DETERMINING PERMITTIVITY OF DIELECTRIC CYLINDER FROM THE SCATTERED FIELD DATA
(short paper)

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Abstract

The inverse scattering problem is to recover input parameters (material parameters, size of the object, initial data of the field, etc.) from the scattering field data. In the present paper we describe the procedure to recover permittivity of the dielectric cylinder. A limited number of measurements of scattered field intensity along the line on the shadow side of the cylinder are used. Numerical results were calculated for several values of dielectric permittivity. The results demonstrate the possibility of reconstruction of permittivity for the dielectric cylinder.

1. INTRODUCTION

The wave scattering process is one of the key research methods in physics. The study of the amplitude and phase of scattered waves, and their dynamics in space and time allows to fully describing the process under study. At that, forward and inverse problems are distinguished in the theoretical description of scattering processes. In the first case, the scattered field is sought based on the initial data. If the inverse problem, then scattered field is assumed to be given, and the problem of determining the initial values of the field, or other initial parameters, is formulated. Definitely, the solution of the inverse problem provides fundamentally different data and possibilities compared to the forward one. The rigorous mathematical foundations for the theory of the inverse problem were developed in the second half of the last century. The main results could be found in the monograph [1]. However, much less progress has been achieved so far for inverse time-harmonic electromagnetic wave scattering problems. The reasons are due to the complexity of the combined system of equations for the electric and the magnetic fields, necessity to impose additional assumptions on the incident illumination and the nature of the scatterer, the difficulties originating from the much more complicated regularity theory of solutions to Maxwell's equations as compared to solutions of elliptic partial differential equations. Nevertheless, the development of computational technologies, along with the relevance and importance of the problem of electromagnetic wave backscattering, has stimulated significant progress in the problem over the past two decades. It worth to note the methods using study of inverse wave scattering is non-intrusive to the object under study. To implement it we only need to collect scattered fields outside the object non-destructively. Therefore, it has a wide range of applications in various fields, such as: radar and sonar imaging [2,3], geophysics [4], and medicine imaging [5], remote sensing [6], biomedical imaging and diagnosis [7].

Iterative methods are usually width used to solve inverse scattering problems (ISP). Iterative procedure employing an equivalent Neumann series solution in each iteration step has been presented in [8] for solving the two-dimensional inverse scattering problem. Iterative algorithm for the reconstruction of the three-dimensional complex-valued refractive index of an object proposed in [9]. The ISP is reformulated as an optimization problem that is iteratively solved thanks to a conjugate gradient method in Ref. [10]. Another popular method is linear sampling method (LSM) which consists of solving the far-field integral equation for the unknown angle function of far field pattern [11,12]. Domain derivatives employed in [13] to solve inverse electromagnetic scattering problems for perfect conducting or for penetrable obstacles. A variational approach developed to the interior transmission problem appeared in the study of the ISP proposed [14]. Reconstruction of the refractive index from experimental backscattering data using a globally convergent inverse method has been realized in [15]. Also, it worth to note that the so called photonic nanojet discovered in this century [16]is an example of the success revisit of even well-known effects using modern computing technologies. In [17,18] it is shown that the solution of ISP for objects capable of generating nanojets leads to new implemented practical applications.

While (semi) analytical methods can provide a qualitative analysis of the final results, numerical methods appear to be more reliable, offer wider possibilities, and, in addition, allow controlling the accuracy of the results. Therefore, the choice of the solution method moved from semi-analytical methods to numerical ones, along the development of computational technologies. The method we present is a symbiosis of both approaches: approximately representing the original equation in operator form, we expressed the scattered field in terms of the initial data of the problem. Thus, having the scattered field data, we are able to determine any parameter from the group of initial data. As a demonstration of the possibilities of implementing the method, we analyze the dependences of the scattered field (determined along some line) for different values of the cylinder size and permittivity. The advantage of the presented approach is a convenient and physically meaningful functional relationship between the final and initial data; compactness of setting the initial data - they are set along the line opposite to the scattering object, but not around it; controlled accuracy of the results.

2. SCATTERING OF ELECTROMAGNETIC WAVE BY DIELECTRIC OBJECT: SOLUTION FOR INVERSE SCATTERING PROBLEM

The electromagnetic field which has nonzero *Ez, Hx, Hy* components is referred to as TM polarized field. For that case the Maxwell's equations are transformed to Helmholtz equation for electric field:

$$\nabla^2 E_z(x,y) + k^2 \varepsilon E_z(x,y) = 0 \tag{1}$$

where $k=2\pi/\lambda$, λ - wavelength, ε - dielectric permittivity, which equal an unity and εc out and inside of the cylinder respectively. Introducing free space Green function $G(\vec{r},\vec{r}\,') = \frac{1}{4i} H_0^{(2)}(k|\vec{r}-\vec{r}\,'|)$ for similar equation [19], we get:

$$E_z(\vec{r}) = E_z^{inc}(\vec{r}) - \frac{ik^2}{4}\int(\varepsilon_c - 1)E_z(\vec{r}\,')H_0^{(2)}(k|\vec{r}-\vec{r}\,'|)\,d\vec{r} \tag{2}$$

Here $\vec{r} = (x,y)$, $H_0^{(2)}$ is the Hankel function of zero order. The next step is use of approach developed in [20]. We divide the cross section of the dielectric cylinder into *N* cells so that the dielectric constant and the electric field intensity can be assumed constant over each cell. Then an integral equation (1) for the electric field will be transformed to the next one:

$$E_m = E_m^{inc} - \frac{ik^2}{4}\sum_{n=1}^{N}(\varepsilon_n - 1)E_n \int_{cell\ n} H_0^{(2)}(k\rho)dx'\,dy' \qquad (3)$$

where E_n and ε_n – represent electric field intensity and the dielectric constant at the center of cell n, $\rho = \sqrt{(x'-x_m)^2 + (y'-y_m)^2}$. Replacing a cell by a circle with the same square we can get an exact solution for the integral over Hankel function:

$$\frac{ik^2}{4}\int_0^{2\pi}\int_0^{a} H_0^{(2)}(k\rho)\rho d\rho d\varphi = \begin{cases} i/2\left(\pi ka H_1^{(2)}(ka) - 2i\right), & m = n \\ i\pi ka/2 J_1(ka) H_0^{(2)}(k\rho_{mn}), & m \neq n \end{cases} \qquad (4)$$

where ρ and φ are polar coordinates based on a coordinate origin at the centre of cell n, a is the radius of a circular cell, J_1 represents the Bessel function of the first order.

Let’s introduce notations for the left parts of (3) and (4): E_L, $H_{L,c}$ and E_c, $H_{c,c}$ if $\vec{r}$ are inside or out of the cylinder cross section C. Thus from (3) we have 2 equations:

$$E_c = E_c^i - H_{c,c}(\varepsilon_n - 1)E_c, \quad \vec{r} \in C \qquad (5)$$

$$E^L = E_L^i - H_{L,c}(\varepsilon_n - 1)E_c, \quad \vec{r} \notin C \qquad (6)$$

where E_c^i and E_L^i means the incident field determined for the points lying along line placed on some fixed distance L from the cylinder and inside the cylinder respectively. It should be mentioned that all quantities in (5-6) denoted by E are the vectors, while rest are the matrices. Solving (5) for E_c and then using it in (5) we get

$$E_L = E_L^i - H_{L,c}[1/(\varepsilon_n - 1) + H_{c,c}]^{-1} E_c^i \qquad (7)$$

Hence, we find for the dielectric constant:

$$1/(\varepsilon_n - 1) = \left(E_L^i - E_L\right)^T \left(H_{L,c}E_c^i - H_{L,c}H_{c,c}H_{L,c}^{-1}\left(E_L^i - E_L\right)\right) / \left(\left(E_L^i - E_L\right)^T \left(E_L^i - E_L\right)\right) \qquad (8)$$

There are no restrictions on the position where scattering field should be measured. Those, they can be chosen arbitrarily: along a certain closed line around the cylinder, a semicircle at a distance, etc. As mentioned above, we considered the location of these values along a line on the side of the cylinder opposite to the incident wave direction.

3. RESULTS AND DISCUSSIONS

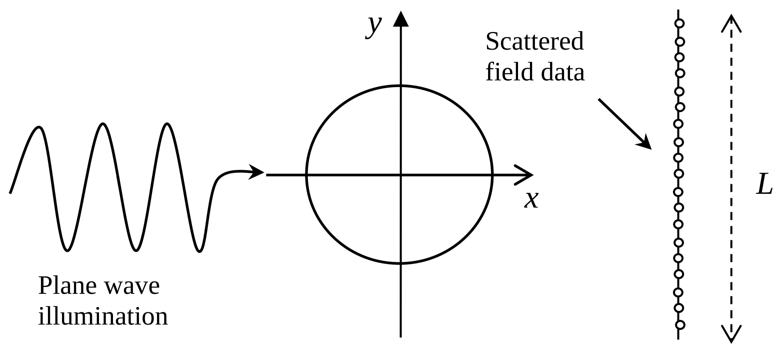

Figure 1. Top view of the dielectric cylinder illuminated by a plane wave.

The sketch of the considered scattering process is shown in Figure 1. A plane wave irradiates a dielectric cylinder. The incident direction is chosen as the *x*-axis. The scattered field is fixed along the line of length *L* on the opposite side. Equation (8) can be used to calculate the dielectric constant for the known cylinder size, incident waveform, and scattered field data. However, possible random measurement errors can lead to inadequate calculation of the ε_n by (8). In order to find the more suitable value of dielectric constant we initiate the fitting process of the ε_n to the values of the scattered field. For this purpose, we will use formulas (5-6). First, we calculate internal field E_c from (5). As it is seen the internal field is completely determined by given geometry and material of the scattering body, ibid. cylinder for our case. Then we substitute E_c into (6) to calculate the scattered field intensity $S_i = |E_L(\vec{r}_i)|^2$ (i=1..N) for the N points lying along line placed on some fixed distance L from the object (Fig.1). Thus, having calculated values of S_i for a number of dielectric constant values, we compare them with the actually measured S_i^m. To minimize random measurements errors, it is better to compare the standard deviation for the normalized values of S_{ni} and S_{ni}^m taking the relations of S_i and S_i^m to their maximum for each value of dielectric constant. As a result, the problem of determining the ε_n is reduced to finding the minimum among σ_j calculated for the given values of ε_j:

$$\sigma_j = \sqrt{\sum_{i=1}^{N} \left(S_{ni}(\varepsilon_j) - S_{ni}^m(\varepsilon)\right)^2 / N} \qquad (9)$$

where ε_j – fixed value of dielectric constant, and ε - unknown. Comparing these values σ_j we select the optimal value of ε from the condition of the minimum deviation σ_j.

One of the important calculation parameters is the choice of the number of scattering field data N. Generally, the described procedure works for any N, but it is obvious that for a sufficiently small N the final result will lead to an inadequate value of ε. Let us estimate the approximate value for N. Let us choose the length $L=10\ \lambda\ (>5rc)$ and the step between the points $\Delta x=0.1\ \lambda$. Since $L=N\Delta x$, it means N = 100. It is easy to see that as the radius increases, the minimum acceptable number of points increases, which, accordingly, leads to an increase in the computation time. We used in all calculations $L=16\lambda$, $\Delta x=0.01\lambda$.

In order to examine the outlined procedure, we calculated S_{ni} for different values of dielectric constant ε and for two values (λ and 2λ) of cylinder radius (Figure 2). The correspondence of the line colors to the dielectric constants is the same for both cases and is indicated in the inset of Figure 2b. We can see a significant difference in the curves for different dielectric constants. That is, comparison to the real measured curve will lead to the optimal choice due to the use of the principle of minimum deviation, according to (9). The smaller the radius, the greater the change in the intensity of the scattered wave, which manifests itself in a greater number of inflections in Figure 2a.

At that, for a smaller radius, a more unstable grouping of curve bendings is seen near to the scattering axis. At the same time, for a larger radius, one can even see a certain order for the change in the curves: an increase in their half-width as ε increases for $\varepsilon<2$, and reverse change for $\varepsilon>2$. This may be due to the displacement of the focus inside the cylinder as $\varepsilon>2$ [21].

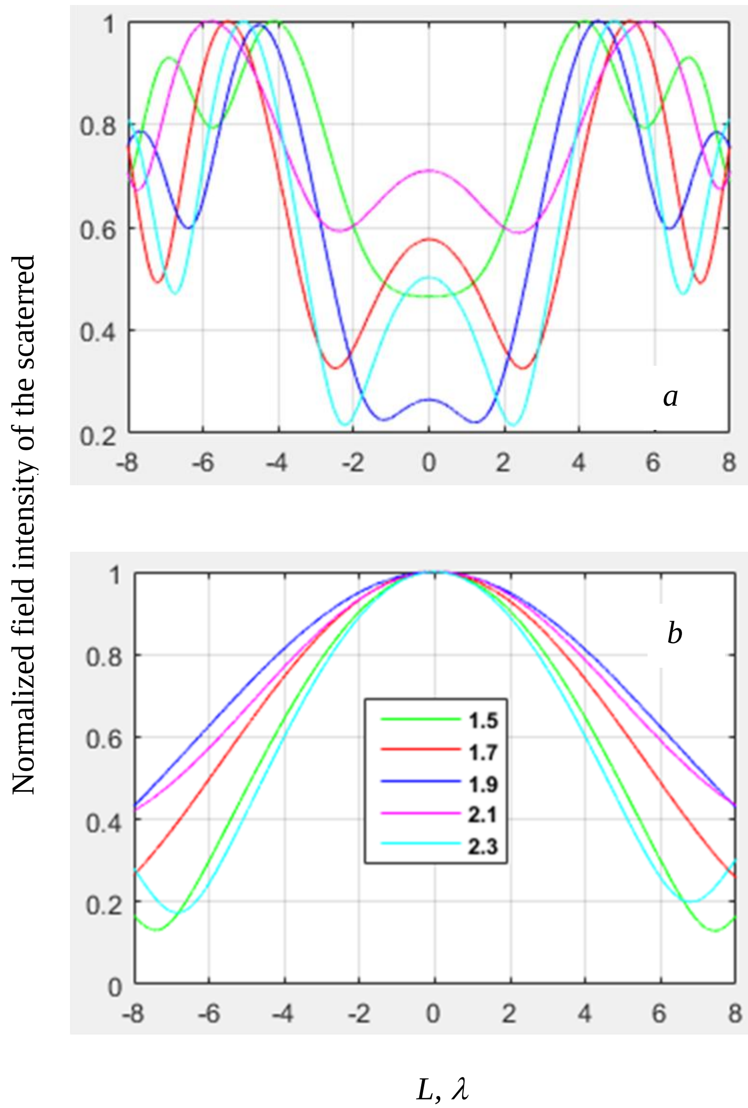

Figure 2 Normalized field intensity along line *L* (see Fig.1) for dielectric cylinder of radius λ (a), and 2λ (b). Inset shows the values of dielectric permittivity for different color lines

4. CONCLUSION

The forward scattering problem is to compute scattered field for given size and material parameters of the scatterer. Respectively, the inverse scattering problem is to recover input data from the scattering field data. One of or a set of the following elements can be considered as the input data: material parameters, size, initial data of the field. In the present paper the possibility to recover permittivity from knowledge of the scattering field data have been considered. By replacing the original equation for the scattered field with an approximate one, it is possible to express the sought scattered field in terms of the incident one. Then, having data for both fields, it is possible to quickly and efficiently determine the permittivity. The procedure has been demonstrated for dielectric cylinder. Potentially the proposed approach is suitable to determine size of the object as well.

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