

APPLICATION OF FRACTAL METHODS TO THE ANALYSIS OF BIOCRYSTAL IMAGES

(selected from CEMA'23 Conference)

N. Ampilova^{*}, V. Lyamin^{*} and I. Soloviev^{*}

^{*} St. Petersburg State University, Comp. Sci. Dept
7/9 University emb., 199034 St. Petersburg, Russia

E-mail: n.ampilova@spbu.ru

Abstract

The analysis of digital images having complex structure includes the application both processing methods and mathematical algorithms. As a rule, the processing helps to find the representation of an image which is more suitable for application of mathematical methods. Depending on the type of images the obtaining of an appropriate presentation may be the essential part of the study. In the paper we apply the application of fractal and multifractal methods to compare crystals of medical preparations with a control image which does not contain a preparation (placebo). The images under study were prepared for analysis by a special program and transformed to grayscale palette. For several classes containing 5-6 images the applied methods demonstrated the separation of obtained characteristics, which means the difference in structures of medical preparations and placebo.

1. INTRODUCTION

In many cases digital images may be interpreted as phase portraits of complex processes in different moments of time. The analysis of these representations by mathematical methods gives a possibility to describe the properties of observed processes by using various characteristics (features) of the images.

As the distribution of pixel intensities is the main information about a digital image, specifications of the image are formulated in these terms. Many mathematical methods of analysis use a normed distribution on the cells of a given size, which is a probabilistic measure defined on the image. The choice of a measure may be performed by different ways depending on a problem. The simplest way takes the ratio of sum of pixel intensities in a cell to the common sum of intensities for the image. Such a method

results in an averaging of intensities, and sometimes smooths out the difference between parts of the image. The more preferable way is to take the ratio of the sum of intensities from a given interval (for a cell) to the sum of intensities from this interval for the image.

The experience of many researchers confirms that for images having complex structure the choice of appropriate methods is not just a technique, but an art. We should perform a filtration and improving the image quality, and not destroy its specific structure. This helps to highlight and emphasize certain properties of images and to facilitate their visual perception by an expert. In addition, the application of mathematical methods of analysis implies that the images under study are presented in a standard form convenient for solving the problems of a certain class. This work may be performed with the help of both graphic editors and specially devised programs.

In medical applications this approach furnishes insights into the clarification of the diagnosis. The combining filtration with following morphological analysis demonstrates reliable results in operating with radiograph and ultrasound images [3, 4].

Mathematical methods result in obtaining characteristic which may be used for further image classification. So, Haralick's texture methods describe the image structure in terms brightness ratio for neighboring pixels [5]. Morphological methods reveal specific patterns, for example contours of various forms [6].

Methods of fractal analysis describe the image by using fractal dimensions. As one numerical value cannot characterize the image structure, we should apply some modifications to obtain vector characteristics. In this sense, the "blanket technics" is very productive. It is applied to calculate the Minkovsky dimension for gray level images [2]. An image is represented as a surface in 3-dimension space, where two coordinates indicate the position of a pixel, and the third one is the pixel intensity. There is one-to-one correspondence between the surface and the given image. The surface is given by the gray level function, which is defined for integer values and then extended to real numbers. Such a definition corresponds to an approximation of the surface by unit squares parallel to the image plane.

Then we construct upper and bottom “blankets” for the surface by a definite rule, such that every next iteration is on the distance 1 from the previous one.

Actually, this procedure means a sequential smoothing of the surface. On every step we calculate the volume of the body between upper and bottom blankets and then calculate an approximate value of the surface area and as a result the Minkovsky dimension of the surface. However, the most informative characteristic is not this dimension, but the ratio logarithm of the area on the next step to the logarithm of the step number. For a given set of steps we obtain a vector value (called further by characteristic vector). These vectors are different for images of different classes.

Blanket technics was successfully applied for study of biomedical preparations [1] and SAR images as well [8,9].

Multifractal methods are useful if analyzed images may be interpreted as a union of several fractal subsets. In this case we can obtain the set of fractal dimensions of the subsets (multifractal spectrum) or Renyi spectra measuring changes of the initial distribution of pixel intensities at its sequential renormalizations.

Our experience when analyzing biomedical images allows proposing a combined application of fractal and multifractal methods to obtain reliable results [10].

In this work we developed a technology for differentiation crystals of medical preparations obtained by the thesiography method from the image of control pattern which does not contain the preparation. The preprocessing included editing of images from a microscope by a special program followed by filtration and transformation to halftone representation. Mathematical methods used in this paper are calculation of characteristic vector (blanket technics) and Renyi spectra. Experiments were performed for 3 classes of medical preparation images. All the results demonstrated the separation of calculated features.

The paper has the following structure. The next section contains the description of material and methods to obtain images. In section 3 we briefly describe the preliminary processing of the images from a microscope for the application of mathematical methods. The next part is devoted to the fractal and multifractal methods. The results of experiments are demonstrated in section 5.

2. MATERIAL AND METHODS

2.1. Materials

The images were obtained by the light microscope MIKMED – 6, which consists of eyepiece LOMO WF 10X/22, objectives Plan 4/0,1 Plan 10/0,25 Plan 40/0,65, and video-camera MC – 6,3 (USB-3,0).

The samples were prepared on laboratory slides. Medical preparations of plant, animal and mineral origin in dilution from 1 to 1000 CH by C. Haneman were manufactured in pharmacy POLEVET (oil). Sterile brine solution (NaCl – 0.9%) was used.

2. 2. Method of studying

The research of the crystallization of biosubstrates was conducted by thesiography method. At first the preparation under study was dynamized (by the method of “shaking”). Brine solution (from 1 to 3 drop depending on the substrate quantity) was dripped on a slide. Then 1 drop of the substrate was set in the drop of brine solution. The result of crystallization was fixed after 24 hours. The image of brine solution (placebo) was taken as the control one. The obtained crystals of biosubstrates were compared with the control image.

3. IMAGE EDITING

3.1. Transformation of images to a standard form

To apply methods of image analysis we should convert the images to some appropriate form. For this purpose we cut out the part of the image which contains crystal. The cutting is produced so that there was a “little” space between crystal and the image boundary. One may use graphic editors such as Gravit Designer, Photo Pos Pro, Paint. It is important to do it so the center of the crystal coincide with the center of the image. Then, following the formula below, we perform a rotation of the image so that the sides of the crystal faces become parallel to the sides of the image.

$$x_1 = x_0 + (x - w/2) \cos \varphi - (y - h/2) \sin \varphi$$

$$y_1 = x_0 + (x - w/2) \sin \varphi - (y - h/2) \cos \varphi$$

where (x_1, y_1) — new coordinates, (x, y) — old coordinates, (x_0, y_0) the center of rotation, and φ — the angle of rotation, and (w, h) define the size of the new image. This

transformation is performed by a C# program. After the processing of initial images by the described way we choose the image of minimal size and cut all the others by this size. Then all the images are transformed to grayscale palette. The illustrating example is given below.

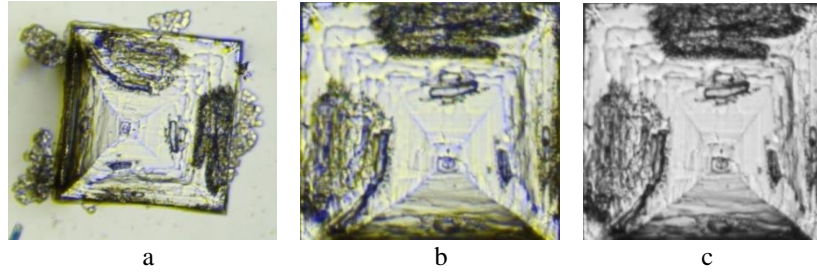


Figure 1. a) a crystal, b) after rotation and cutting, c) grayscale image

3.2. Image preprocessing

We considered global and local equalization, the Laplace operator, linear and nonlinear filtering, unsharp masking.. The experiments show that global and local equalization practically did not give expected results, as well as filtering. So we applied the Laplace operator and unsharp masking. The first method is isotropic, i.e. invariant under rotation, and unsharp masking allows flexible parametric adjustment.

4. MATHEMATICAL METHODS

4.1. Blanket construction

A detailed description of this method can be found in [2, 11], so we will provide here only the information necessary to describe the algorithm of implementation.

Let $G = \{X_{ij}, i = 0, 1, \dots, K, j = 0, 1, \dots, L\}$ be a gray-scale image and X_{ij} be the gray level of the (i, j) -th pixel. This is a gray-level surface for the image, which can be viewed as a fractal for a certain measure range.

Consider $F \subset \mathbb{R}^n$. Then δ -parallel body F_δ is a set of points being at the distance of no more than δ from F , i.e

$$F_\delta = \{x \in \mathbb{R}^n: |x - y| \leq \delta, y \in F\}.$$

We denote by $Vol(F_\delta)$ n -dimensional volume of F_δ . If for some constant s at the limit $Vol(F_\delta)/\delta^{n-D} \rightarrow 0$ is positive and bounded, then the number D is called the Minkovski dimension of the set F .

Blankets u_δ, b_δ for a gray level surface are constructed as follows

$$\begin{aligned} u_\delta(i, j) &= \max \{u_{\delta-1}(i, j) + 1, \max_{|(m,n)-(i,j)| \leq 1} u_{\delta-1}(m, n)\} \\ b_\delta(i, j) &= \min \{b_{\delta-1}(i, j) - 1, \min_{|(m,n)-(i,j)| \leq 1} u_{\delta-1}(m, n)\} \\ u_0(i, j) &= b_0(i, j) = X_{ij} \end{aligned}$$

A point $F(x, y)$ is included in a δ -parallel body if $b_\delta(i, j) < F(x, y) < u_\delta(i, j)$. The definition of a blanket is based on the fact that the blanket for a surface on the step δ includes all the points of the blanket for a surface constructed on the step $\delta - 1$ together with the points that are at the distance of 1 from this blanket.

The volume of a δ -parallel body is calculated by u_δ and b_δ :

$$Vol(F_\delta) = \sum_{i,j} (u_\delta(i, j) - b_\delta(i, j)).$$

The surface area may be calculated using one of two formulas

$$A_\delta = \frac{Vol_\delta}{2\delta}$$

$$A_\delta = \frac{Vol_\delta - Vol_{\delta-1}}{2}.$$

The Minkovsky dimension is defined as $D \approx 2 - \frac{\ln A_\delta}{\ln \delta}$

To obtain the image characteristics, we use a vector $\{(\ln \delta, \ln A_\delta)\}$, the size of which is determined by the number of different values of δ .

4.2. Renyi spectra

The detailed information about multifractal and Renyi spectra may be found in [6]. Consider the set $M \subset R^n$, and its partition into $N(\varepsilon)$ cells with side (or volume) ε . We define the probability measure $p(\varepsilon) = \{p_i(\varepsilon)\}$, $i = 1, \dots, N(\varepsilon)$, $\sum_{i=1}^{N(\varepsilon)} p_i(\varepsilon) = 1$.

Also consider the generalized statistical sum (or the sum of the moments of the measure)

$$S(q, \varepsilon) = \sum_{i=1}^{N(\varepsilon)} p_i^q(\varepsilon)$$

As usual in multifractal technique we assume that for the measure and statistical sum the power law holds:

$$p_i(\varepsilon) \sim \varepsilon^{\alpha_i}, \quad S(q, \varepsilon) \sim \varepsilon^{\tau(q)}$$

where $\tau(q)$ is a function of class C^1 . The symbol " \sim " is understood as follows:

$$\alpha_i = \lim_{\varepsilon \rightarrow 0} \frac{\ln p_i(\varepsilon)}{\ln \varepsilon}, \quad \tau(q) = \lim_{\varepsilon \rightarrow 0} \frac{\ln S(q, \varepsilon)}{\ln \varepsilon}.$$

Under these assumptions, the set of generalized Renyi dimensions is defined as

$$D_q = \lim_{\varepsilon \rightarrow 0} \frac{1}{q-1} \frac{\ln S(q, \varepsilon)}{\ln \varepsilon}$$

These values describe the changing of initial measure $\{p_i(\varepsilon)\}$ for a range of q .

For $q=0$ we obtain capacity dimension, and for $q=1$ by applying the Lopital rule -
- the so called information dimension (or the dimension of a measure support)

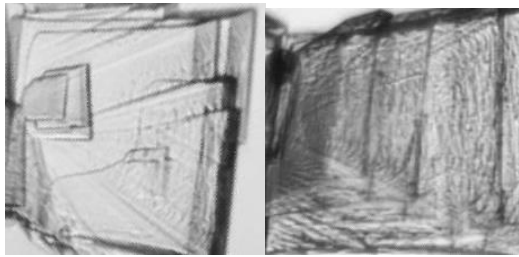
$$D_1 = \lim_{\varepsilon \rightarrow 0} \frac{\sum_i p_i(\varepsilon) \ln p_i(\varepsilon)}{\ln \varepsilon}.$$

5. RESULTS OF EXPERIMENTS

In all the illustrations we use following denotations

Table 1. The correspondence between preparation used and their denotation in graphs

Preparation	Denotation
Placebo (brine solution)	Sample 0
Conium 1000 CH	Sample 1
Acidum phosphoricum 30 CH	Sample 2
Lac defloratum 1000 CH	Sample 3



a

b

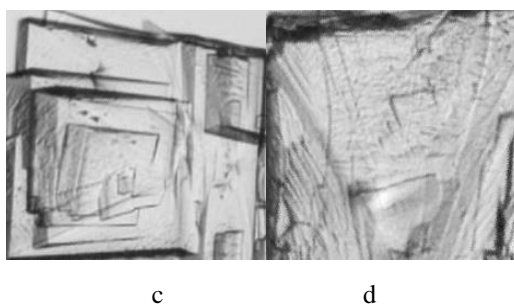


Figure 2. a) Sample 0, b) Sample 1, c) Sample 2, d) Sample 3

The picture shows images of crystals of biosubstrates and placebo. On Fig.3 graphs of characteristic vectors and Renyi spectra for Sample 1 and Sample 0 are given. Graphs are separated. The point of intersection in Renyi spectra ($q=0$) means that all the images have close value of capacity dimension.

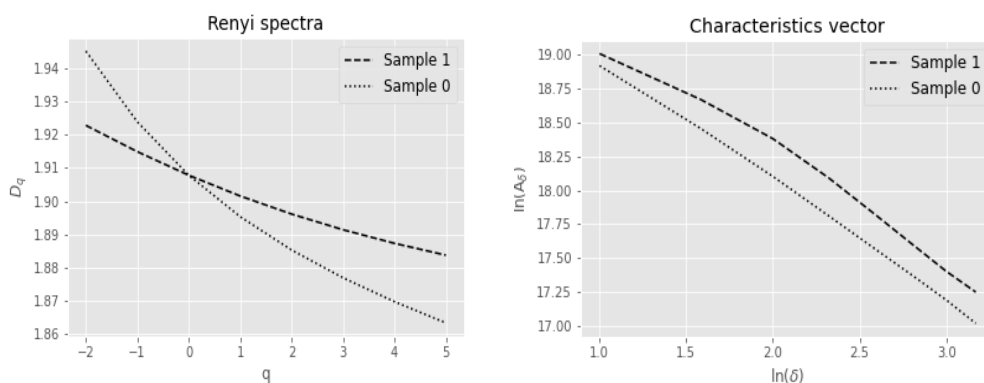


Figure 3 Comparing results for Conium 1000 CH and Placebo a) Renyi spectra and b) characteristic vectors

The following graphs demonstrate the difference between the structure of the crystal of *Acidum* phosphoricum 30 CH and placebo.

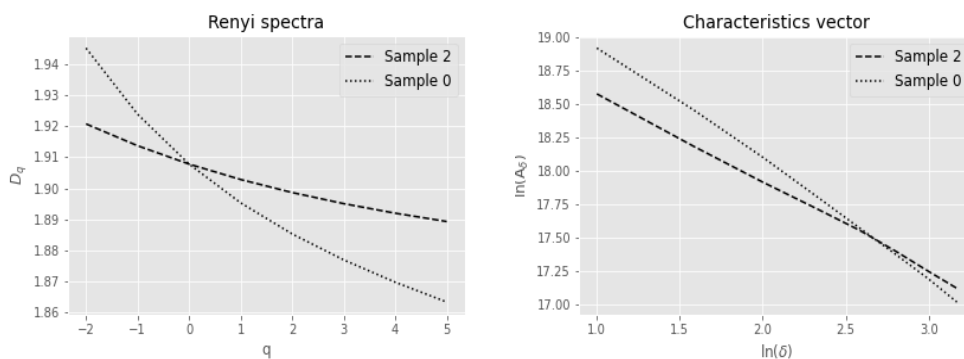


Figure 4. Comparing results for *Acidum* phosphoricum 30 CH and placebo a) Renyi spectra and b) characteristic vectors

In this case, Renyi spectra give a better result than characteristic vectors. Analogical graphs for the crystals of Lac defloratum 1000 CH and placebo are given in Fig.5.

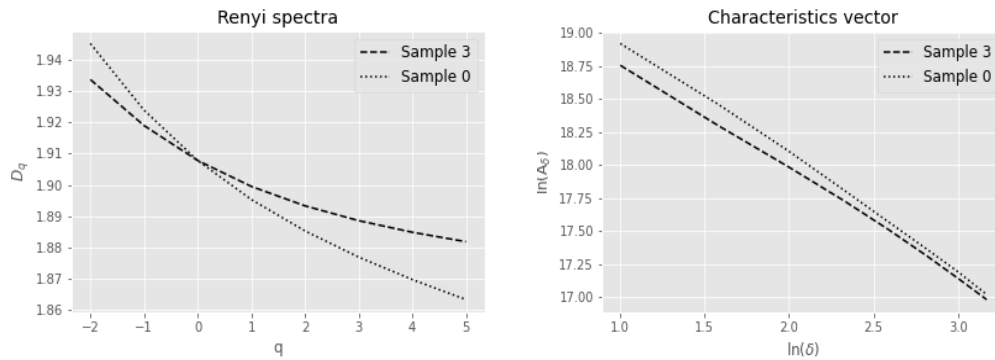


Figure.5. Comparing results for Lac defloratum 1000 CH and placebo

6. CONCLUSION

Our experiments show that the combining of an appropriate image preprocessing and methods of fractal analysis allows obtaining graphs that are separated.

The fact that Renyi spectra have a point of intersection near $q=0$ means that all the images studied have close values of capacity dimension. Hence to obtain classifying signs one need to use vector characteristics.

REFERENCES

- [1] N. Ampilova, I. Soloviev , J.-G. Barth. “Application of fractal analysis methods to images obtained by crystallization modified by an additive”. Journal of Measurements in Engineering, Vol. 7, Issue 2, 2019, p. 48-5
<https://doi.org/10.21595/jme.2019.20436>
- [1] K. J. Falconer, "Fractal geometry", Chichester : Wiley,1990.
- [2] V.Georgieva, S. Vassilev. “Kidney segmentation in ultrasound images via active countours”, Proc. 11 Int. Conf. CEMA16, 13-15 Oct. 2016, Sofia, Bulgaria. p.43-46
- [3] V. Geogieva, P.Petrov, B.Iantovics. “X-ray image processing for tissue involent-based caries detection”, Proc. 14 Int. Conf. CEMA19, 17-19 Oct. 2017, Sofia, Bulgaria. p.22-26.

- [4] R. M. Haralick, K. Shanmugan, I. Dinstein. “Textural Features for Image Classification”, IEEE Transactions on systems, man and cybernetics, volume SMC-3. - IEEE, 1979. - №6. - P. 610-621. - ISSN 00189472.
- [5] R. Gonzalez, R Woods. “Digital image processing”, 2002
- [6] S.P. Kuznetsov, "Dynamical chaos", M, Izd.fiz-mat lit.,2001 (in Russian)
- [7] A. Malamou, C.Pandis, P.Frangos and P. Stefaness. “SAR image terrain classification using the modified fractal signature (MFS) method”, Proc. 9 Int. Conf. CEMA14, 16-18 Oct. 2014, Sofia, Bulgaria. p.8-11.
- [8] G. Pouraimis, A. Kotopoulis, N. Ampilova, I. Soloviev, E. Kalitsis and P. Frangos, “Sea state determination using normalized one-dimensional radar signatures at X-band and fractal techniques”, Proc. 14 Int. Conf. CEMA19, 17-19 Oct. 2019, Sofia, Bulgaria, p. 27-31.
- [9] I. Soloviev, “Application of multifractal methods for the analysis of crystal structures”, JAE 2022, JAE-Journal of Applied Electromagnetism /<http://jae.ece.ntua.gr>), 24(2),19-30.
- [10] Y. Y. Tang, C. Y. Suen, "Modified fractal signature (MFS): a new approach to document analysis for automatic knowledge acquisition", IEEE Transactions on Knowledge and Data Engineering,1997, 9, № 5,p. 747-762.