RADIATION BY A NANO–RING

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Abstract

The paper considers the secondary radiation of an electron moving along the circumference of a nano – ring under the action of crossed laser radiation fields. The electron is radially between two concentric potential barriers, and its movement along the ring is considered classical. The time-averaged equation of electron motion, thanks to the Kapitza method, is reduced to solving the pendulum equation. The nonlinear equation solution is found for each regime of electron motion. The main characteristics of the electron motion are determined. The period-averaged value of the total reradiation energy flux is found for the rotation regime. The corresponding (power) radiation patterns are provided. Quantitative estimates for rotational motion have been carried out.

Keywords: Nanotube, nano – ring, electromagnetic waves, laser radiation.

1. EQUATION OF MOTION OF AN ELECTRON

Consider a ring of radius R, which represents a quantum well between two concentric potential barriers, an example of which can be carbon nanotubes [1,2]. Let us assume that the electron's motion along the circumference is classical, and the electron's free path length is much larger than the length of the circumference. Suppose that at the initial moment $t = 0$, the electron has an energy W and moves under the influence of crossed laser radiation fields $\{E_0 \sin \omega t\,, E_0 \sin(\omega t + \alpha)\}$.

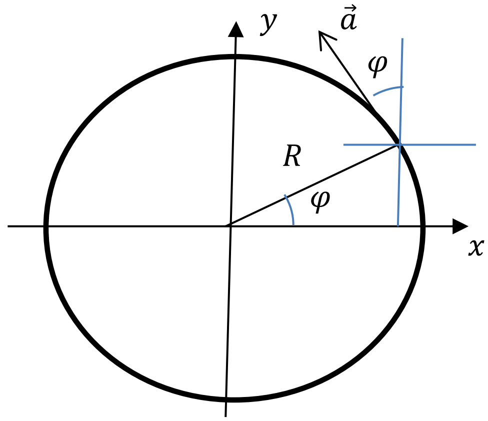


Figure 1. Radiation by a nano–ring.

By projecting the forces of the electric fields acting on the electron onto the tangent of the circle at time t, according to Newton's second law, we obtain the equation of motion for the electron:

$$\ddot{\varphi} = \overline{\Omega}^2 (\sin\varphi \sin \omega t - \cos\varphi \sin(\omega t + \alpha)),$$

$$\overline{\Omega}^2 = \mathrm{eE_0/mR}, \quad (1)$$

where m is the effective mass of the electron, ω is the laser frequency, and α is the phase difference.

We will look for a solution to the equation in the form of a sum of a slow and fast oscillating parts

$$\varphi(t) = \Phi(t) + \xi(t), \quad (2)$$

using the Kapitza method [3,4].

This form of the desired solution is physically justified due to the inertia of the electron, since it should weakly respond to fast external pulsations. From the above equation (1), it follows that

$$\ddot{\Phi}(t) + \ddot{\xi}(t) = \overline{\Omega}^2 (\sin(\Phi(t) + \xi(t)) \sin \omega t - \cos(\Phi(t) + \xi(t)) \sin(\omega t + \alpha). \quad (3)$$

This equation contains pulsating and slowly changing terms. These terms can be separated by expanding the trigonometric functions in a Taylor series up to the first-order term at the point $\varphi(t) = \Phi(t)$.

$$\sin \varphi(t) \cong \sin \Phi(t) + \xi(t) \cos \Phi(t),$$

$$\cos \varphi(t) \cong \cos \Phi(t) - \xi(t) \sin \Phi(t). \quad (4)$$

By integrating over fast time, the function $\Phi(t)$ can be considered constant. As a result, using this expansion, two related equations can be obtained using (4) from (3):

$$\ddot{\xi} = \overline{\Omega}^2(\sin\Phi \sin\omega t - \cos\Phi \sin(\omega t + \alpha)), \tag{5}$$

$$\ddot{\Phi} = \overline{\Omega}^2 \xi \frac{\partial}{\partial\Phi}(\sin\Phi \sin\omega t - \cos\Phi \sin(\omega t + \alpha)). \tag{6}$$

Integrating equation (5), we get

$$\xi(t) = -\frac{\overline{\Omega}^2}{\omega^2}(\sin\Phi \sin\omega t - \cos\Phi \sin(\omega t + \alpha)), \tag{7}$$

assuming integration constants to be zero due to small amplitudes ξ as $\Omega/\omega \ll 1$. Then, the "slow" motion equation (6) can be conveniently written as

$$\ddot{\Phi}(t) = -\frac{\omega^2}{2}\frac{\partial}{\partial\Phi}\xi^2(t). \tag{8}$$

Averaging its right-hand side over time, we finally obtain the equation of "slow" motion of the electron

$$\ddot{\Phi}(t) = -\frac{\omega_0^2}{2}\sin 2\Phi(t), \tag{9}$$

where $\omega_0^2 = -\cos\alpha \, \frac{\overline{\Omega}^4}{2\omega^4}$, corresponds to the linear frequency of electron oscillations at small amplitudes ($0.5 \sin 2\Phi(t) \approx \Phi(t) \ll 1$).

It should be noted that the averaged equation of motion of the Kapitza pendulum [3] has the same form. The theory of the Kapitza pendulum is used, for example, in the theory of free-electron lasers.

2. SOLUTION TO THE PENDULUM EQUATION

Suppose the initial conditions for the equation (9)

$$\Phi(0) = 0, \;\; \Omega_0 \equiv \dot{\Phi}(0) = \frac{1}{R}\sqrt{\frac{2W}{m}}, \tag{10}$$

where Ω_0 is the initial angular velocity of the electron when the initial angle $\Phi(0) = 0$, and W is the initial kinetic energy of the electron.

Multiplying both sides of equation (9) by $\dot{\Phi}$ and integrating, we find

$$\Omega(t) \equiv \dot{\Phi}(\mathrm{t}) = \sqrt{\Omega_0^2 - \omega_0^2 \sin^2\Phi(t)}. \tag{11}$$

Further, let's consider the types of solutions that depend on the value of the parameter

$$\kappa = \frac{\omega_0}{\Omega_0}. \tag{12}$$

2.1 Rotational regime

Suppose $\kappa < 1$, the initial velocity of the electron is greater than the linear oscillation velocity. Integrating equation (11) with respect to time, we obtain the solution of the pendulum in the form of an incomplete elliptic integral of the first kind

$$\Omega_0 t = \int_0^{\Phi} \frac{d\Phi}{\sqrt{1-\kappa^2 \sin^2 \Phi}}, \tag{13}$$

or the equation can be represented in the form of

$$\sin \Phi(\mathrm{t}) = \mathrm{sn}(\Omega_0 t, \kappa), \tag{14}$$

where $\mathrm{sn}(u, \kappa)$ is the Jacobi elliptic sine function.

From (13), the period of electron rotation around the circle can be easily found.

$$T = \frac{4}{\Omega_0} \int_0^{\pi/2} \frac{d\Phi}{\sqrt{1-\kappa^2 \sin^2 \Phi}} = \frac{4}{\Omega_0} K(\kappa), \tag{15}$$

where $K(\kappa)$ is the complete elliptic integral of the first kind, which has an asymptotic expansion.

$$K(\kappa) \cong \frac{\pi}{2}\left(1 + \frac{1}{2^2}\kappa^2 + \left(\frac{1\cdot 3}{2\cdot 4}\right)^2 \kappa^4 + \cdots\right). \tag{16}$$

2.2 Oscillation regime

Let $\kappa > 1$ and the initial velocity be less than the velocity of linear oscillations. In order for the integrand in (15) to be real, we introduce a new integration variable

$$\xi = \arcsin(\kappa \sin \Phi). \tag{17}$$

Substituting (17) into (13), we obtain a similar solution

$$\omega_0 t = \int_0^{\xi} \frac{d\xi}{\sqrt{1-\kappa^{-2} \sin^2 \xi}}. \tag{18}$$

Alternatively, it can be expressed through the elliptic sine

$$\sin \Phi(\mathrm{t}) = \kappa^{-1} \mathrm{sn}(\omega_0 t, \kappa^{-1}). \tag{19}$$

To find the oscillation period, we can use equation (18)

$$T = \frac{4}{\omega_0} K(\kappa^{-1}). \tag{20}$$

2.3 Motion along the separator

Assuming $\kappa = 1$, the electron in this mode has an unstable equilibrium position, which it reaches as $t \to \infty$. Indeed, taking (13) into account and calculating the integral

$$\Omega_0 t = \int_0^{\Phi} \frac{d\Phi}{\sqrt{1-\sin^2 \Phi}} \quad (\omega_0 = \Omega_0), \tag{21}$$

we obtain

$$\Phi(\mathrm{t}) = arcsin\,\mathrm{th}(\omega_0 \mathrm{t}). \tag{22}$$

As expected, the period of oscillation in this mode tends to infinity

$$T = \frac{4}{\omega_0} K(1) \to \infty. \tag{23}$$

3. ELECTROMAGNETIC WAVE RE–RADIATION

We write the dipole moment (relative to the center of the ring) as

$$\vec{p} = -eR\{\cos\Phi,\ \sin\Phi\} = -p\vec{n} \tag{24}$$

and determine the derivatives of the dipole moment:

$$\dot{\vec{p}} = p\,\vec{n} \times \vec{\Omega}, \qquad \ddot{\vec{p}} = p(\Omega^2\vec{n} + \vec{n} \times \dot{\vec{\Omega}}). \tag{25}$$

Using expressions (11) and (9), we preliminarily calculate the square of expression (25).

$$\ddot{p}^2 = p^2(\Omega^4 + \dot{\Omega}^2) = p^2\left((\Omega_0^2 - \omega_0^2\sin^2\Phi)^2 + \frac{1}{4}\omega_0^4\sin^2 2\Phi\right). \tag{26}$$

Then, if we average over angles

$$< \ddot{p}^2 >= \frac{1}{2\pi}\int_0^{2\pi} \ddot{p}^2\, d\Phi = p^2\left(\Omega_0^4 - \Omega_0^2\omega_0^2 + \frac{1}{2}\omega_0^4\right), \tag{27}$$

we can easily find the average value of the total power of radiation from the ring in the far field

$$P = \int \vec{S}\, d\vec{\sigma} = \frac{2p^2}{3c^3}\left(\Omega_0^4 - \Omega_0^2\omega_0^2 + \frac{1}{2}\omega_0^4\right), \tag{28}$$

where $\left|\vec{S}\right| = \frac{\ddot{p}^2\sin^2\vartheta}{4\pi c^3 r^2}$ is the magnitude of the Poynting vector, ϑ is the angle between the normal to the ring and the radius vector $\vec{r}$. Hence, we obtain the angular distribution of the radiation intensity in the solid angle $d\widetilde{\Omega}$ [4]

$$\langle I \rangle = \langle (\vec{S}d\vec{\sigma})/(d\widetilde{\Omega}\,) \rangle = \langle S \rangle r^2 = \langle \ddot{p}^2 \rangle/(4\pi c^3\,)\sin^2\vartheta, \tag{29}$$

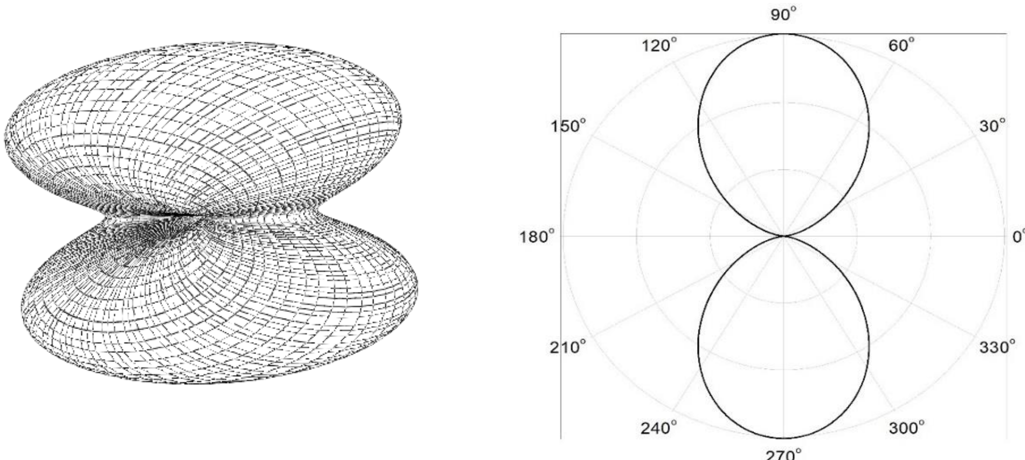
$$I = \ddot{p}^2/(4\pi c^3\,)\sin^2\vartheta. \tag{30}$$

4. NUMERICAL ESTIMATES

Let us choose the value of the laser angular frequency to be $\omega = 10^{13}$ Hz. This corresponds to a linear angular frequency of $\omega_0 = 37$ GHz at small electron oscillation amplitudes. We will further assume that the amplitude of the laser electric field is $E = 2 \cdot 10^3$ V/cm, the radius of the ring is $R = 5 \cdot 10^{-5}$ cm, and the initial energy of the electron is $W = 2 \cdot 10^{-3}$ eV.

For the above parameters, the electron will perform rotational motion ($\kappa = 0{,}7$) with an initial angular velocity of $\Omega_0 = 53$ GHz. The average radiation frequency of the electron is $\omega_{slow} = 45$ GHz, and the rotation period is $T = 1{,}4 \cdot 10^{-10}$ s.

Figure 2 shows the power radiation patterns of the nanoring.



a) b)

Figure 2. Radiation patterns of a nano–ring: a) in a spherical system (24), b) averaged radiation pattern at azimuthal angles Φ in (29).

4. CONCLUSION

In this problem, we have examined the frequency conversion of laser radiation by an electron moving in a circular path. Solutions to the equation of motion of the electron have been obtained using the Kapitza method in various regimes : rotation, oscillation, and motion along the separatrix. The corresponding periods of the electron's motion have been determined for each solution, although the period tends to infinity for motion along the separator.

For the rotational regime, the average value of the total energy flux of secondary radiation has been calculated over the period. The corresponding radiation directivity diagrams have been determined using the Poynting vector. Quantitative estimates have been made for rotational motion.

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