A NEW NUMERICAL APPROACH TO ELECTROMAGNETIC EIGENVALUE PROBLEM AND WAVE SCATTERING BY CONDUCTING COMPLEX-SHAPED GEOMETRIES: GAUSSIAN BASIS AND REGULARIZED HANKEL FUNCTIONS

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Abstract

In the present study, a new methodology for solving an eigenvalue problem and the two-dimensional E-polarized electromagnetic wave diffraction by the arbitrary shaped perfect electric conducting (PEC) scatterers is proposed. The approach is based on the Gaussian basis function and the Regularized Hankel's function. The study provides the theoretical background of the newly proposed approach in detail. By expanding the current density on the surface with the summation of Gaussian functions and approximating the Hankel function with regularization leads to having a simpler, compact, and novel approach to investigate the behavior of the electromagnetic field in the vicinity of the obstacles. Also, the numerical results including the comparison with the other methods are provided. The outcomes reveal that the proposed method can be employed for such a class of diffraction problems to solve the problem, numerically.

1. INTRODUCTION AND PROBLEM FORMULATION

The numerical and semi-numerical methods in Electromagnetic diffraction problems are rapidly evolving branches after high-performance computers become cheap and easily accessible. There exist many methods which give approximate solutions to the scattering problems with acceptable and manageable accuracy like the Method of Moments, Finite-difference time-domain method, Finite element method, the Method of Auxiliary Sources (MAS), and Orthogonal Polynomials, etc. [1-9]. The Method of Moments gives the possibility to reduce the scattering problem to an integral equation or coupled set of an integral equation which finally are expressed as the matrix equation. The method of Moments has a singularity problem when the boundary conditions are required on the surface. To avoid the singularity, Method of Moments uses the regularization technic or tries to find an analytical solution for the self-terms in the matrix equation [1-4]. On the other hand, the method of auxiliary sources for example considering the analytical nature of the field at the boundary and by the analytical continuation of the scattered field, the sources are shifted inside or outside of the corresponding surface [5-7]. Then, the singularity problem is resolved. However, the method of auxiliary sources has the problem when the geometry of the scatterer contains the edge or corners, in that case, the scattered field is not analytical on the surface and auxiliary sources cannot be shifted inside [10-12]. That is why in previous works, the corresponding authors made small changes in the original geometries and have smooth surfaces to obtain results with high accuracy. The change at the corners leads to having a deviation between the solution of the geometry with a smooth surface and the original one. The same procedure was followed for finding eigenvalues and eigenfields of the corresponding geometries. In the present study, these two issues are overcome by introducing the Regularized Hankel function [11,12].

The solution of a fairly wide class of physical problems is reduced to the solution of singular integral equations. As a rule, the kernel of the integral equation has a singularity since at zero value of the argument the value of the kernel becomes infinite. This is purely mathematical infinity, and special mathematical methods have been created to solve this problem (regularization method, etc.). But in physics, as you know, there are no quantities with infinite values. And for all other values of the argument, the value of the kernel has a clear physical meaning. The question arises whether it is possible to initially create such a function, which will have a completely adequate physical meaning, including zero values as the argument. In the presented article, such a problem is posed relative to Green's function for a two-dimensional problem.

In this study, the goal is to introduce a new methodology that will give the possibility to solve the two-dimensional diffraction, eigenvalue, and eigenfield problem for arbitrary shape scatterers. The main idea in this approach is that the Hankel function (corresponding to Green’s function in two-dimensional space), which represents the electric field created by the current density due to the Dirac function is replaced by the regularized Hankel function, which corresponds to the Gaussian current density. The regularized function at the far-field is the same as the Hankel function but at a small distance is a different function that does not have a singularity in the center. This gives the ability to put the sources directly on the surface and require the boundary condition in the same points. As a result, we avoid the singularity problem by regularizing the corresponding function. Therefore, the diffraction problem by arbitrary shape scatterers both smooth and the ones with corners or edges can be solved. Previously, such problems with corners cannot be solved with MAS. Mathematically, it has been shown that a function can be expressed approximately in terms of the linear combinations of translations of Gaussian function [13,14]. This fact triggers us to develop a new and approximate solution for two-dimensional E- polarized electromagnetic diffraction problems. Here, the Method of Moments is employed with expanding the current density on the scatterer surface as a summation of the Gaussian function. As a result, an integral equation is obtained to be solved for matrix coefficients evaluation. Here, to avoid repetitive calculation of the same integral, the corresponding integral is calculated only once, then it is directly employed during the computation by using the regularization. Here, the regularized Hankel's function is proposed. The function is represented by analytical expressions with similar behavior for different arguments. This gives the ability to solve the problem faster.

In the following chapter, the formulation of the problem is presented. The theoretical background would be given in detail. Then, the numerical results obtained by the proposed method are provided. Later, the comparison with the other methods is presented. In the end, the conclusion is drawn.

2. FORMULATION OF THE PROBLEM

In this section, a mathematical background is provided in detail. Due to proposing a new approach, the mathematical derivations would be given from the starting point. Here, we consider the two-dimensional electromagnetic diffraction problem by the PEC object with an arbitrary shape. The investigation covers open, closed, and semi-closed surfaces such as strip, cylinder, and semi-closed circular strip, respectively. All objects are located in a vacuum. As an excitation source, we can use a plane wave or the cylinder source (both in E-polarized case) with time dependency $e^{-i\omega t}$ where $i=\sqrt{-1}$ is the angular frequency and t is time. The electromagnetic field radiated by the sources excites current on the scatterer's surface [1,3]. In the present study, all objects have an infinite length in z-direction. Therefore, the electric field has only one component perpendicular to the XoY plane and is oriented along Z-axis [2,3]. To find the scattered electric field (E_sc), the induced current on the scatterer is convolved with the corresponding Green's function given in (3). Then, the scattered field is found by (1) [9]:

$$E_{sc}(x,y) = i\omega A = \frac{\omega\mu}{4}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} J(x',y')H_0^{(1)}\left(k\sqrt{(x-x')^2+(y-y')^2}\right)dx'dy' \ , \qquad (1)$$

where $H_0^{(1)} -$ is Hankel's function of zero-order and the first kind and corresponds to the Green's function of the equation, (x',y') stands for the source point, $k=2\pi/\lambda$ is wavenumber and μ stands for the magnetic permeability. To solve (1), there are many methods are developed [1-4]. Here, the Method of Moments (MoM) approach is employed [1,2]. To solve the integral equation given in (1), the current density is expressed as a summation of the basis functions $f_i(x',y')$ with corresponding constant weights given as a_i. Here, N is the number of source points [13,14].

$$J(x',y') = \sum_{i=1}^{N} a_i\, f_i(x_i,y_i,x',y') \qquad (2)$$

where, $f_i(x_i,y_i,x',y') = \sqrt{2}(\alpha k)^2 e^{-(\alpha d k)^2}$ and $d=\sqrt{(x'-x_i)^2+(y'-y_i)^2}$.

Here, d is the distance between the source point (x',y') and the basis function's center location (x_i,y_i). Note that, all Gaussian functions have the same variance and only translation of them is used in the study. The novelty of the study is to employ the Gaussian function as a basis while expressing the current density induced on the scatterer as given in (6) [13,14]. In fact, (x_i,y_i) are N discrete points on the scatterer's surface and α should be calibrated while comparing the results with strict analytical solutions or the numerical solutions with high accuracy. Keep in mind that, with such an approach, Gaussian function decays fast and is negligible beyond the square area $|x'-x_i| > \lambda, |y'-y_i| > \lambda$. Therefore, we can assign a zero value to the basis function outside this square area.

$$f_i(x',y') = \begin{cases} \sqrt{2}(\alpha k)^2 e^{-(\alpha dk)^2}, |x'-x_i| \le \lambda, |y'-y_i| \le \lambda \\ 0, \quad |x'-x_i| > \lambda, |y'-y_i| > \lambda \end{cases} \tag{3}$$

where α is the free parameter to be optimized and λ is the free-space wavelength of the incident wave. The details would be given in the numerical part of the present study. Then, if we put the expression (2) into (1), the induced current and corresponding electric fields will be the smooth functions on the surface as :

$$E_{sc}(x,y) = \frac{\omega\mu}{4}\sqrt{2}(\alpha k)^2 \sum_{i=1}^{N} a_i \int_{y_i-0.5\lambda}^{y_i+0.5\lambda} \int_{x_i-0.5\lambda}^{x_i+0.5\lambda} e^{-(\alpha k\sqrt{(x'-x_i)^2+(y'-y_i)^2})^2} H_0^{(1)}(k\sqrt{(x-x')^2+(y-y')^2})dx'dy' \tag{4}$$

To apply the boundary condition, the total field ($E(x,y)$) should be obtained mathematically. The corresponding field is the sum of the scattered field and the incident field as $E(x,y) = E_{inc}(x,y) + E_{sc}(x,y)$). On the surface of the scatterer, the boundary of the tangential component of the total electric field should be zero. Because the electric field has only one component and this component is tangential to the scatterer. Therefore, the boundary condition becomes $E(x,y)|_\tau = [E_{inc}(x,y) + E_{sc}(x,y)]|_\tau = 0$. The corresponding equation is provided in (5).

$$E_{inc}(x,y)|_\tau = -E_{sc}(x,y)|_\tau \tag{5}$$

where τ is the tangential vector on the surface which is directed in the z-direction due to having an E-polarized incident wave. Then, the integral equation is obtained after (5) is satisfied:

$$E_{inc}(x,y) = -\frac{\omega\mu}{4}\sqrt{2}(\alpha k)^2 \sum_{i=1}^{N} a_i \int_{y_i-0.5\lambda}^{y_i-0.5\lambda} \int_{x_i-0.5\lambda}^{x_i-0.5\lambda} e^{-\left(\alpha k\sqrt{(x'-x_i)^2+(y'-y_i)^2}\right)^2} H_0^{(1)}\left(k\sqrt{(x-x')^2+(y-y')^2}\right)dx'dy' \tag{6}$$

Here, with the point matching technics including N points ($j = 1,2, \dots N$) on the surface of the scatterer, N equations are obtained by boundary condition as given in (7).

$$E_{inc}(x_j, y_j) = -\frac{\omega\mu}{4}\sqrt{2}(\alpha k)^2 \sum_{i=1}^{N} a_i \int_{y_i-0.5\lambda}^{y_i+0.5\lambda} \int_{x_i-0.5\lambda}^{x_i+0.5\lambda} e^{-(\alpha k\sqrt{(x'-x_i)^2+(y'-y_i)^2})^2} H_0^{(1)}(k\sqrt{(x_j-x')^2+(y_j-y')^2}) dx'dy' \tag{7}$$

We can express the corresponding equations as a matrix equation below:

$$Z_{ij} * a_i = B_j \tag{8}$$

where

$$B_j = E_{inc}(x_j, y_j),$$

$$Z_{ij} = -\frac{\omega\mu}{4}\sqrt{2}(\alpha k)^2 \int_{y_i-0.5\lambda}^{y_i+0.5\lambda} \int_{x_i-0.5\lambda}^{x_i+0.5\lambda} e^{-\left(\alpha k\sqrt{(x'-x_i)^2+(y'-y_i)^2}\right)^2} H_0^{(1)}\left(k\sqrt{\left(x_j-x'\right)^2+\left(y_j-y'\right)^2}\right) dx'dy'.$$

Here, (x_i, y_i) are the points where the source is located and double integral is taken around this point.

After finding unknowns by inversion, the current density can be obtained by (2). Similarly, the scattered Electric field can be found with (4). It is clear that for each (x_i, y_i) in the double integral above, the value of the integral would be the same since $e^{-(\alpha k\sqrt{(x'-x_i)^2+(y'-y_i)^2})^2}$ is constant because the integration is taken around (x_i, y_i). Therefore, we can simplify the integral by taking the integration range around the center of the reference frame and the integral denote as regularized Hankel function as (9).

$$RH(x, y) = \int_{-0.5\lambda}^{0.5\lambda} \int_{-0.5\lambda}^{0.5\lambda} e^{-\left(\alpha k\sqrt{(x')^2+(y')^2}\right)^2} H_0^{(1)}(k\sqrt{(x-x')^2+(y-y')^2}) dx'dy' \tag{9}$$

Here, RH stands for the abbreviation of the regularized Hankel's Function. While ordinary Hankel's function gives the electric field of the current density represented by Dirac's delta function, the regularized one represents the electric field of the current density represented by the Gaussian function. It should be noted that Dirac's function is the limit case of the Gaussian function. Finally, the scattered field can be expressed as:

$$E_{sc}(x, y) = \frac{\omega}{4}\sum_{i=1}^{N} a_i RH(k\rho) \tag{10}$$

where $\rho = \sqrt{(x-x')^2+(y-y')^2}$.

To simplify more, (11) is provided.

$$RH(k\rho) = \int_{-0.5\lambda}^{0.5\lambda} \int_{-0.5\lambda}^{0.5\lambda} e^{-\left(\alpha\sqrt{(kx')^2+(ky')^2}\right)^2} H_0^{(1)}(k\rho) d(kx')\, d(ky') \tag{11}$$

To have the faster numerical realization of this method it is better to avoid the calculation of (11), repetitively. This integral has a singular kernel but the value of the integral should be finite because it describes the electric field created with smooth Gaussian current. We can evaluate the integral (11) numerically by using the regularization in the kernel. If we plot this integral for different argument $(k\rho)$, we will get a function which at a big distance behaves like a Hankel's function, and the smaller distance it is a smooth function with no singularity in the center (Fig. 1). That's why we call it the regularized Hankel's function. Because we know the shape of the regularized Hankel's function we can approximate it with some analytical functions. Here, (12) stands for the approximation of (18) with less than 1% error:

$$\widetilde{RH}(k\rho) = \begin{cases} J_0(k\rho) + im\left(A\dfrac{H_0^{(1)}(k\rho)}{\log(0.015k\rho)} - 0.23i\right)i, if\ k\rho \in (0,2), A = 4.74e^{i\pi\frac{180}{190}} \\ J_0(k\rho) - 1.74i, \qquad k\rho = 0 \\ H_0^{(1)}(k\rho), k\rho \in (2,\infty) \end{cases} \tag{12}$$

where J_0 is the Bessel function with zeroth order.

For $k\rho = 0$, the function has uncertainty, type $\frac{\infty}{\infty}$. It can be resolved. Figure 1(a) shows the plot on which we have both function RH (red) and $\widetilde{RH}$ (blue) for different $k\rho$. Figure 1(b) shows the amplitude of the imaginary part of Hankel's function (red) and the imaginary part of the $\widetilde{RH}$ function (Blue). As we see at the point $k\rho = 2$. $\widetilde{RH}$ the function goes smoothly to the ordinary Hankel's Function. RH does not have a singularity in the point $k\rho = 0$.

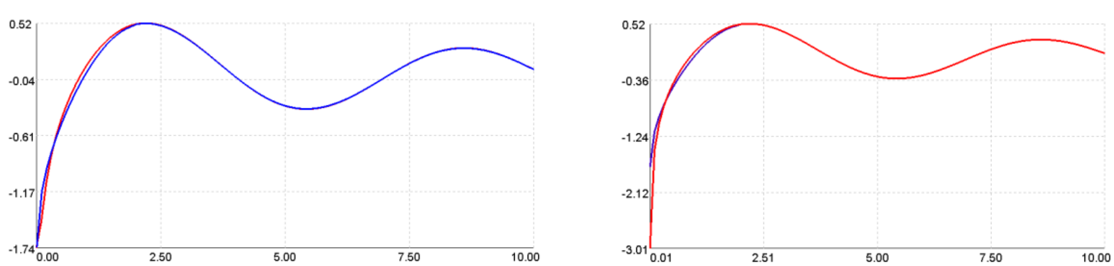


Figure 1. The absolute value of Hankel function and $\widetilde{RH}$ for different $k\rho$

Because the regularized Hankel's function does not have a singularity in the center while requiring the boundary condition on the scatterer's surface, we don't need the regularization technic for the self-terms (when we require boundary condition in the point where the source is located). Therefore, we can put the sources directly on the scatterer's surface. Also, this method works fine with the surface which has sharp corners and edges which is not possible for MAS.

3. RESULTS OF NUMERICAL EXPERIMENTS

In this part, numerical outcomes such as total radar cross-sections, near-field distribution are provided for different scattering and eigenvalue problem geometries. The program package is created and for different geometries, the diffraction and the eigenvalue problems can be obtained with the corresponding program. The α- parameter given in (2) is chosen to be 10 because this value gives the best match between analytical or another numerical approach for different shapes of the scatterer and different frequencies. The closed cavity resonators such as square, H-shaped and asteroid geometries have non-zero Eigen fields at the resonant frequencies [15-17]. To obtain resonance characteristics, an integration over a circular small contour inside the cavities is taken as given in (13). The idea of finding the eigenvalues by solving the scattering problem is proposed in [16,17].

$$R(k) = \int_0^{2\pi} |E_{sc}|^2 d\phi \qquad (13)$$

First, let's consider the square. It should be noted that, for square cavities, an analytical solution exists [15]. Fig. 2(a) shows the geometry of the square with the dimension 1X1. Fig. 2(b) shows the frequency characteristics of the corresponding geometry regarding the wavenumber. Here, the excitation is done by a line source located at (x_inc,y_inc)=(100,0). The sharp resonances corresponding to the eigenvalues of the square are observed.

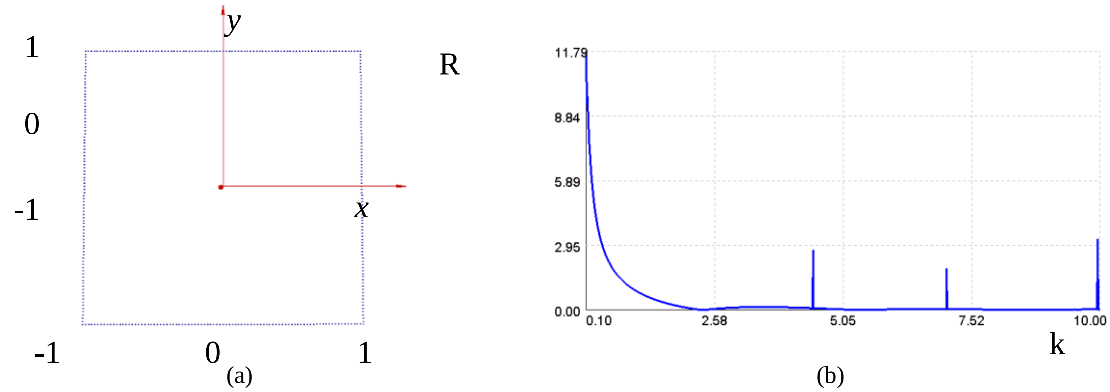


Figure 2. The geometry of the problem and frequency characteristics of the geometry (b).

At Table 1, the comparison between the analytical and proposed methods for the eigenvalue of the square scatterer is provided. As it is seen, the deviation is less than 0.5% for the first three resonances.

Table 1. Resonance values for the square geometry

Resonances	The Analytical Outcomes for Eigenvalues	Outcomes for Eigenvalues by the proposed approach
First	4.4733	4.4407
Second	7.0234	7.048
Third	9.95	9.9336

Fig. 3(a) shows the near-total field distribution on the first resonance k=4.47 of the square. An electromagnetic wave goes around the square and casts a shadow behind. Inside the square, we see the eigenvalues field. Fig. 3(b) shows the field distribution at the no resonant frequency k=3 where the field inside is practically zero.

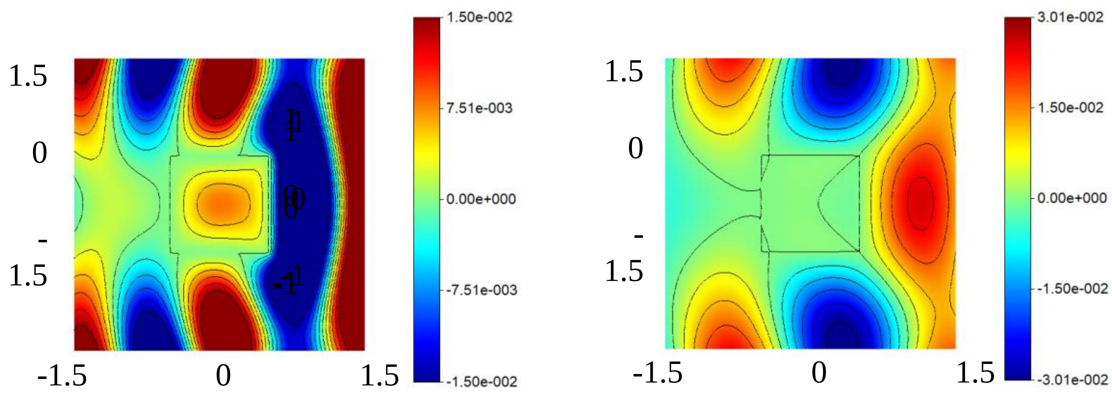
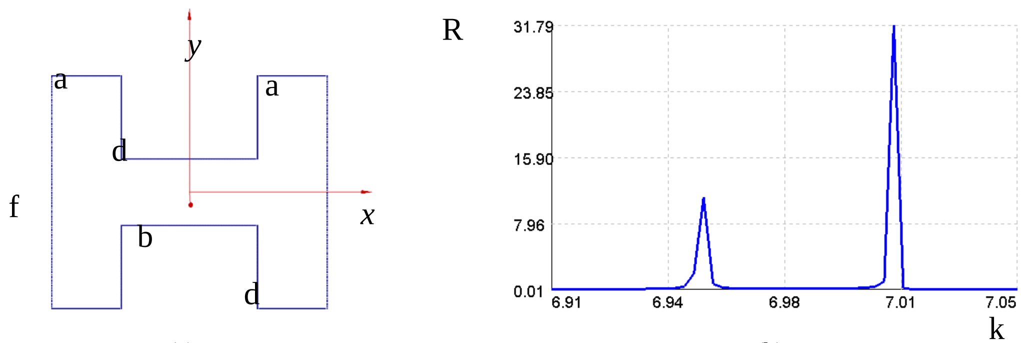


Figure 3. Total Electric field distributions for square geometry ($k = 4.47$ for (a) and $k = 3$ for (b)).

Let us consider now the H-shaped waveguide, find its resonances, and draw corresponding Eigen fields. Fig. 4(a) shows the geometry of the object. Fig. 4(b) shows the TRCS in the vicinity of the first two resonances.

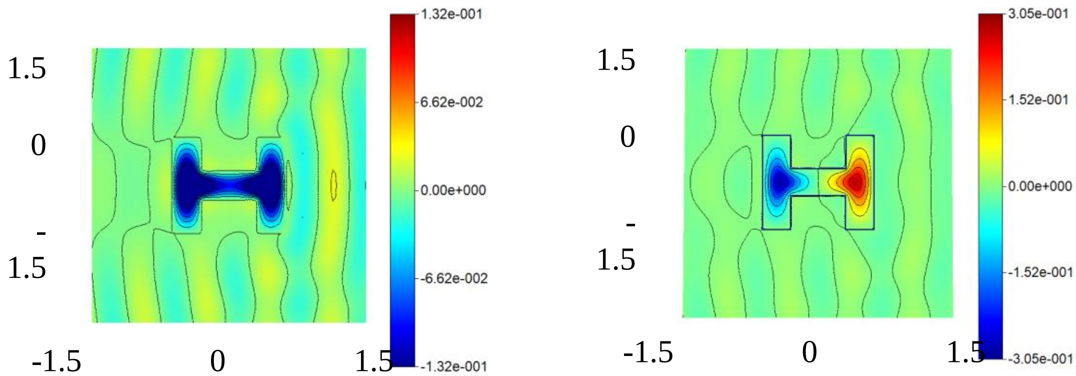


(a)　　(b)

$(a = 0.4, b = 0.8, d = 0.5, f = 1.4)$

Figure 4. The geometry of the scatterer (a) and the frequency characteristics of the geometry (b).

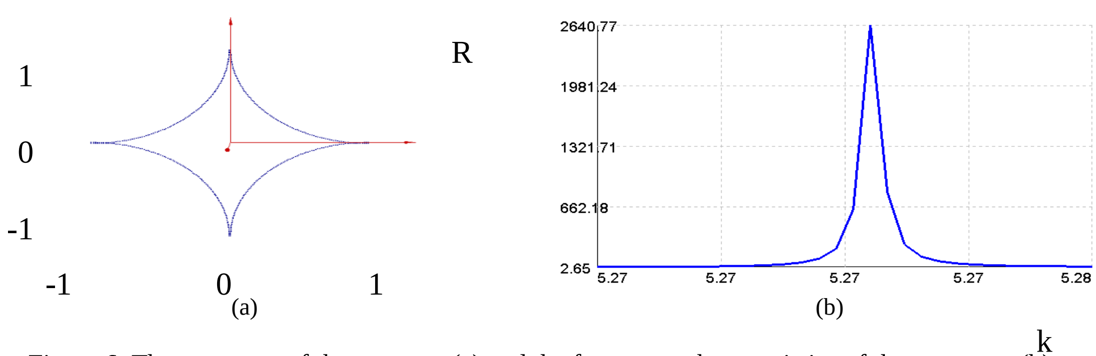
Fig. 5(a) shows the resonant field distribution at k=6.956 and Fig 5(b) shows the resonant field distribution at k=7.013.



(a)　　(b)

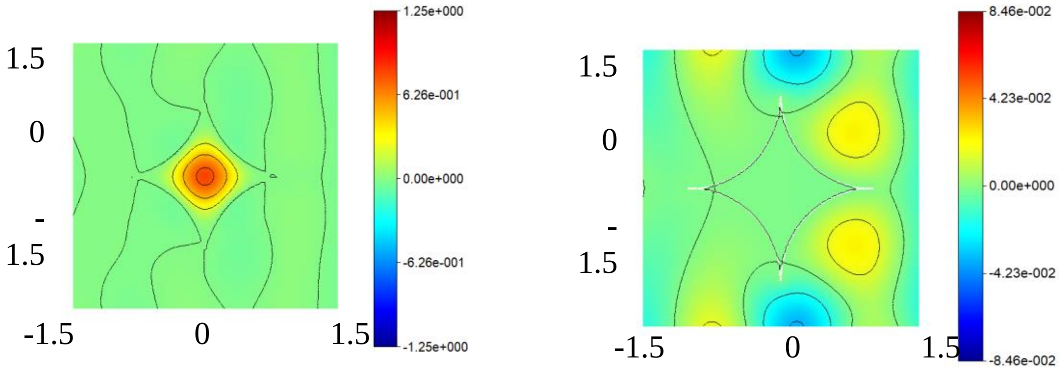
Figure 5. Total Electric field distributions for H-shaped geometry ($k = 6.956$ for (a) and $k = 7.013$ for (b)).

Fig. 6(a) shows the geometry of the next object which we call ‘Asteroid’. Fig. 6(b) shows the TRCS in the vicinity of the first resonance.



(a)　　(b)

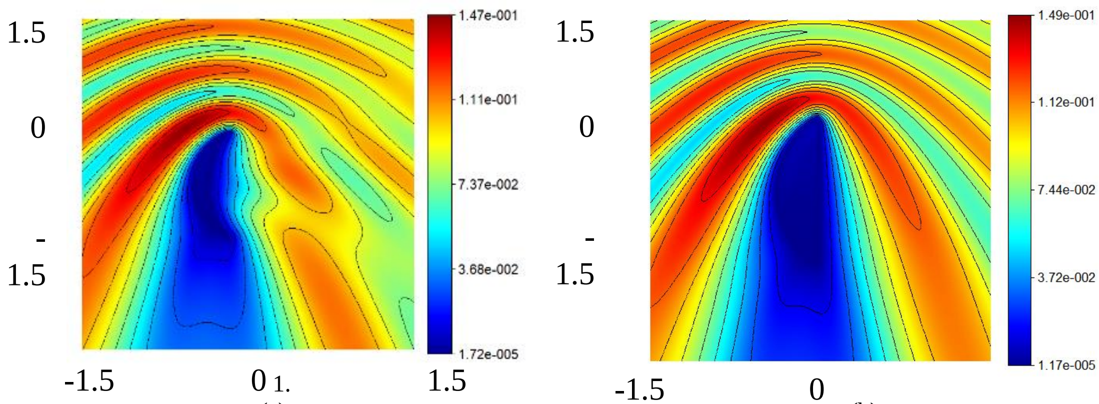
Figure 6. The geometry of the scatterer (a) and the frequency characteristics of the geometry (b).



(a) (b)

Figure 7. Total Electric field distributions for asteroid geometry ($k = 5.273$ for (a) and $k = 4$ for (b)).

Fig. 8 shows the total near field distribution at nonresonant frequencies for moon shape, flat-concave lens shape, wedge shape, strip, double side concave lens shape, and the ellipse shape scatterers. Here, r_i, θ_i^{ap}, d_i stand for the radius of the circular geometries, the aperture angle, and length of the strip on the plane, respectively ($i = 1,2$). For the incident line, the source location is denoted with (x_{inc}, y_{inc}). For the wedge geometry given in Fig 8(c), θ corresponds to the angle between two strips.

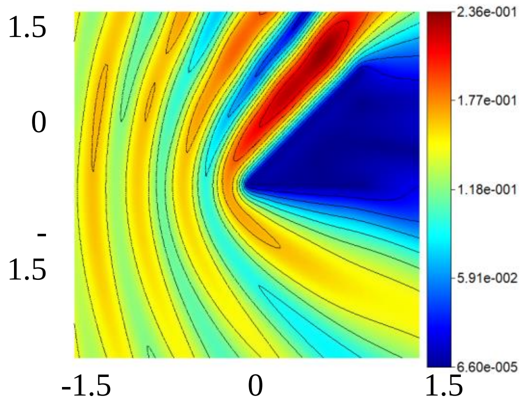


(a) (b)

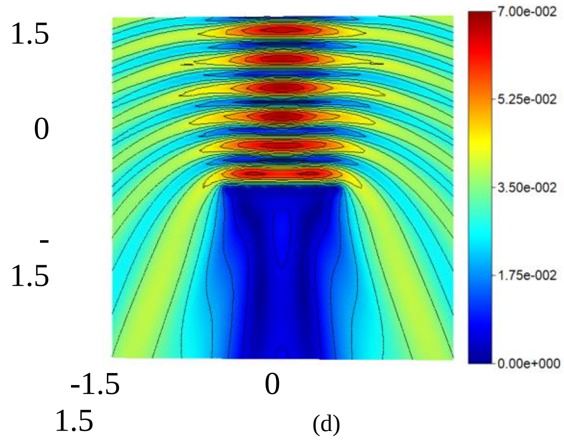
1.5

$k = 4, r_1 = 1, r_2 = 1.5,$
$\theta_1^{ap} = \frac{3\pi}{2.5}, \theta_2^{ap} = \frac{8\pi}{5}, x_{inc} = 0, y_{inc} = 20$

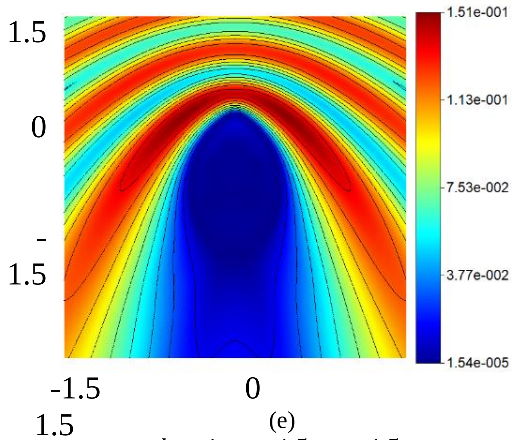
$k = 4, r = 1.5, d = 2.4,$
$\theta_1^{ap} = \frac{4\pi}{3}, x_{inc} = 0, y_{inc} = 20$



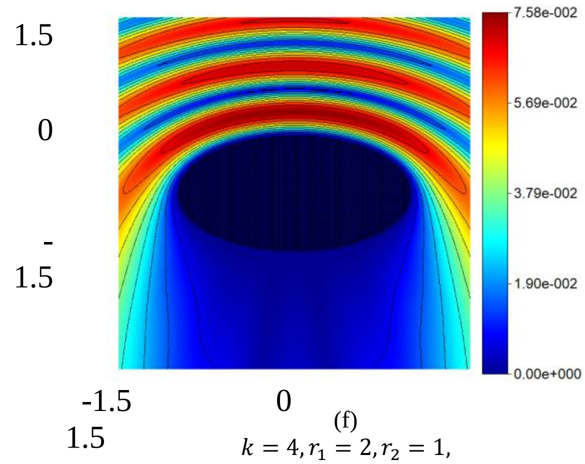
(c)
$k = 4, d_1 = 2, d_2 = 2\sqrt{2},$
$\theta = 45^{\circ} x_{inc} = -10, y_{inc} = 0$



(d)
d- is the size of the strip
$k = 4, d = 2, x_{inc} = 0, y_{inc} = 100$



(e)
$k = 4, r_1 = 1.5, r_2 = 1.5,$
$\theta_1^{ap} = \frac{4\pi}{3}, \theta_2^{ap} = \frac{4\pi}{3} x_{inc} = 0, y_{inc} = 20$



(f)
$k = 4, r_1 = 2, r_2 = 1,$
$\theta_1^{ap} =, \theta_2^{ap} = x_{inc} = 0, y_{inc} = 100$

Figure 8. Total Electric field distributions for (a) moon shape, (b) Plano-convex lens, (c) wedge, (d) strip, (e) convex lens, (f) ellipse.

4. COMPARISON WITH THE OTHER METHODS

Fig. 9 shows the comparison results of the frequency characteristics obtained with the proposed method (red) and the method of orthogonal polynomials (blue) [9]. The comparison is obtained for the diffraction by the PEC circular arc with the radius 1 and the aperture angle of 60 degrees. The source is located in the center. As it is noticed the first resonance coincides regarding the wavenumber and amplitude. The second resonant frequency value is also matched. However, the deviation of amplitude is observed. This can be explained by the fact that as the excitation, in the proposed approach, the regularized Hankel is employed. In contrast, the ordinary Hankel function is used for the other approach.

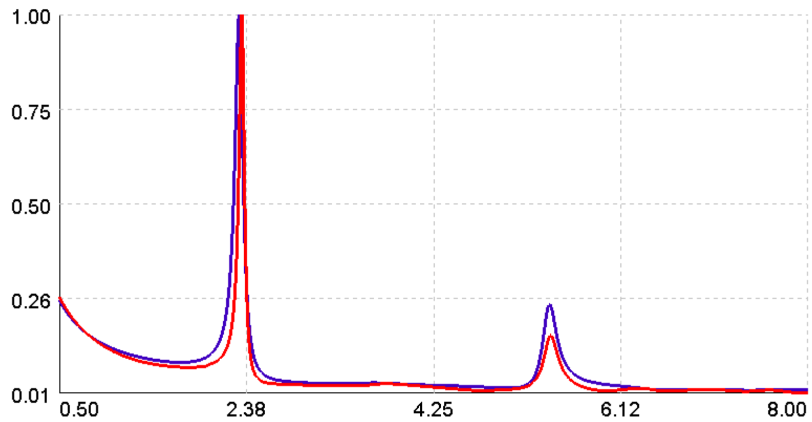
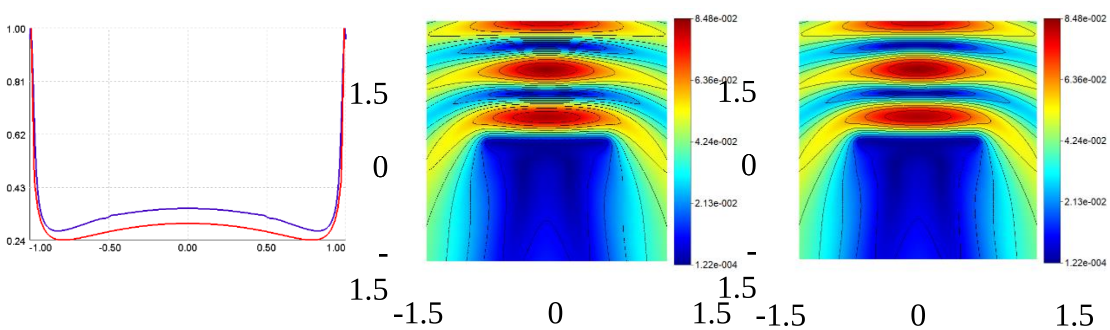


Figure 9. Normalized frequency characteristics of the circular arc by the proposed method (red) and the orthogonal polynomials approach (blue).

Fig. 10 shows both comparisons of the current density and the near field distribution due to the PEC strip with the size 2 at $k = 4$. Here, the source is located at $x_{inc} = 0, y_{inc} = 10$. The red graph is obtained with MoM using pulse basis function and the blue one is obtained with the present method. The deviation in the current density is observed since the regularized Hankel function is employed in the proposed approach. Fig. 10(b) and (c) provide the total near electric field distributions obtained by the proposed and MoM (pulse function, point matching) approaches, respectively.

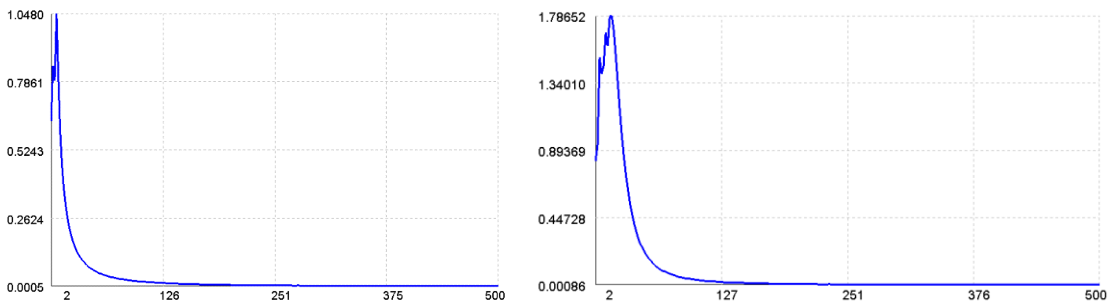


(a) (b) (c)

Figure 10. Comparison of two methodologies (MoM and proposed approach) (a) Absolute Value of the Current Densities and Electric Field Distributions by the proposed approach (b) and MoM (c).

5. CONVERGENCE

We also investigated the convergence of our result for the diffraction by the circular PEC arc with radius 1 and the aperture size of 60 degrees. We estimated the satisfaction of the boundary condition in the point of the arc contour in comparison with the incident field amplitude. Fig. 11 shows the corresponding graph. As we see, by increasing the number of discretization points the graphs approach the zero value. In terms of the incident field amplitude, the boundary condition in the point on the arc is 99.95 % is satisfied when $N = 500$. Also, for the higher frequency, the same procedure is followed and the error in the boundary condition is found 0.09 % for $N = 500$. It should be denoted that, less than 1 percent error can be obtained after $N > 30$.



(a) (b)

Figure 11. The error in boundary condition satisfaction (%) versus the collocation points (N) (a) $k = 4$ (b) $k = 8$.

4. CONCLUSION

In this article, a new methodology for the two-dimensional diffraction problem of the E-polarized electromagnetic wave by the arbitrarily-shaped perfect electric conducting scatterer is presented. Gaussian basis function is introduced for the current density representation and the Regularized Hankel’s function is defined which gives the ability to put the secondary sources directly on the scatterer. The present study has the advantage in comparison with other methods such as MoM and MAS since the boundary condition is satisfied in the same points where the secondary sources are located. As a result, the singularity problem is avoided. Such an approach gives the ability to solve the diffraction problem by an arbitrary shape scatterer consisting of both smooth parts as well as the corners and edges. The comparison with other methods reveals that the gives highly accurate results are obtained by the proposed method. Furthermore, the introduction of the Gaussian function as the basis function gives the ability to extend the proposed approach for two-dimensional H-polarization and also three-dimensional cases.

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