ON APPLICATION OF SIMULATION FOR INVESTIGATION OF LOW-FREQUENCY MAGNETIC FIELDS EFFECT

N. Ampilova*, I. Soloviev*, P. Frangos**

*St. Petersburg State University, Comp. Sci. Dept.
7/9 University emb., 199034 St. Petersburg, Russia
n.ampilova@spbu.ru

**School of Electrical and Computing Engineering, National Technical University of Athens,
9, Iroon Polytechniou Str., 157 73 Zografou, Athens, Greece
pfrangos@central.ntua.gr

Abstract

Low frequency and low intensity magnetic and electric fields are used in physiotherapy for a long time. But due to the fact that the action of these fields on living tissues is very complicated and at the moment insufficiently studied, nowadays there is no simple method to measure their curative effect. In practice the natural way of investigations is the experimental one: to observe the state of a patient during curative sessions and collect statistical data. We propose to combine practical approach with the study of processes in external environment when low intensity magnetic fields act. With this purpose we calculate and visualize the trajectories of ions in electromagnetic field generated by a magnetotherapy device. Such an approach leads to obtaining a series of images that show trajectories depending on the choice of the device parameters. That results in visual representation of typical phase portraits generated by the applied mathematical model. Visual perception helps to compare the number of procedures, parameters of the used device and the changing of patient state, and thus to form an expert knowledge in this area.

1. INTRODUCTION

To gain a better insight into the problem of effects of low intensity magnetic field on living organism one may use mathematical and computer modeling. The most natural and simple model is the distribution of intensities of magnetic field generated by a coil. In this case the calculation of the field value is based on the superposition principle: the common magnetic field is the sum of the fields generated by the contours of a coil. For several coils the magnetic field is the sum over all the coils. Induction and self-induction are supposed to be negligible quantities. Numerical methods allow calculating non-uniform magnetic field, i.e. the field changing in the space. The calculations and visualization of the resulting field for several coils in 3D were performed in [1, 3, 4], and the results of calculation were applied to a special magnetoterapy device — "magnetobed", which is actively used in medical practice.

The next step in the studying processes in an environment when electromagnetic field acts is to consider the model of the movement of charged particles (ions or cations). As a first approximation, we may use the model where an ion moves in accordance with the Newton second law, and the Lorentz force acts on it. The problem is to calculate and visualize the trajectories of different ions for given configuration of electric and magnetic fields. This model does not consider the environment where the field acts, so at first we assume that the motion is in air environment.

Medical practice shows that the action of magnetic field in mineral water improves the curative effect for patients with diabetes. To apply the described model for mineral water we should use the magnetic permeability of this environment in formulas for calculation of magnetic induction. But at the moment the dependence of the permeability on mineral water composition is insufficiently studied. So, to make the problem easier it was assumed that the magnetic permeability of mineral water is approximately the same as of ordinary water, which is a natural assumption in this mathematical model. We also assume that in a small size an ion motion in the environment can be thought of as the motion in vacuum [2].

Thus, to model an ion motion we used the method of calculation of the magnetic field of a coil, which is described above. The field is calculated in the points of a space grid. We approximated differential equations of the ion movement by discrete ones by using a second order difference scheme. If a next point of trajectory is not in the grid of points for which values of magnetic field are calculated, a linear interpolation is performed to calculate the value of the field in the point. Various cases of configuration of electrical and magnetic fields were modelled, in particular periodic effects of the fields. In this regime there are two cases — commensurable and incommensurable frequencies. The most complex phase portraits were obtained for incommensurable ones.

It should be noted that the proposed difference scheme leads to the solving linear system by Cramer's method, which is simpler than numerical integration by the Runge-Kutta method. All calculations were compared with calculation in MATLAB package. The results are similar. All the experiments were applied to a special device used in clinics.

Such a method allows us to select a set of parameters and obtain visualization of ion movement. For visualization we used ParaView package and MATLAB. In fact, an imitation model has been designed that may be used for construction some medical devices and choice the most appropriate parameters. The model includes performing the calculations, saving obtained data in files of a required format, call the ParaView to visualize magnetic field or combine visualization of the field and the ion trajectory.

This approach may help to estimate the effectiveness of the magnetotherapy by monitoring the patient state and comparing obtained data with the parameters of the device. Visualization makes this analysis more clear. Moreover, in doing so we model the processes in the external environment, where electromagnetic field acts, and this study contributes to a better comprehension of processes in internal environment – living tissue. This is practical and useful method.

To develop this technique we have to consider more complex models which take into account the interaction between particles and the structure of the environment. The questions concerning the influence of geomagnetic and artificial magnetic fields on transfer processes in aqueous media such as electrolytes and biological objects are discussed in the monography [2]. Our future investigations suggest a more detailed study of this subject.

2. MATHEMATICAL MODEL DESCRIPTION

The detailed derivation of equations for the movement of charged particle in electrical and magnetic fields is given in many textbooks, for example in [5]. We describe them briefly. Consider a charged particle with a charge q and mass m. Let $\vec{E}(x, y, z, t)$ be the intensity of electrical field in the point (x, y, z) at the moment t, and $\vec{B}(x, y, z, t)$ be the magnetic field induction. The force acting the ion in electrical field is equal to $q\vec{E}$, and the Lorentz force in magnetic field equals $q\vec{v} \times \vec{B}$. Then writing the second law of Newton we obtain $m\frac{d\vec{v}}{dt} = q(\vec{E} + \vec{v} \times \vec{B})$. Assuming that $\vec{B}$ is co-directed with Oz, and hence $B_z = B, B_x = B_y = 0$, we obtain the following system of equations:

$$\begin{cases} m\ddot{x} = q\big(E(x,y,z,t)\sin\gamma\cos\beta + \dot{y}\,B(x,y,z,t)\big) \\ m\ddot{y} = q(E(x,y,z,t)\sin\gamma\sin\beta - \dot{x}B(x,y,z,t)) \\ m\ddot{z} = qE(x,y,z,t)\cos\gamma \end{cases} \tag{1}$$

The positions of vectors electric and magnetic fields are illustrated in Fig.1

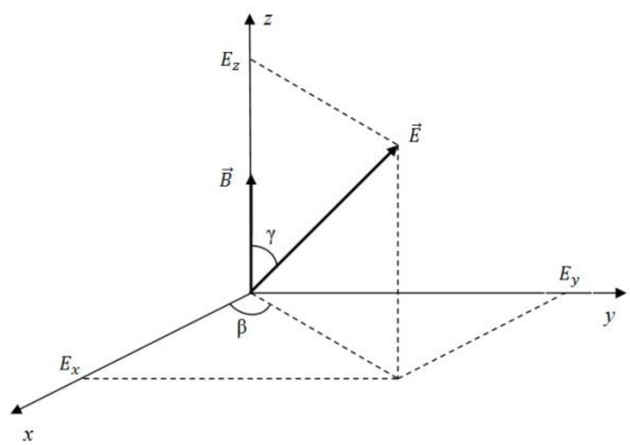

Figure 1. The disposition of vectors $\vec{\boldsymbol{B}}$ and $\vec{\boldsymbol{E}}$

It is well known that system (1) is integrable if there is no electrical field and the magnetic field is uniform and constant. Then the trajectory of an ion is screw line. But for non-integrable cases we have to apply approximate calculations. We use the second order difference system. Let $[t_0, T]$ be the time interval on which a trajectory is calculated, $t_i = t_0 + ih, i = 0, \dots n, x(t_i) = x_i, y(t_i) = y_i$, $z(t_i) = z_i$. Assume that:

$$\ddot{x} \approx \frac{x_{i+1} - 2x_i + x_{i-1}}{h^2},$$
$$\ddot{y} \approx \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}, \tag{2}$$
$$\ddot{z} \approx \frac{z_{i+1} - 2z_i + z_{i-1}}{h^2},$$

and

$$\dot{x} \approx \frac{x_{i+1} - x_{i-1}}{2h}, \dot{y} \approx \frac{y_{i+1} - y_{i-1}}{2h}, \dot{z} \approx \frac{z_{i+1} - z_{i-1}}{2h}. \tag{3}$$

Substituting (2) and (3) in (1) we obtain the second order system of difference equations:

$$\begin{cases} x_{i+1} - K_i y_{i+1} = 2x_i - x_{i-1} - K_i y_{i-1} + L_i \sin\gamma \cos\beta \\ K_i x_{i+1} + y_{i+1} = 2y_i - y_{i-1} + K_i x_{i-1} + L_i \sin\gamma \sin\beta \\ z_{i+1} = 2z_i - z_{i-1} + L_i \cos\gamma, \end{cases} \tag{4}$$

where $K_i = \frac{qh}{2m} B(x_i, y_i, z_i)$ if current is constant, and $K_i = \frac{qh}{2m} B \cos\omega t_i$ in the case of variable current with frequency ω. By analogy $L_i = \frac{qh^2}{m} E(x_i, y_i, z_i)$, if electrical field depends on the point, and $L_i = \frac{qh^2}{m} E \cos\omega t_i$ for periodic field. The system (4) is linear, coordinates of z are found independently from x and y, and on every step x_{i+1}, y_{i+1} may be calculated by the Cramer method, because $\Delta = 1 + K_i^2 \neq 0$.

3. EXPERIMENTS AND RESULTS

We applied the implemented program to model ion trajectories both for arbitrary value of parameters and in a special magnetotherapy device. The device has two coils which are active simultaneously and electrodes. Magnetic and electrical fields may be periodic and have both commensurable and incommensurable frequencies.

3.1. The device description

Device parameters are the following: coils have external radius 58 mm, height — 34 mm; current intensity is 3A, the number of turns is 20, the number of windings is 25. The size of the region (in mm) is 600x300x500, the size of a cell of the lattice (in mm), where magnetic field is calculated, is 4x4x4, the step on time h (in sec) is 10^{-6}.

The beginning of the first coil has coordinates (0, 150, 250), the axis direction is (1, 0, 0). The beginning of the second coil has coordinates (600, 150, 250), the axis direction is (-1, 0, 0).

Figure 2. Magnetotherapy device

The experiments were performed for various types of ions and combinations of magnetic and electrical fields.

3.1.1 Natrium ion in magnetic field

Natrium ion has mass $m = 3.817 * 10^{-23} g$, charge $q = 1$. We take $B = 10^{-3} T$. Consider the case when only magnetic field generated by 2 coils acts. Calculate the ion trajectory with initial data $(x_0, y_0, z_0) = (100{,}150{,}250)$, $(x_1, y_1, z_1) = (100{,}150{,}250.01)$. The trajectory was calculated in 200000 points.

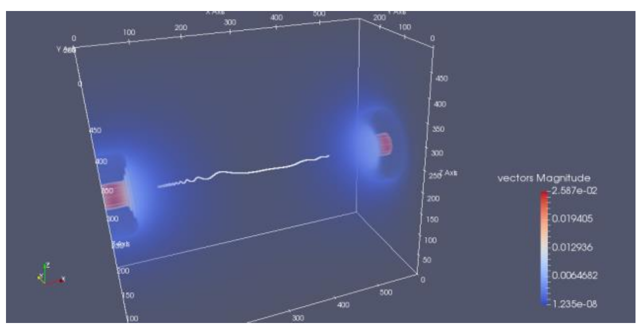

Figure 3. The trajectory of natrium ion in magnetic field

3.1.2 Natrium ion trajectory in magnetic and electrical fields

Modify the previous example and add electrical field. Assume that vector $\vec{E}$ has coordinates (0.1, 0, 0) and $(x_0, y_0, z_0) = (x_1, y_1, z_1) = (100{,}150{,}240)$. The number of points in the trajectory is 3709.

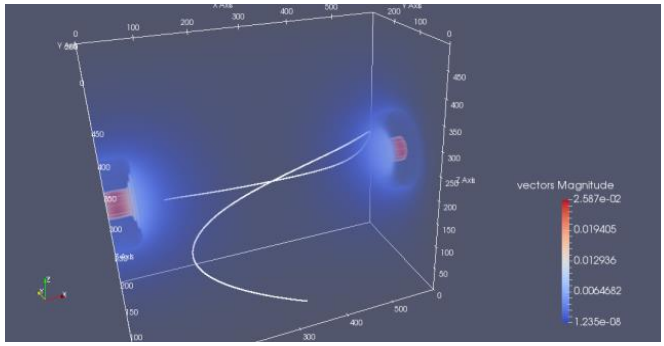

Figure 4. The trajectory of natrium ion in magnetic and electrical fields

3.1.3 Natrium ion in periodic magnetic field

Now we consider the periodic magnetic field with frequency 10 Hz, $(x_0, y_0, z_0) =$ (100,150,250), $(x_1, y_1, z_1) = (100{,}150{,}250.01)$.

The number of points is 73688.

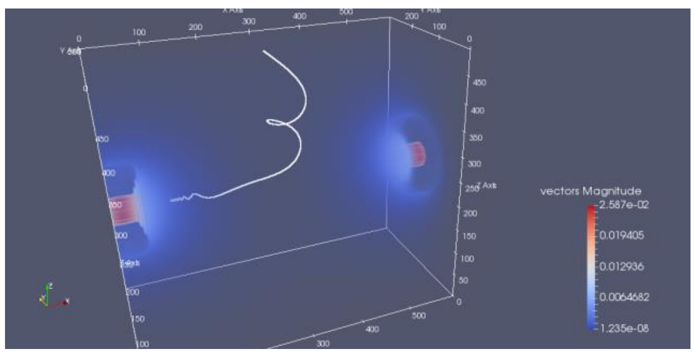

Figure 5. The trajectory of natrium ion in periodic magnetic field

3.1.4 Sulfate ion in magnetic field

Consider the motion of sulfate ion with mass $m = 16 * 10^{-23}$ g, $q = -2$ in magnetic field. The number of points is 200000.

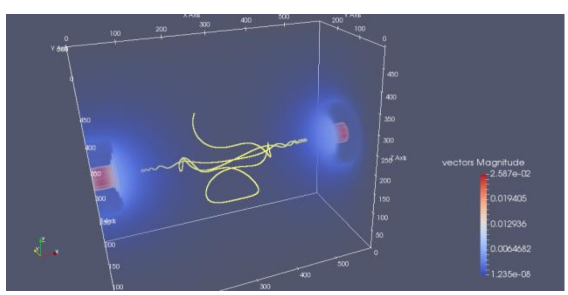

Figure 6. The trajectory of sulfate ion in magnetic field

3.2. Natrium ion in periodic magnetic and electrical fields

Now we consider periodic magnetic field with frequency 200π, and electric field with frequency 80π. The number of points is 150000.

On Fig.7 we present the results of calculations performed in accordance with system (4).

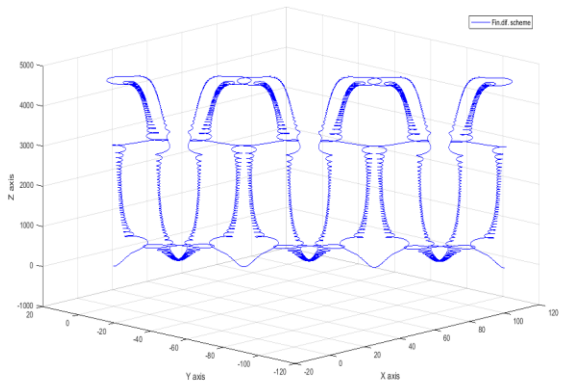

Figure 7. Natrium ion in periodic fields: commensurable frequencies

The next picture shows the results when the frequency of magnetic field is 200π, and the frequency of electrical field is $2\pi \cdot 20e$. Note that we may observe such a motion on rather long distance on z-coordinate (near 6 m). Hence in real device we would see only small part of the trajectory and the difference between these cases will not be considerable for visual perception.

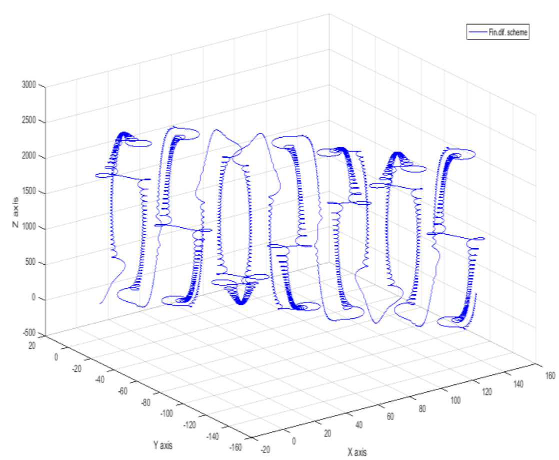

Figure 8. Natrium ion in periodic fields: incommensurable frequencies

4. CONCLUSION

The designed and implemented program is the imitation model for calculation of distribution of electric and magnetic fields and the motion of ions in these fields in magnetotherapy devices. It allows us to model and visualize the typical trajectories of different ions for given parameters of the used device. The obtained set of images may help in matching of given parameters to the results of monitoring the patient state. Thus, simulation and visualization of results are proved to be appropriate methods for interpretation of statistical data.

The future investigations of the state of environment under action of magnetic field require taking into consideration not only an ion motion, but also the interaction between particles, boundary conditions, the environment composition and many other parameters. This problem is the subject of our future investigations.

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