modelling and visualization in biology and medicine [1,5]. Experimental data show that low frequency electromagnetic fields may have a beneficial effect on the processes occurring in living systems. Over the past 20 years magnetobiology [4] is progressing rapidly. This branch of biology originates from biophysics and contributes to the elaboration of the methods of treatment of various deceases. In particular, in medicine the magnetotherapy method is widely used, and in this case the modelling and visualization of magnetic fields help to set and control the device parameters. That results in increasing effectiveness and safety of curative sessions.

Computer modelling and visualization of low frequency magnetic field in a magnetotherapy device was performed in [2, 7], where the calculations were made for a given configuration of coils in 3D space. The calculation of magnetic induction was performed in the points of a given space grid, and to obtain the induction value in an arbitrary point various methods of interpolation were used. In view of the large volume of calculation several types of optimization were implemented. Basing on the obtained results the authors of [3,9] developed an application for modelling the motion of a charged particle in the low frequency magnetic field generated by a given configuration of coils. In all the papers visualization was performed with using Paraview software, which complicated the technology considerably.

Thus at the moment there is a program software for calculation and visualization of the magnetic field generated by different configurations of coils used in magnetotherapy devices, and the modelling of the motion of a charged particle in this field. The main problem is the authors used specialized graphic packages to visualize the results of calculation. Such an approach complicates the practical using of the software implemented, because a user has to organize file transfer between different program systems “by hand”.

In this work we present a more flexible variant of the solving the problem above, which combines both all the calculations and visualization in one environment. Besides that, a new algorithm for calculation and visualization of low frequency magnetic

field is designed and implemented. It is based on the fact that in any point of the field the magnetic induction vector lies on the tangent to a magnetic field line. So, we do not use a predefined space configuration for the lattice of points, but being in a given point we obtain the next point for calculation by moving along the tangent on a small distance *h.* On this approach we do not need to use an interpolation, which leads to a reducing run-time.

In our study we have implemented an approach where the calculation of the field and trajectories of charged particles and the visualization of the results are performed in a uniform programming environment — the cross-platform software package Unity [10,11]. This environment is widely used for design of the projects with step-by-step improvement, for example in the cycles of prototyping or testing. There is a wide range of tools for the editing objects in the built-in editor and tuning the editor by using a scenario which adds new functional peculiarities and control elements.

Unity applications can be deployed to any desktop or mobile platform with little optimization and minor changes in code. This technique allowed us to optimize resource–intensive mathematical operations, which led to an optimization as a whole.

The system assumes further development. It is planned to visualize the motion of a charged particle in various physical environments, to improve the interface, the editor for the environment, coils and charged particles. The implemented program product may be easily used by physicians for study the results of magnetotherapy for various parameters of physiotherapy devices.

The paper is organized as follows. The next section contains a brief description of the mathematical model and algorithms for its solving. In section 3 the description of Unity environment is given. The last section shows the results of experiments.

2. MATHEMATICAL DESCRIPTION OF THE MODEL

2.1. Magnetic field calculation

Our model includes the calculation of the low-frequency magnetic field generated by one or several coils which are placed in the space according to a given configuration and the modelling of the motion of a charged particle in this field. The first problem is well known model task, and the calculation is performed in the cylindrical system of coordinates by the formulas using elliptic integrals. In the process of calculation we have to use the Cartesian coordinates, so the transitions between two systems are performed. The magnetic field is traditionally calculated in the points of a given space grid, so in an arbitrary point an interpolation is performed. This way of solving was due to the fact that for visualization a special software was used, which required the array of values of magnetic inductions in a lattice given. This approach was implemented in our previous works [2,3,7,9]. In this paper we propose a new method in which we construct the grid on a section plane. Each vector of magnetic induction in a point M lies on the tangent to the magnetic field line passing through M, hence we start from a point of the grid, calculate the magnetic induction and obtain the next point as the point on the tangent on a given distance from the initial point M. The implementation of such an algorithm became possible in the special environment which unites calculations and visualization.

In the modelling of the charged particle motion we use the second law of Newton, being the Lorentz force acts on the particle. We use the second order system of difference equations which is obtained by application of the method of central differences for derivatives. Such an approach leads to more effective results and does not require using software packages. As these tasks were discussed in papers [2,3] in details, we give here only main formulas for easier understanding.

The scheme for calculation of magnetic field for one contour of a coil is shown on Fig.1.

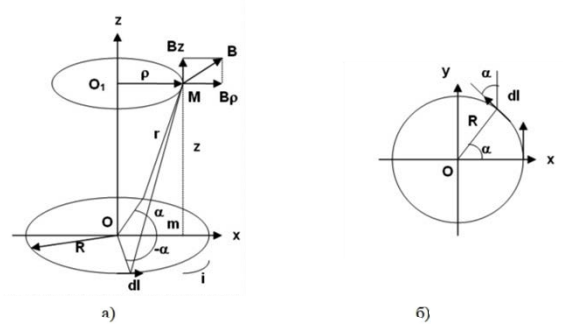

Figure 1. Calculation of magnetic induction in a point M on a contour

Here magnetic field lines are in the plane passing through the axis Oz. The magnetic induction vector $\vec{B_M}$ in a point M is defined by scalar components $B_\rho = rot_\rho A$ and $B_z = rot_z A$, (as $B_\alpha = 0$) and are calculated as

$$B_\rho = \frac{\mu_0 i}{2\pi}\frac{z}{\rho\sqrt{(R+\rho)^2+z^2}}\left(\frac{R^2+\rho^2+z^2}{(R-\rho)^2+z^2}L - K\right), \quad (1)$$

$$B_z = \frac{\mu_0 i}{2\pi}\frac{1}{\sqrt{(R+\rho)^2+z^2}}\left(\frac{R^2-\rho^2-z^2}{(R-\rho)^2+z^2}L + K\right). \quad (2)$$

where *L* and *K* are elliptic integrals of the first and second type.

For an arbitrary configuration of coils the value of magnetic induction in a point M is calculated as the sum of magnetic induction taken over all the contours (using formulas 1,2) of all the coils, and all needed coordinate transformations are performed. All these actions are collected in a program block (procedure) *MagnIndPoint*().

2.2. Visualization of magnetic field

The algorithm for calculation of magnetic induction is the follows.

a) Chose a plane containing OZ axis and orthogonal the plane XY.

b) Construct a lattice of points (M_i) in the given part of this plane and calculate the magnetic inductions vectors B_{M_i}.

c) Let $M(x,y,z)$ be a point of the lattice and B_M be the magnetic induction vector in M. This point lies on a magnetic field line and B_M is on the tangent in this point to the line. The directing vector of the tangent is defined by the coordinates (B_x, B_y, B_z). To obtain the next point for calculation of magnetic induction we take the point $M_1(x_1, y_1, z_1)$ on the tangent which is on a small distance h from $M(x,y,z)$. The coordinates of $M_1(x_1, y_1, z_1)$ are found from the system

$$\frac{x_1 - x}{B_x} = \frac{y_1 - y}{B_y} = \frac{z_1 - z}{B_z},$$

$$(x_1 - x)^2 + (y_1 - y)^2 + (z_1 - z)^2 = h^2.$$

Thus, we have

$$x_1 = x \pm \frac{hB_x}{\alpha(x,y,z)},$$

$$y_1 = y \pm \frac{hB_y}{\alpha(x,y,z)},$$

$$z_1 = z \pm \frac{hB_z}{\alpha(x,y,z)}$$

where

$$\alpha(x,y,z) = \sqrt{B_x^2 + B_y^2 + B_z^2}.$$

We take the both values for (x_1, y_1, z_1) and repeat the calculations from item a). The construction is completed when we come to the boundary of the given area or the number of steps is achieved. The number of section planes is a parameter of the program.

2.3. Motion of a charged particle

In what follows we used the mathematical model that includes the action of electrical field as well. The detailed derivation of equations for the movement of charged particle in electrical and magnetic fields is given in many textbooks, for example in [8]. We describe them briefly. Let q and m be charge and mass of a particle. Denote by $\vec{E}(x,y,z,t)$ and $\vec{B}(x,y,z,t)$ the intensities of electrical and magnetic field induction in the point (x,y,z) at the moment t. The force acting the ion in electrical field is equal to $q\vec{E}$, and the Lorentz force in magnetic field equals $q\vec{v}\times\vec{B}$. Writing the second law of Newton we obtain $m\frac{d\vec{v}}{dt} = q(\vec{E} + \vec{v}\times\vec{B})$. Assuming that $\vec{B}$ is co-directed with Oz, and hence $B_z = B, B_x = B_y = 0$, we obtain the following system of equations

$$m\ddot{x} = q\big(E(x,y,z,t)\sin\gamma\cos\beta + \dot{y}\,B(x,y,z,t)\big)$$

$$m\ddot{y} = q(E(x,y,z,t)\sin\gamma\sin\beta - \dot{x}B(x,y,z,t)) \quad (3)$$

$$m\ddot{z} = qE(x,y,z,t)\cos\gamma$$

The system was investigated in [3,9] and the algorithm for approximate calculation based of using difference equations was proposed and implemented. The second order discrete system obtained from (3) has the form

$$x_{i+1} - K_i y_{i+1} = 2x_i - x_{i-1} - K_i y_{i-1} + L_i \sin\gamma\cos\beta$$

$$K_i x_{i+1} + y_{i+1} = 2y_i - y_{i-1} + K_i x_{i-1} + L_i \sin\gamma\sin\beta$$

$$z_{i+1} = 2z_i - z_{i-1} + L_i\cos\gamma, \quad (4)$$

where $K_i = \frac{qh}{2m}B(x_i, y_i, z_i)$ if current is constant, and $K_i = \frac{qh}{2m}B\cos\omega t_i$ in the case of variable

current with frequency ω. By analogy $L_i = \frac{qh^2}{m} E(x_i, y_i, z_i)$, if electrical field depends on the point, and $L_i = \frac{qh^2}{m} E \cos \omega t_i$ for periodic field. The system (4) is linear, z_i are calculated independently from x and y, and on every step x_{i+1}, y_{i+1} may be found by the Cramer method, because the determinant of the system $\Delta = 1 + K_i^2 \neq 0$.

So, in our work we realized this algorithm in Unity engine. The initial data are parameters of a particle *q* and *m*, the initial points $\boldsymbol{M_0} = (\boldsymbol{x_0}, \boldsymbol{y_0}, \boldsymbol{z_0})$, $\boldsymbol{M_1} = (\boldsymbol{x_1}, \boldsymbol{y_1}, \boldsymbol{z_1})$ (in global Cartesian coordinate system) of the trajectory and values of magnetic induction in these points. For each point of the trajectory the magnetic induction is calculated by the program *MagnIndPoint*.

3. TECHNOLOGICAL TOOLS OF THE MODELLING

3.1. Short description

Unity environment is a cross-platform tool for development of 3D applications. Such applications may be set practically on all known platforms [11]. Unity supports DirectX and OpenGL, which results in obtaining high-level solutions by the use of modern graphical tools.

Unity also supports script languages, so the programming in this system should present no problems because there is no need to use a large number of extraneous solutions. In the kernel of Unity there are many built-in additional tools for design, so the programming in the environment is simple and clear enough.

The working process is effected by way of the Unity visual editor. The editor links scenes, and combines the project resources and the code into interactive objects, which allows optimizing the process of implementation of professional visual applications. It should be noted that in this environment a flexible approach to object-oriented programming is realized: objects are created by union but inheritance, and there are the links controlled between the objects.

3.2. Optimization

To optimize the calculation of magnetic induction we at first splat into streams the calculation of coordinates and magnetic induction. Then we splat the calculation of functions, magnetic induction for all the contours of a coil, total value of magnetic induction in a point, coordinate transformations and visualization. Depending on a system configuration such an optimization leads to the reducing run-time approximately 7-8 times. For a system of 3 coils (10 contours) the run time before optimization is 3-4 min, after optimization 25-30 sec.

4. EXPERIMENTS

Device parameters are the following: current intensity is $3\mathrm{A}$, coils have 7 turns, the number of windings is 25. The size of the region (in mm) is 600x600x300 the step on time *h* (in sec) is 10^{-6}.

The experiments were performed for various types of ions and combination of magnetic and electrical fields. In experiments was modelled the motion of 3 particles

$Na^+: q = 3.817 * 10^{-26}, m = 1.60217 * 10^{-19};$

$SO_4^-: q = -7.634 * 10^{-26}, m = 1.59468 * 10^{-22}$

$\mathrm{e}: q = -1.602 * 10^{-19}, m = 9.1 * 10^{-31}$

for different configuration of magnetic field. All the particles start the motion from the initial point (5,10,15). On Fig. 2-3 the motion of the particles is shown for different position of coils.

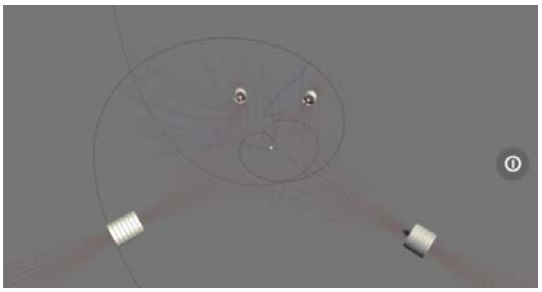

Figure 2. Four coils with coordinates (-272, 0, -272), (141, 0, 332), (-130, 0, 348), (284, 0, -284)

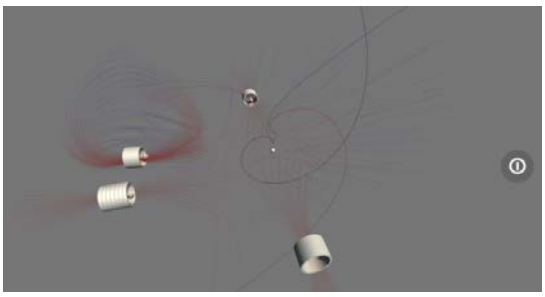

Figure 3. Four coils with coordinates (-331, 0, -74), (0, 0, 317), (-322, 0, 51), (284, 0, -280)

CONCLUSION

Development of tools for computer modelling of magnetic field distribution and the motion of various particles in the field is the important task which may help in studying the influence of low frequency magnetic field on living tissues and estimate the effectiveness of various curative sessions. Further investigations lead to a natural using other mathematical models which may be applicable not only for air but for different environments as well.

REFERENCES

[1] V. Aleksandrov. Electromagnetic fields and ecology. St. Petersburg, 2005. (in Russian).

[2] N. Ampilova, D. Dimitrov, B. Kudrin. Mathematical modelling of low frequency magnetic field in systems for magnetotherapy. Proc. 8 Int. Conf. CEMA13, Sofia, Bulgaria, October 17-19, 2013, p. 48-51.

[3] N. Ampilova, F. Uvarichev, I. Soloviev. On the estimation of magnetotherapy curative sessions effects. Proc. 13 Int. Conf. CEMA18, 18-20 Oct. 2018, Sofia, Bulgaria, p. 62-65, ISSN: 1314-2100, Изд. KING 2001, Sofia.

[4] V. Binhi. Magnetobiology: Underlying physical problems. San Diego, Academic Press, 2002.

[5] V. Binhi, A. Savin. Effects of weak magnetic fields on biological systems: physical aspects. PHYS-USP, 2003, 46 (3), p. 259–291.

[6] N. Ida. Numerical modelling for electro-magnetic non-destructive evaluation. New-York, Springer Publishing Company, 1994.

[7] B. Kudrin, I. Soloviev. On interpolation methods of low frequency magnetic field in systems for magnetotherapy. Proc. 9 Int. Conf. CEMA14, Sofia, Bulgaria, October 16-18, 2014, p. 154-157.

[8] B. Lennert, "Dynamics of charged particles", North Holland, 2004.

[9] F. Uvarichev, N. Ampilova, I. Soloviev. On the modelling ion motion in electromagnetic field. Proc. 10 Int. Conf. EPE-2018, ID 2627, p.1-4, ISBN 978-1-5386-5061-5 IEEE Catalog number CFP 1847S-USB.

[10] Unity. URL: https://unity.com

[11] Unity Multiplatform. URL: https://unity3d.com/unity/features/multiplatform.