EXACT MODEL MATCHING BY DYNAMIC MEASUREMENT OUTPUT FEEDBACK FOR LINEAR TIME-INVARIANT SYSTEMS

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Abstract

In this paper the problem of exact model matching by dynamic measurement output feedback for linear time-invariant systems is studied. Explicit necessary and sufficient conditions are given for the problem to have a solution over the Euclidean ring of proper rational functions. A simple procedure is given for the computation of the controller that solves the problem. The design procedure consists of solving a linear equation in proper rational matrices over the Euclidean ring of proper rational functions.

1. INTRODUCTION

The exact model matching with stability by dynamic measurement output feedback was studied in [1] and [2] where necessary and sufficient conditions have been established for the existence of solution. In [3] a sufficient condition has been established for the solution of exact model matching problem with stability and an efficient computational method to determine it is given. In [4] it is studied and completely solved the exact model matching problem with stability by constant measurement output feedback for a class of linear time-invariant systems. In particular, necessary and sufficient conditions are established for the existence of solution and an algorithm to determine it is given. The purpose of this paper is to present a simple solution of the exact model matching problem by dynamic measurement output feedback for linear time-invariant systems over the Euclidean ring of proper rational functions. In particular, necessary and sufficient conditions are established which guarantee the existence of solution over the Euclidean ring of proper rational functions and a procedure is given for the computation of this solution. Our approach has certain advances over the known results in literature [1], [2] and [3], since it proves that the existence of solution of the exact model matching problem by dynamic measurement output feedback for linear time-invariant systems over the Euclidean ring of proper rational functions depends on the infinite zero structure of proper rational matrices of mathematical model of open-loop system. The above clearly demonstrates the contribution of this paper with respect to existing results.

2. PROBLEM STATEMENT

Let us consider a linear time-invariant system with external model described by the following equations

$$\begin{bmatrix} \mathbf{y}_c(z) \\ \mathbf{y_m}(z) \end{bmatrix} = \begin{bmatrix} \mathbf{A}(z) & \mathbf{B}(z) \\ \mathbf{C}(z) & \mathbf{D}(z) \end{bmatrix} \begin{bmatrix} \mathbf{u}(z) \\ \mathbf{w}(z) \end{bmatrix} \tag{1}$$

where $\mathbf{u}(z) \in \mathbf{R}^m$ is the vector of control inputs $\mathbf{w}(z) \in \mathbf{R}^q$ is the vector of disturbance inputs, $\mathbf{y}_m(z) \in \mathbf{R}^p$ is the available measurement output vector, $\mathbf{y}_c(z) \in \mathbf{R}^s$ is the vector of outputs to be controlled and $\mathbf{A}(z)$, $\mathbf{B}(z)$, $\mathbf{C}(z)$ and $\mathbf{D}(z)$ are proper rational matrices of appropriate dimensions. The exact model matching problem by dynamic measurement output feedback is defined as follows. Find the controller:

$$\mathbf{u}(z) = \mathbf{F}(z)\mathbf{y}_m(z) \tag{2}$$

where $\mathbf{F}(z)$ is proper rational matrix of appropriate dimensions, so that the transfer function $\mathbf{T}_c(z)$ relating $\mathbf{w}(z)$ and $\mathbf{y}_c(z)$ of the compensated system:

$$\mathbf{T_c}(z) = \mathbf{A}(z)\mathbf{F}(z)[\mathbf{I} - \mathbf{C}(z)\mathbf{F}(z)]^{-1}\,\mathbf{D}(z) + \mathbf{B}(z) \tag{3}$$

is a prescribed proper and stable rational matrix $\mathbf{H}(z)$. In particular, we will examine if there exists a dynamic output feedback (2) such that

$$\mathbf{T}_c(z) = \mathbf{H}(z) \tag{4}$$

If so, give necessary and sufficient conditions for existence and a procedure to calculate $\mathbf{F}(z)$.

3. BASIC CONCEPTS AND PRELIMINARY RESULTS

Let $\mathbf{R}_{\mathrm{p}}(\mathrm{z})$ be the Euclidean ring of proper rational functions in z. A matrix $\mathbf{W}(\mathrm{z})$ whose elements are proper rational functions is called proper rational matrix. A square matrix $\mathbf{U}(\mathrm{z})$ over $\mathbf{R}_{\mathrm{p}}(\mathrm{z})$ is said to be biproper if its inverse exists and is also proper. A conceptual tool for the study of the structure of rational matrices is the following standard form. Every proper rational matrix $\mathbf{W}(\mathrm{z})$ of dimensions $p \times m$ with rank$\mathbf{W}(\mathrm{z})= r$, can be expressed as:

$$\mathbf{W}(\mathrm{z}) = \mathbf{U}_1(\mathrm{z})\, \mathbf{M}(\mathrm{z})\, \mathbf{U}_2(\mathrm{z}) \tag{5}$$

The matrices $\mathbf{U}_1(\mathrm{z})$ and $\mathbf{U}_2(\mathrm{z})$ are biproper and the matrix $\mathbf{M}(\mathrm{z})$ is given by:

$$\mathbf{M}(\mathrm{z}) = \begin{bmatrix} \mathbf{M}_r(\mathrm{z}) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \tag{6}$$

where $\mathbf{M}_r(\mathrm{z}) = \operatorname{diag}\left[\mathrm{z}^{-\delta 1}, \dots\dots, \mathrm{z}^{-\delta \mathrm{r}}\right]$ and $\delta_1 \leq \ldots \leq \delta_r$ are nonnegative integers uniquely determined by $\mathbf{W}(\mathrm{z})$. This is the Smith-McMillan form over $\mathbf{R}_{\mathrm{p}}(\mathrm{z})$ [5], [6] and the non-negative integers δ_i for $i = 1, 2, \ldots, r$, determine the structure of the infinite zero of $\mathbf{W}(\mathrm{z})$.

The following Lemmas are taken from [5] and are needed to prove the main theorem of this paper.

Lemma 1. Let $\mathbf{P}(\mathrm{z})$ and $\mathbf{Q}(\mathrm{z})$ be rational matrices with elements in $\mathbf{R}_{\mathrm{p}}(\mathrm{z})$. Then the equation:

$$\mathbf{P}(\mathrm{z})\mathbf{X}(\mathrm{z}) = \mathbf{Q}(\mathrm{z}) \tag{7}$$

has a solution over $\mathbf{R}_{\mathrm{p}}(\mathrm{z})$ if and only if the matrices $\mathbf{P}(\mathrm{z})$ and $[\mathbf{P}(\mathrm{z}), \mathbf{Q}(\mathrm{z})]$ have the same infinite zero structure.

Lemma 2. Let $\mathbf{N}(\mathrm{z})$ and $\mathbf{R}(\mathrm{z})$ be rational matrices with elements in $\mathbf{R}_{\mathrm{p}}(\mathrm{z})$. Then the equation:

$$\mathbf{Y}(\mathrm{z})\mathbf{N}(\mathrm{z}) = \mathbf{R}(\mathrm{z}) \tag{8}$$

has a solution over $\mathbf{R}_{\mathrm{p}}(\mathrm{z})$ if and only if the matrices $\mathbf{N}(\mathrm{z})$ and $\begin{bmatrix}\mathbf{N}(\mathrm{z})\\ \mathbf{R}(\mathrm{z})\end{bmatrix}$ have the same infinite zero structure.

4. MAIN RESULTS

Let rank$[\mathbf{A}(\mathrm{z})]=r$ then there exists biproper matrices $\mathbf{U}_1(\mathrm{z})$ and $\mathbf{U}_2(\mathrm{z})$ [6] which reduce $\mathbf{A}(\mathrm{z})$ to the Smith–McMillan form over $\mathbf{R}_{\mathrm{p}}(\mathrm{z})$:

$$\mathbf{A}(\mathrm{z}) = \mathbf{U}_1(\mathrm{z})\begin{bmatrix}\mathbf{A}_r(\mathrm{z}) & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}\mathbf{U}_2(\mathrm{z}) \tag{9}$$

where $\mathbf{A}_r(\mathrm{z}) = \mathrm{diag}\left[\mathrm{z}^{-\delta 1}, \dots\dots, \mathrm{z}^{-\delta r}\right]$. The non-negative integers δ_i for $i = 1, 2, \dots, r$, determine the structure of the infinite zero of $\mathbf{A}(\mathrm{z})$. Let also:

$$\mathbf{L}(\mathrm{z}) = \mathbf{H}(\mathrm{z}) - \mathbf{B}(\mathrm{z}) \tag{10}$$

Denote

$$\mathbf{U}_1^{-1}(\mathrm{z})\,\mathbf{L}(\mathrm{z}) = \begin{bmatrix}\mathbf{L}_1(\mathrm{z})\\ \mathbf{L}_2(\mathrm{z})\end{bmatrix} \tag{11}$$

The matrix $\mathbf{L}_1(\mathrm{z})$ in (11) has r rows.

The main result of this paper is given below, in particular the main theorem of this section gives necessary and sufficient conditions for the existence of a dynamic measurement output feedback that solves the exact model matching problem over the Euclidean ring of proper rational functions.

Theorem. The exact model matching problem by dynamic measurement output feedback has a solution over $\mathbf{R}_{\mathrm{p}}(\mathrm{z})$ if and only if the following conditions hold:

(a) The matrices $\mathbf{A}(\mathrm{z})$ and $[\mathbf{A}(\mathrm{z}), \mathbf{L}(\mathrm{z})]$ have the same infinite zero structure.

(b) The matrices $\mathbf{D}(\mathrm{z})$ and $\begin{bmatrix}\mathbf{D}(\mathrm{z})\\ \mathbf{A}_r^{-1}(\mathrm{z})\mathbf{L}_1(\mathrm{z})\end{bmatrix}$ have the same infinite zero structure.

Proof: To prove necessity, write the transfer function of the closed – loop system given by equation (3) as follows:

$$\mathbf{T}_{\mathrm{c}}(\mathrm{z}) = \mathbf{A}(\mathrm{z})\mathbf{X}(\mathrm{z})\,\mathbf{D}(\mathrm{z}) + \mathbf{B}(\mathrm{z}) \tag{12}$$

where the matrix $\mathbf{X}(\mathrm{z})$ is given by:

$$\mathbf{X}(\mathrm{z}) = \mathbf{F}(\mathrm{z})[\mathbf{I} - \mathbf{C}(\mathrm{z})\mathbf{F}(\mathrm{z})]^{-1} \tag{13}$$

Suppose that the exact model matching problem by dynamic measurement output feedback has a solution. Then equation (12) can be rewritten as follows:

$$\mathbf{A}(\mathrm{z})\mathbf{X}(\mathrm{z})\mathbf{D}(\mathrm{z}) + \mathbf{B}(\mathrm{z}) = \mathbf{H}(\mathrm{z}) \tag{14}$$

Using equation (10), equation (14) can be rewritten as follows:

$$\mathbf{A}(\mathrm{z})\mathbf{X}(\mathrm{z})\mathbf{D}(\mathrm{z}) = \mathbf{H}(\mathrm{z}) - \mathbf{B}(\mathrm{z}) = \mathbf{L}(\mathrm{z}) \tag{15}$$

If we define $\mathbf{Y}(\mathrm{z}) = \mathbf{X}(\mathrm{z})\mathbf{D}(\mathrm{z})$ then equation (15) can be rewritten as follows:

$$\mathbf{A}(\mathrm{z})\mathbf{Y}(\mathrm{z}) = \mathbf{L}(\mathrm{z}) \tag{16}$$

Since by assumption the exact model matching problem by dynamic measurement output feedback has a solution, the equation (15) has a solution for $\mathbf{X}(\mathrm{z})$ over $\mathbf{R}_{\mathrm{p}}(\mathrm{z})$ and therefore equation (16) has also a solution for $\mathbf{Y}(\mathrm{z})$ over $\mathbf{R}_{\mathrm{p}}(\mathrm{z})$. Hence, according to Lemma 1 the matrices $\mathbf{A}(\mathrm{z})$ and $[\mathbf{A}(\mathrm{z}), \mathbf{L}(\mathrm{z})]$ have the same infinite zero structure. This is the condition (a) of the Theorem. Using (9) equation (15) can be rewritten as follows:

$$\mathbf{U}_1(\mathrm{z}) \begin{bmatrix} \mathbf{A}_r(\mathrm{z}) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \mathbf{U}_2(\mathrm{z})\, \mathbf{X}(\mathrm{z})\, \mathbf{D}(\mathrm{z}) = \mathbf{L}(\mathrm{z}) \tag{17}$$

Or equivalently:

$$\begin{bmatrix} \mathbf{A}_r(\mathrm{z}) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \mathbf{U}_2(\mathrm{z})\, \mathbf{X}(\mathrm{z})\, \mathbf{D}(\mathrm{z}) = \mathbf{U}_1^{-1}(\mathrm{z})\, \mathbf{L}(\mathrm{z}) \tag{18}$$

Using equation (11), equation (18) can be rewritten as follows:

$$\begin{bmatrix} \mathbf{A}_r(\mathrm{z}) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \mathbf{U}_2(\mathrm{z})\, \mathbf{X}(\mathrm{z})\, \mathbf{D}(\mathrm{z}) = \begin{bmatrix} \mathbf{L}_1(\mathrm{z}) \\ \mathbf{L}_2(\mathrm{z}) \end{bmatrix} \tag{19}$$

From equation (19) we have that:

$$[\,\mathbf{A}_r(\mathrm{z})\ \mathbf{0}]\mathbf{U}_2(\mathrm{z})\,\mathbf{X}(\mathrm{z})\,\mathbf{D}(\mathrm{z}) = \mathbf{L}_1(\mathrm{z}) \tag{20}$$

Or equivalently:

$$\mathbf{\Phi}(\mathrm{z})\,\mathbf{D}(\mathrm{z}) = \mathbf{A}_r^{-1}(\mathrm{z})\mathbf{L}_1(\mathrm{z}) \tag{21}$$

Where the matrix $\mathbf{\Phi}(\mathrm{z})$ over $\mathbf{R}_\mathrm{p}(\mathrm{z})$ is given by:

$$\mathbf{\Phi}(\mathrm{z}) = [\mathbf{I}_r \quad \mathbf{0}]\,\mathbf{U}_2(\mathrm{z})\,\mathbf{X}(\mathrm{z}) \tag{22}$$

Since by assumption $\mathbf{X}(\mathrm{z})$ exists and is proper as well, equation (21) has a solution for $\mathbf{\Phi}(\mathrm{z})$ over $\mathbf{R}_\mathrm{p}(\mathrm{z})$. Hence according to lemma 2 the matrices $\mathbf{D}(\mathrm{z})$ and $\begin{bmatrix} \mathbf{D}(\mathrm{z}) \\ \mathbf{A}_r^{-1}(\mathrm{z})\mathbf{L}_1(\mathrm{z}) \end{bmatrix}$ have the same infinite zero structure. This is the condition (b) of the Theorem.

The sufficiency of conditions (a) and (b) can be proved as follows. Condition (a) guarantees that the matrix $\mathbf{L}_2(\mathrm{z})$ in (19) is zero. Since $\mathbf{L}_2(\mathrm{z}) = \mathbf{0}$ and the rational matrix $\mathbf{A}_r(\mathrm{z})$ is nonsingular, equation (15) is equivalent to equation (21). Then according to Lemma 2, condition (b) guarantees that the equation (21) has a solution for $\mathbf{\Phi}(\mathrm{z})$ over $\mathbf{R}_\mathrm{p}(\mathrm{z})$. Since $\mathbf{\Phi}(\mathrm{z})$ is a solution over $\mathbf{R}_\mathrm{p}(\mathrm{z})$ of equation (21), from relationship (22) it follows that:

$$\mathbf{X}(\mathrm{z}) = \mathbf{U}_2^{-1}(\mathrm{z})\begin{bmatrix} \mathbf{\Phi}(\mathrm{z}) \\ \mathbf{0} \end{bmatrix} \tag{23}$$

is a solution over $\mathbf{R}_\mathrm{p}(\mathrm{z})$ of equation (15) or equivalently of equation (12). Furthermore from (13) it follows that:

$$\mathbf{F}(\mathrm{z}) = [\mathbf{X}(\mathrm{z})\mathbf{C}(\mathrm{z}) + \mathbf{I}]^{-1}\mathbf{X}(\mathrm{z}) \tag{24}$$

The matrix $\mathbf{F}(\mathrm{z})$ given by (24) with $\mathbf{X}(\mathrm{z})$ given by (23) is proper and satisfies equation (3) and therefore the exact model matching problem by dynamic measurement output feedback has a solution over $\mathbf{R}_\mathrm{p}(\mathrm{z})$. This completes the proof.

5. COMPUTATION OF THE CONTROLLER

In this section a procedure is given for the calculation of proper solution of exact model matching problem by dynamic measurement output feedback.

Given: $\mathbf{A}(\mathrm{z}), \mathbf{B}(\mathrm{z}), \mathbf{C}(\mathrm{z}), \mathbf{D}(\mathrm{z})$ and $\mathbf{H}(\mathrm{z})$

Find: $\mathbf{F}(\mathrm{z})$

Step 1. Let rank $\mathbf{A}(\mathrm{z}) = r$. Find biproper matrices $\mathbf{U}_1(\mathrm{z})$ and $\mathbf{U}_2(\mathrm{z})$ [6], which reduce $\mathbf{A}(\mathrm{z})$ to the Smith–McMillan form over $\mathbf{R}_\mathrm{p}(\mathrm{z})$.

$$\mathbf{A}(\mathrm{z}) = \mathbf{U}_1(\mathrm{z}) \begin{bmatrix} \mathbf{A}_r(\mathrm{z}) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \mathbf{U}_2(\mathrm{z})$$

where $\mathbf{A}_r(\mathrm{z}) = \mathrm{diag}\left[\mathrm{z}^{-\delta 1}, \dots \dots, \mathrm{z}^{-\delta r}\right]$. Let $\mathbf{L}(\mathrm{z})$=$\mathbf{H}(\mathrm{z})$-$\mathbf{B}(\mathrm{z})$. Denote

$$\mathbf{U}_1^{-1}(\mathrm{z})\ \mathrm{L}(\mathrm{z}) = \begin{bmatrix} \mathbf{L}_1(\mathrm{z}) \\ \mathbf{L}_2(\mathrm{z}) \end{bmatrix}$$

where $\mathbf{L}_1(\mathrm{z})$ has r rows.

Step 2. Check the conditions (a) and (b) of the Theorem. If these conditions are satisfied go to step 3. If not go to step 6.

Step 3. Solve equation (21) and find $\mathbf{\Phi}(\mathrm{z})$.

Step 4. Set

$$\mathbf{X}(\mathrm{z}) = \mathbf{U}_2^{-1}(\mathrm{z}) \begin{bmatrix} \mathbf{\Phi}(\mathrm{z}) \\ \mathbf{0} \end{bmatrix}$$

Step 5. Set

$$\mathbf{F}(\mathrm{z}) = [\mathbf{X}(\mathrm{z})\mathbf{C}(\mathrm{z}) + \mathbf{I}]^{-1}\mathbf{X}(\mathrm{z})$$

Step 6. Our problem has no solution.

6. CONCLUSIONS

In this paper the solution of exact model matching problem by dynamic measurement output feedback over the Euclidean ring of proper rational functions is studied. Necessary and sufficient conditions have been established for the problem to have a solution. A simple procedure is given for the computation of the dynamic controller that solves the problem. The design procedure consists of solving a linear equation in proper rational matrices over the Euclidean ring of proper rational functions. In our point of view the main results of this paper are useful for further understanding of exact model matching problem with stability by dynamic measurement output feedback.

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